circle(S, B): ($|B| \le 3$)

// find min circle enclosing S with extra constraint that bdry passes thru pts in B

0. if $S = \emptyset$ or $|B| = 3$ return ...
1. pick $p \in S$ randomly
2. C = circle($S - \{p\}$, B)
3. if p outside C then
4. C = circle($S - \{p\}$, $B \cup \{p\}$) (*)
5. return C

$|B| \le 2$

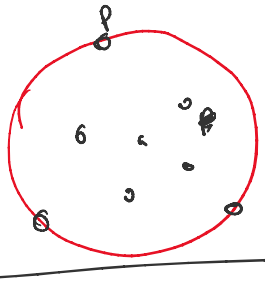
Rand. Analysis:

let $T_b(n)$ = expected runtime for $|S| = n$, $|B| = b$

for any fixed S, B,

$Pr[(*) \text{ is done}] = Pr[p \text{ is on boundary of circle}(S,B)]$

$\le \frac{3}{n}$



$$T_b(n) \le T_b(n-1) + O(1) + \frac{3}{n} \cdot T_{b+1}(n-1)$$

base case: $T_b(0) = O(1)$

$T_3(n) = O(1)$

$\Rightarrow T_2(n) \le T_2(n-1) + O(1)$

$\Rightarrow T_2(n) = O(n)$

$O(1) + 3 \cdot T(n-1)$

$\Rightarrow T_2(n) = O(n)$

$\Rightarrow T_1(n) \le T_1(n-1) + O(1) + \frac{3}{n} \cdot O(n)$

$\Rightarrow T_1(n) = O(n)$

$\Rightarrow T_0(n) \le T_0(n-1) + O(1) + \frac{3}{n} \cdot O(n)$

$\Rightarrow T_0(n) = \boxed{O(n)}$ expected time

Rmks: in d dims, $O((d+1)!\, n)$ expected time

(Megiddo/Dyer: $O(2^{2^d} n)$)

⋮

Problem Given n numbers,
find median ($(n/2)^{th}$ smallest)
(or more generally, k^{th} smallest) (selection)

Known:

$O(n)$ time by Blum, Floyd, ... '73 ⇐

$\sim 16n$ comparisons

$\sim 5.4n$ "

Schönhage et al. '76: $\sim 3n$

Dor, Zwick '95: $\sim 2.95n$ ⇐

Lower bds: Bent, John '80 $\sim 2n$

Dor, Zwick '96 $\sim 2.000\,000\,000\,000\,000\,000\,000\,001\,n$

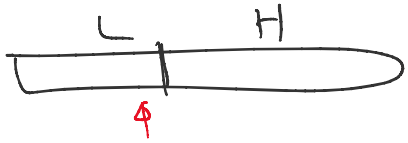
Randomized algms?

Randomized algms?

Method 1: quickselect (variant of quicksort)

select($\{a_1, \ldots, a_n\}$, k): // return k^{th} smallest

1. if $n=1$ return a_1
2. pick "pivot" x from $\{a_1, \ldots, a_n\}$ AT RANDOM
3. $L = \{a_i : a_i \le x\}$, $H = \{a_i : a_i > x\}$
4. if $k \le |L|$ select(L, k)
   else select$(H, k-|L|)$



Rand. Analysis:

$|L|$ is equally likely to be $1, 2, \ldots, n$

$\Rightarrow \Pr\left[\underbrace{\frac{n}{4} < |L| < \frac{3n}{4}}_{\text{good}}\right] \ge \frac{1}{2}$

$\Rightarrow$ expected # iterations before seeing good pivot $= 2$

$$T(n) \le T\left(\tfrac{3}{4}n\right) + 2 \times O(n)$$

$\Rightarrow O(n)$.

(more careful analysis: $\le 3.4n$ comps $\approx 2 + 2\ln 2$)

Method 2: Floyd-Rivest Algm ('75)

1. sampling

Method 2: Floyd-Rivest

idea - random sampling