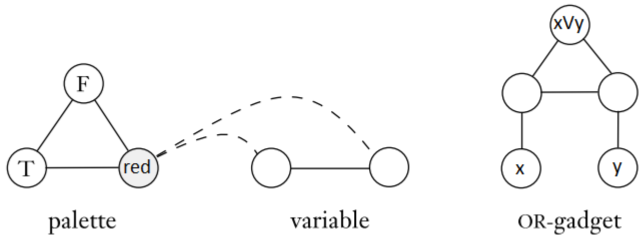
NP-complete, continued

a Let $DOUBLE\text{–}SAT = \{\langle\phi\rangle \mid \phi$ has at least two satisfying assignments$\}$. Show that $DOUBLE\text{–}SAT$ is NP-complete.

b Let $HALF\text{–}CLIQUE = \{\langle G\rangle \mid G$ is an undirected graph having a complete subgraph with at least $m/2$ nodes, where m is the number of nodes in $G\}$. Show that $HALF\text{–}CLIQUE$ is NP-complete. *(Hint: It is usually easiest to reduce from an NP-complete language that is as similar as possible to this one, rather than starting from e.g. $3SAT$ every time.)*

c Show that if P = NP, a polynomial time algorithm exists that produces a satisfying assignment when given a satisfiable Boolean formula. *(Note: The algorithm you are asked to provide computes a function; but NP contains languages, not functions. The P = NP assumption implies that SAT is in P, so testing satisfiability is solvable in polynomial time. But the assumption doesn't say how this test is done, and the test may not reveal satisfying assignments. You must show that you can find them anyway. Hint: given ϕ, use the satisfiability tester repeatedly on slight variations of ϕ.)*

d A *coloring* of a (undirected) graph is an assignment of colors to its nodes so that no two adjacent nodes are assigned the same color. Let $3COLOR = \{\langle G \rangle \mid G \text{ is colorable with 3 colors}\}$. Show that $3COLOR$ is NP-complete. *(Hint: Call our palette of 3 colors "True", "False", and "red", and use the following three subgraphs.)*



e In the following solitaire game, you are given an $m \times n$ board. On each of its mn positions lies either a blue stone, a red stone, or nothing at all. You play by removing stones from the board until each column contains only stones of a single color and each row contains at least one stone. You win if you achieve this objective. Winning may or may not be possible, depending upon the initial configuration. Let $SOLITAIRE = \{\langle G \rangle \mid G \text{ is a winnable game configuration}\}$. Prove that $SOLITAIRE$ is NP-complete. *(Hint: reduce from* $3SAT$*.)*

f (Extreme challenge problem) Define a *string-homomorphism* to be a function $f : \Sigma \to \Gamma^*$ from one alphabet to strings over another alphabet. We can extend f to operate on strings by defining $f(w) = f(w_1)f(w_2)\cdots f(w_n)$, where $w = w_1 w_2 \cdots w_n$ and each $w_i \in \Sigma$. We further extend f to operate on languages by defining $f(A) = \{f(w) \mid w \in A\}$, for any language A. Show that P is closed under every string-homomorphism iff P = NP.