← n →

m ↓

12	21	38	76	27
74	14	14	29	60
21	8	25	10	71
68	45	29	15	76
97	8	12	2	6

12	21	38	76	89
74	14	14	29	60
21	8	20	10	71
68	45	29	15	76
97	8	12	2	6

TM but Not Monge

Monotone

Find min of every row

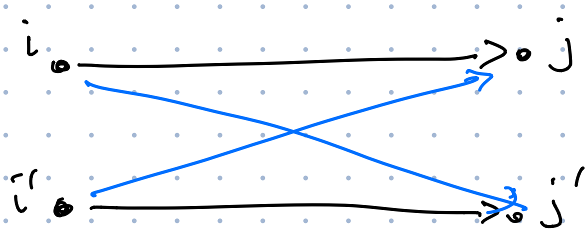
$O(m + n\log m)$

12	21	38	76	27
21	8	25	10	71
97	8	12	2	6

Totally Monotone

Row minima in $O(m+n)$ time

Monge



For any indices $i<i'$ and $j<j'$:

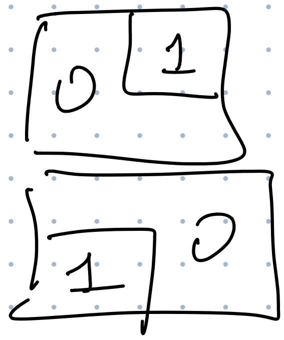
$$M[i,j] + M[i',j'] \le M[i,j'] + M[i',j]$$

suffices to check $i' = i+1$, $j' = j+1$

$O(nm)$

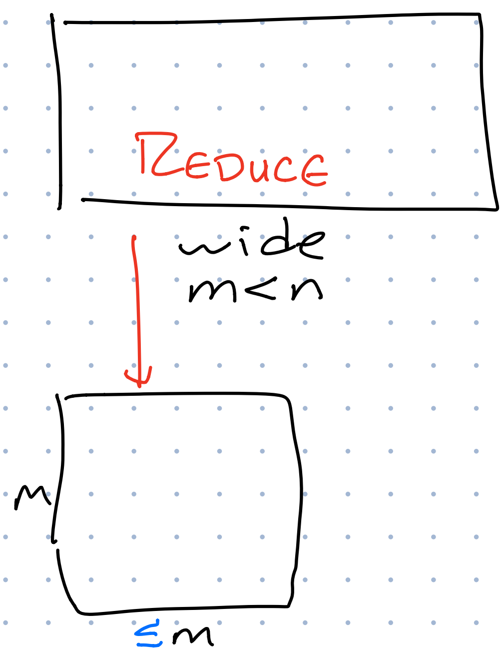
Lemma: Array is Monge iff one of the following

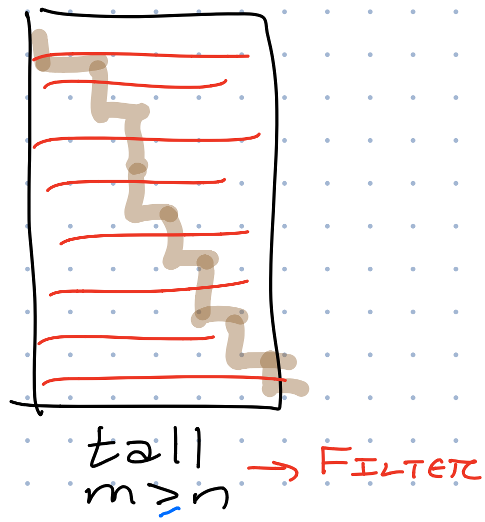
- Every row is const
- Every col is const
- All 0's except UR block of 1s or LL

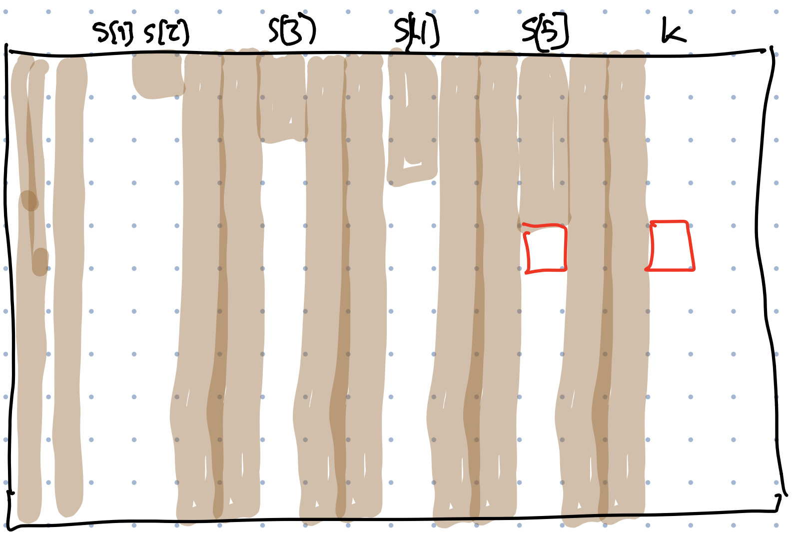


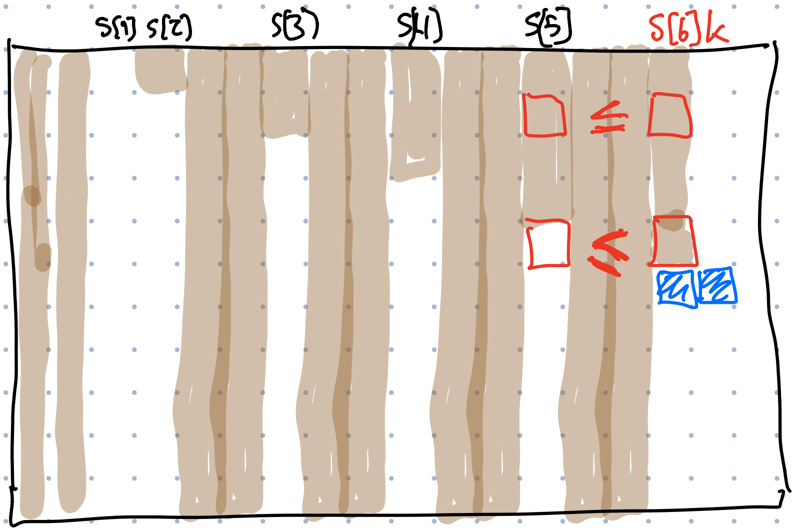
- Monge + Monge
- c · Monge for any c > 0
- Monge^T

Shor
Moran
Aggarwal
Wilbur
Klawe



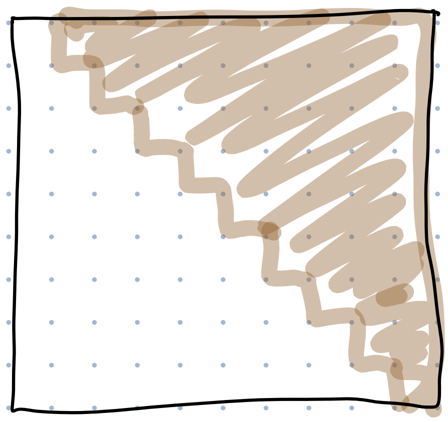


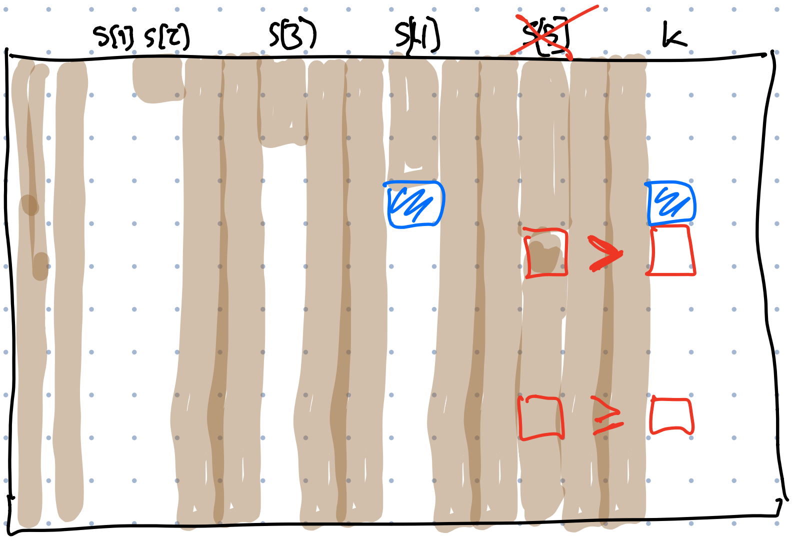




After ≤ 2n iterations

Totes Mono





$$T(m, n) \le \begin{cases} O(n) & m = 1 \\ O(n) + T(m, m) & 1 < m < n \quad \text{REDUCE} \\ O(m) \; \cancel{+ T(\tfrac{m}{2}, n)} \; + T(n, n) & m \ge n \quad \text{FILTER} \end{cases}$$

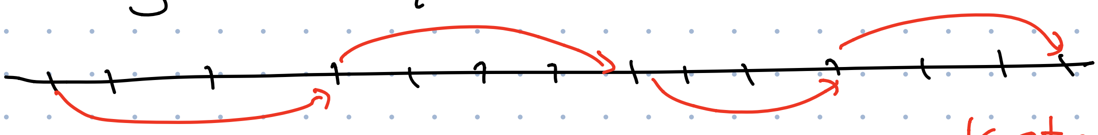
(m: rows, n: cols)

After $O(\log(\frac{m}{n}))$ FILTERS, array is wide

$$O(m) + O(\tfrac{m}{2}) + O(\tfrac{m}{4}) \cdots = O(m)$$

$$O(m + n) \text{ time}$$

Concave Min Weight Subsequence



k steps

$w(i \to j)$ cost of $i \to j$

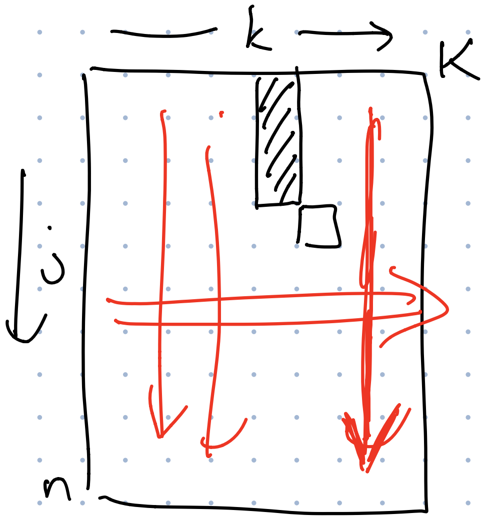
Find k steps minimize $\sum_{j=1}^{k} w(B[j-1] \to B[j])$

Need to find | $B[0..K]$ sorted array of indices
$B[0] = 1$ $B[K] = n$

$OptCost(j, k)$ = min cost of k-link path from 1 to j

(we need $OptCost(1, K)$).

$$OptCost(j,k) = \begin{cases} w(1 \to j) & k = 1 \\ \infty & j < k \\ \min_{i<j} \{ OptCost(i, k-1) + w(i \to j) \} \end{cases}$$

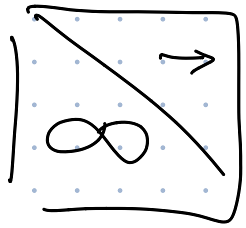


$O(n^2 k)$ time

```
for k = 1 to K
  for j ← 1 to n
    if j < k
      OptCost[j,k] = ∞
    else
      for i = 1 to j-1
        look at OptCost(i,k-1) + w(i→j)
      OptCost[j,k] ← min_i
```

Suppose $w(i \to j)$ is Monge

$$W[i,j] = \begin{cases} w(i \to j) & \text{if } i < j \\ \infty & \text{if } i \geq j \end{cases}$$



$$w(i \to j) = (j-i)^2$$

$$w(i \to j) + w(i+1 \to j+1) \quad \text{vs} \quad w(i \to j+1) + w(i+1 \to j)$$

$$2(i-j)^2 \quad \text{vs} \quad (i-j+1)^2 + (i-j-1)^2$$

$D = i - j \qquad 2D^2 \quad \text{vs} \quad (D+1)^2 + (D-1)^2$

$$< 2D^2 + 2$$

$$w(i \to j) = (S[j] - S[i])^2$$

$M[i,j] = OptCost(i, k-1) + w(i \to j)$ if $w(i \to j)$ is Monge

$$A[i,j] \quad + \quad W[i,j]$$

Monge + Monge = Monge

rows constant

$\Rightarrow O(nk)$ time