Midterm 1 next Tuesday HW4 out next Tuesday

No lecture this Friday — optional review session

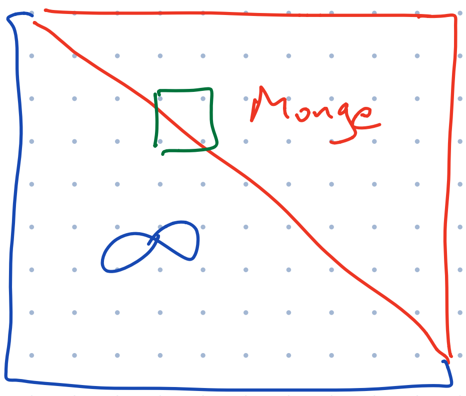
HW0–HW3
not "Advanced"

4 questions 2 hours
One handwritten cheat sheet

Bug in HW 3.1

$\Uparrow$

Bug in last Wed's lecture



$w(i \to j)$ defined when $i < j$

$$w(i\to j) + w(i{+}1 \to j{+}1) \leq w(i \to j{+}1) + w(i{+}1 \to j)$$

$\Downarrow$

Claim: $\square$ is Monge

$\begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix}$ ✓ $\begin{bmatrix} 3 & 7 \\ \infty & 5 \end{bmatrix}$ ✓ $\begin{bmatrix} \infty & 9 \\ \infty & \infty \end{bmatrix}$ ✗ $\begin{bmatrix} \infty & \infty \\ \infty & \infty \end{bmatrix}$ ✓

Randomized Algorithm

worst-case expected time

$$T_A(n) = \max_{|X|=n} E[T_A(X)]$$

Rawlins' nuts and bolts

Randomized "quicksort":

Choose random bolt (uniform, independent)

Partition N's and B's

smaller N, B $\downarrow$ recurse — one match — larger N, B $\downarrow$ recurse

Partition takes $2n-1$ tests.

$$E[T(n)] = 2n-1 + E_k[T(k-1) + T(n-k)]$$

$$\overline{T}(n) = 2n-1 + \frac{1}{n}\sum_{k=1}^{n}\left(\overline{T}(k-1) + \overline{T}(n-k)\right)$$

k is good $\frac{n}{4} \le k \le \frac{3n}{4}$

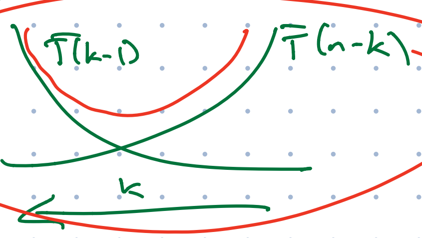
k is bad o/w

$\Pr[k \text{ is good}] = \frac{1}{2}$

overestimate

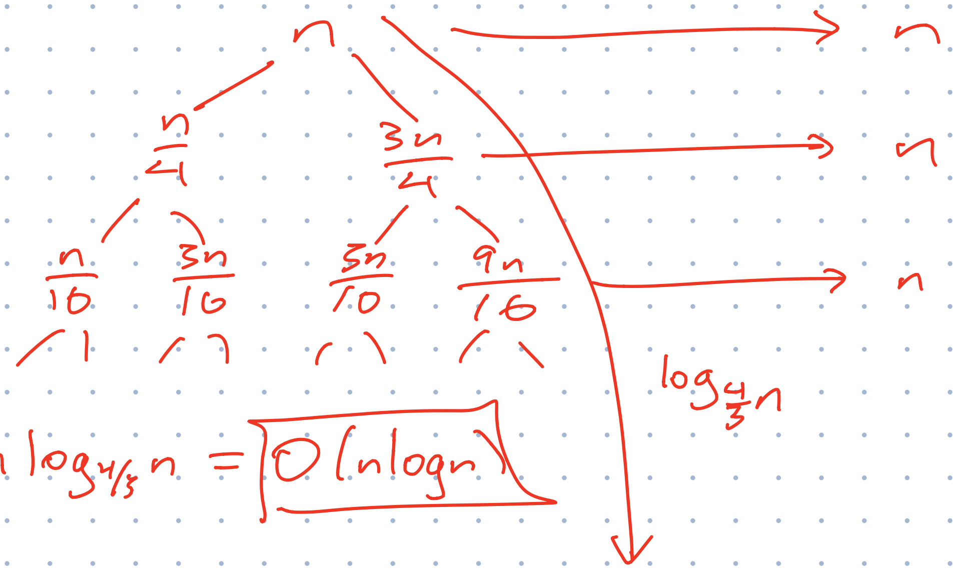
$$\overline{T}(n) = 2n-1 + \frac{1}{n}\sum_{k \text{ good}}\left(\overline{T}(k-1) + \overline{T}(n-k)\right) + \frac{1}{n}\sum_{k \text{ bad}}\left(\overline{T}(k-1) + \overline{T}(n-k)\right)$$

$\overline{T}(n) \le \overline{T}(n+1)$

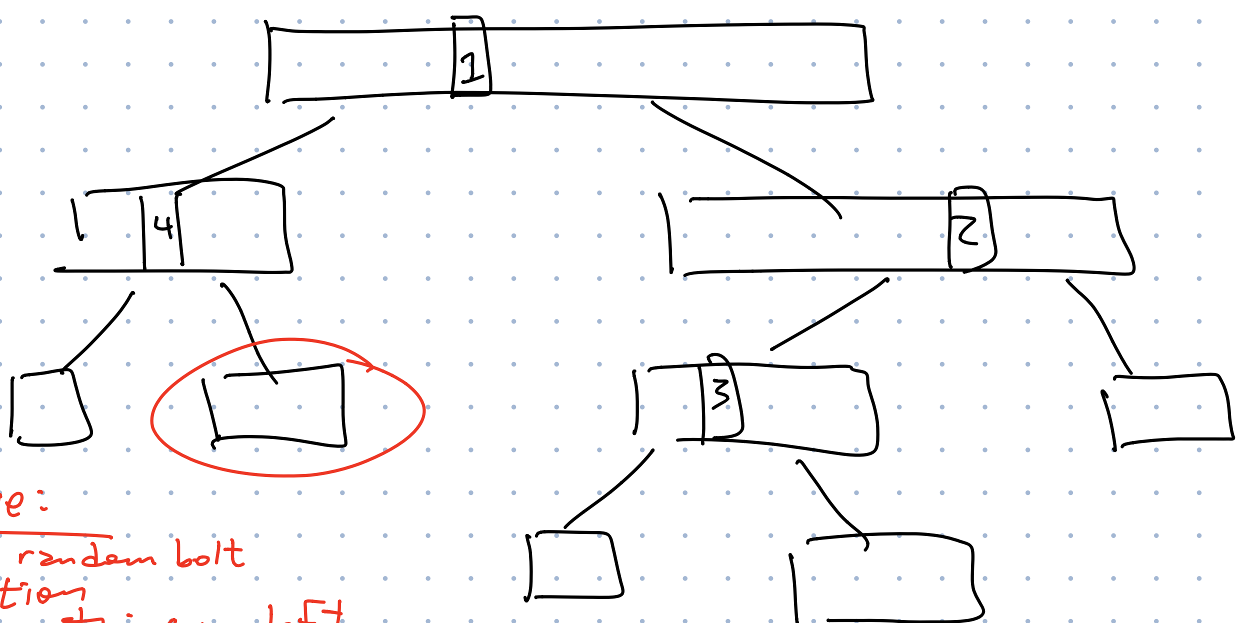


$$T(n) \le 2n - 1 + \begin{cases} \frac{1}{2}\cdot\left(T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right)\right) \\ \frac{1}{2}\,T(n) \leftarrow \end{cases}$$

$$T(n) \le 4n - 2 + T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right)$$



$$T(n) \le 4n \log_{4/3} n = \boxed{O(n \log n)}$$



Recursive:

Choose random bolt
Partition
Do everything on left
then everything on right

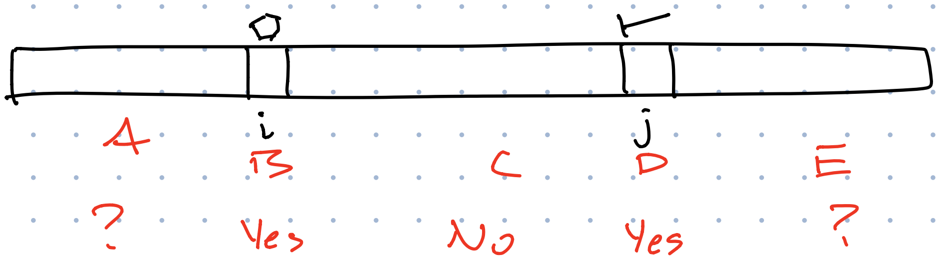
Iterative:

repeat n times:
choose random unmatched bolt
partition its subset

$X_{ij} = 1$ iff we compares $\underbrace{i^{th} \text{ smallest bolt}}_{\text{bolt } i}$ and $\underbrace{j^{th} \text{ smallest nut}}_{\text{nut } j}$

$$E[\text{Time}] = \#\text{tests} = \sum_{i=1}^{n} \sum_{j=1}^{n} E[X_{ij}] = \sum_{i,j} Pr[\text{bolt } i \text{ vs. nut } j]$$

$$E[X_{ij}] = \sum_{x} x \cdot Pr[X_{ij} = x] = 0 \cdot Pr[X_{ij} = 0] + 1 \cdot Pr[X_{ij} = 1]$$



Compare $N_i : B_j$ iff the first bolt chosen in the range $i..j$ is either i or j.

if $i<j$

$$Pr[X_{ij} = 1] = \begin{cases} \dfrac{2}{j-i+1} & \text{if } i<j \\ 1 & \text{if } i=j \\ \dfrac{2}{i-j+1} & \text{if } i>j \end{cases}$$

$$E[\text{time}] = n + 4 \sum_{i=1}^{n} \sum_{j=i+1}^{n} \frac{1}{j-i+1}$$

$$\boxed{4nH_n - 7n + 4H_n}$$