CS 473, Spring 2023
Homework 0 (optional, not graded)

Homework 0 will be not graded. The purpose of this optional homework is to test your familiarity with background material that you should already know. (Solutions will be posted on Jan 26.)

Problem 0.1:

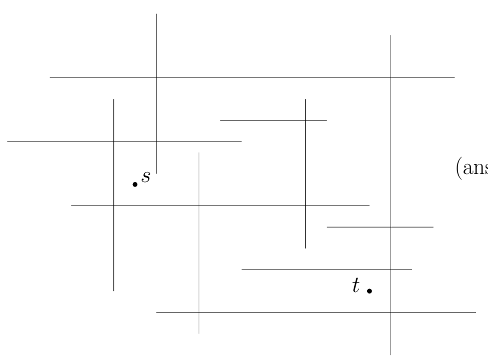
(a) Solve the following recurrence (i.e., give tight Θ bound): $T(n) = 3\,T(n/4) + \sqrt{n}$ if $n \geq 4$, and $T(n) = 1$ if $n < 4$. Use the master theorem.

(b) Consider the following recurrence: $T(n) = 2\,T(\sqrt{n}) + \log n$ if $n \geq 4$, and $T(n) = 3$ if $n < 4$. Use induction to prove that $T(n) = O(\log n \log\log n)$.

Problem 0.2: We are given an $n \times n$ matrix A where all entries are integers from $\{1, 2, \ldots, U\}$, with the property that all rows are monotonically increasing and all columns are monotonically increasing, i.e., $i < i'$ implies $A[i,j] \leq A[i',j]$, and $j < j'$ implies $A[i,j] \leq A[i,j']$. Describe an $O(n \log U)$-time algorithm to find the median element in A (i.e., the $(n^2/2)$-th smallest element).

[Hint: first describe an $O(n)$-time algorithm to count the number of elements less than a given value.]

Problem 0.3: We are given a set of n line segments in 2D, where each line segment is either vertical (with endpoints (x_i, y_i) and (x_i, y_i') for some x_i, y_i, y_i') or or horizontal (with endpoints (x_i, y_i) and (x_i', y_i) for some x_i, x_i', y_i). We are also given two points $s = (x_s, y_s)$ and $t = (x_t, y_t)$. Describe an efficient algorithm to decide whether there is a way to travel from s to t without crossing any of the given line segments. [Hint: consider an $n \times n$ grid, define a graph (with how many vertices and edges?), and apply a standard graph search algorithm.]



(answer is yes)