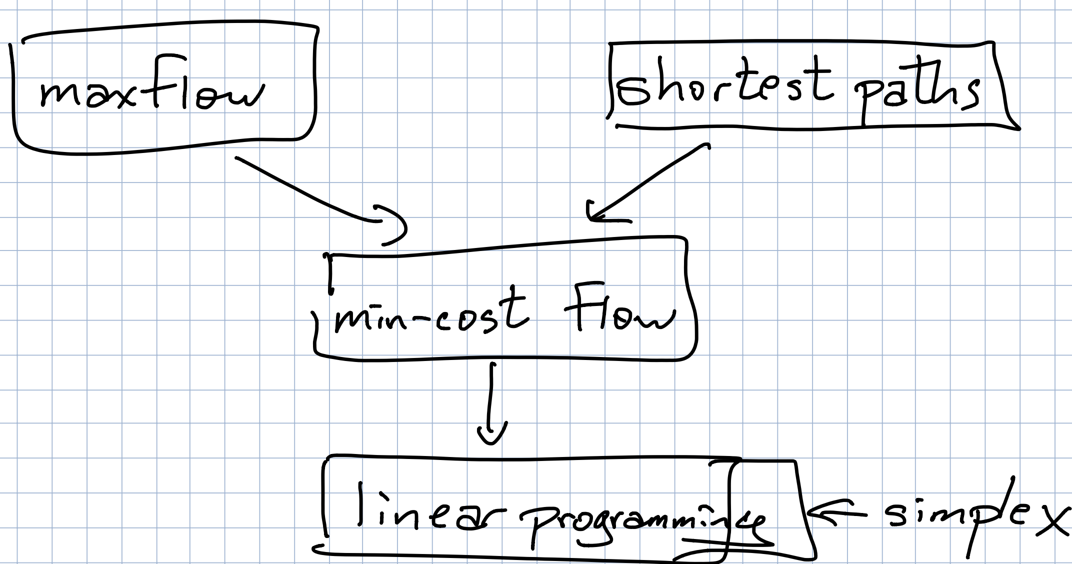
Midterm 2 – 1 week from today

no lecture / opt review session

Regrades back Thu. | Hw7 due today

Conflict exam Wed – register online



Linear Programming

$$\max \sum_{j=1}^{d} c_j x_j$$

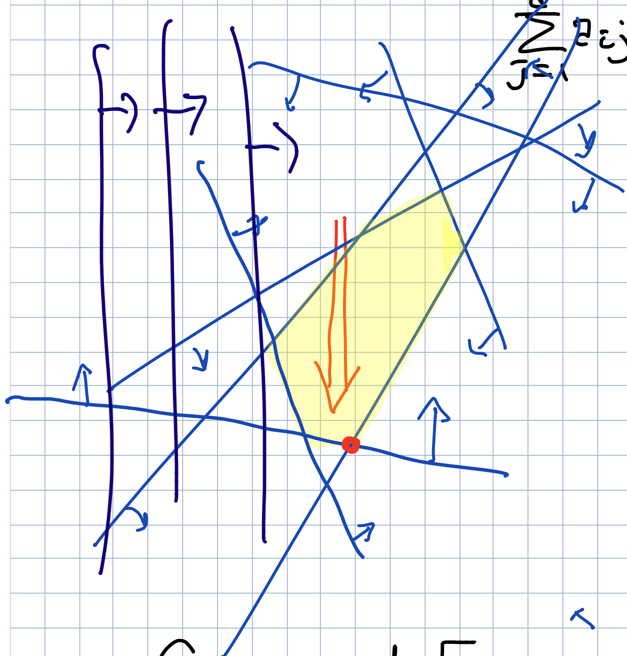
x_j — variable

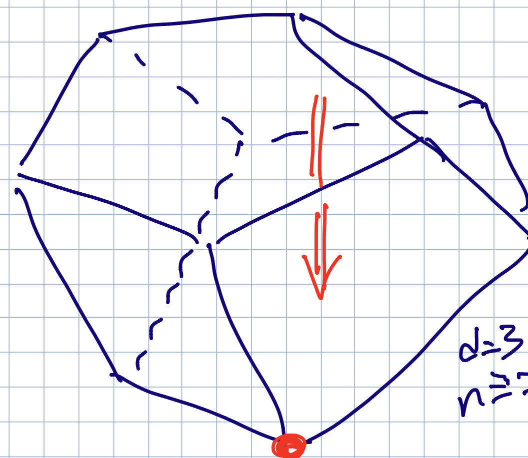
a_{ij}, b_i, c_j — constants

subject to $\sum_{j=1}^{d} a_{ij} x_j \le b_i$ for $i=1..p$

$\sum_{j=1}^{d} a_{ij} x_j \ge b_i$ for $i=p+1..q$

$\sum_{j=1}^{d} a_{ij} x_j = b_i$ for $i=q+1..n$





$d=3$
$n=7$

$x \in \mathbb{R}^d$ $c \in \mathbb{R}^d$

$b \in \mathbb{R}^n$

$A \in \mathbb{R}^{n\times d}$

Canonical Form

$$\boxed{\begin{array}{ll} \max & c\cdot x \\ \text{s.t.} & Ax \le b \\ & x \ge 0 \end{array}}$$

$Ax \le b$ ← $\sum_j a_{ij} x_j \le b_i$ for all i

$x \ge 0$ ← $x_1 \ge 0 \quad x_2 \ge 0 \; \cdots \; x_d \ge 0$

Maximum Flow

maximize $\sum_w f_{s\to w} - \sum_u f_{u\to s}$

subj. to $\sum_w f_{v\to w} - \sum_u f_{u\to v} = 0$ for all $v \neq s,t$

A

$f_{u\to v} \le c_{u\to v}$ for all $u\to v$

$f_{u\to v} \ge 0$ for all $u\to v$

Minimum cut

max ~~minimize~~ $\sum_{u\to v} -c_{u\to v}\cdot x_{u\to v}$

$x_{u\to v} = 1$ means $u\in S$, $v\in T$

$= 0$ otherwise

$x_{u\to v} \ge 0$ for all $u\to v$

~~$x_{u\to v} \le 1$ for all $u\to v$~~

A^T

$x_{u\to v} + S_v - S_u \ge 0$ for all $u\to v$

$S_s = 1$

$S_t = 0$

$S_v = 1$ if $v\in S$

0 if $v\in T$

$0 \xrightarrow{\ge 0} 0$ $\quad 0 \xrightarrow{\ge -1} 1$ $\quad 1 \xrightarrow{\ge 0} 1$ $\quad 1 \xrightarrow{\ge 1} 0$

Π

$$\boxed{\begin{array}{ll} \max & c\cdot x \\ \text{s.t.} & Ax \le b \\ & x \ge 0 \end{array}} \rightleftarrows \boxed{\begin{array}{ll} \min & y\cdot b \\ \text{s.t} & yA \ge c \\ & y \ge 0 \end{array}}$$

$\amalg$

$\parallel$

$$\boxed{\begin{array}{l} \max -b^T \cdot y^T \\ \text{s.t. } -A^T \cdot y^T \le c^T \\ \qquad y^T \ge 0 \end{array}}$$

Fundamental Theorem of LP:

x^* is opt solution for Π
y^* is opt solution for $\amalg$

$\boxed{\text{iff}}$ $c\cdot x^* = y^* A x^* = y^* b$

Weak FTLP:

x is feasible for Π and y is feasible for $\amalg$

then $cx \le \underline{yAx \le yb}$

Proof: $Ax \le b \qquad y \ge 0$

$\Downarrow \qquad \Downarrow$

$yAx \le yb$

$\square$

$$u \longrightarrow v \qquad \mathrm{dist}(v) \le \mathrm{dist}(u) + \ell(u \to v)$$

$$\boxed{\begin{array}{lrcl} \text{maximize} & 4x_1 + x_2 + 3x_3 & & \\ \text{subj. to} & x_1 + 4x_2 & \le & 2 \\ & 3x_1 - x_2 + x_3 & \le & 4 \\ & x_1, x_2, x_3 & \ge & 0 \end{array}}$$

$\sigma^* =$ opt value

$x = (1,0,0) \longrightarrow \sigma^* \ge 4$

$x = (0,0,3) \longrightarrow \sigma^* \ge 9$

$$\boxed{\sigma^* \le \,?}$$

$y_1, y_2 \ge 0$

$$\begin{array}{lrcl} \text{maximize} & 4x_1 + x_2 + 3x_3 & & \\ \text{subj. to} & y_1(x_1 + 4x_2) & \le & 2y_1 \\ & y_2(3x_1 - x_2 + x_3) & \le & 4y_2 \\ & x_1, x_2, x_3 & \ge & 0 \end{array}$$

$$y_1(x_1 + 4x_2) + y_2(3x_1 - x_2 + x_3) \le 2y_1 + 4y_2$$

$$\| $$

$$\underline{(y_1 + 3y_2)}x_1 + \underline{(4y_1 - y_2)}x_2 + \underline{y_2}x_3 \overset{\text{want}}{\ge} \underline{4}x_1 + \underline{\ }x_2 + \underline{3}x_3$$

$$\boxed{\begin{array}{lrcl} \min & 2y_1 + 4y_2 & & \\ \text{s.t.} & y_1 + 3y_2 & \ge & 4 \\ & 4y_1 - y_2 & \ge & 1 \\ & y_2 & \ge & 3 \\ & y_1, y_2 & \ge & 0 \end{array}}$$