Max Flow / min cut

- Augmenting path algo [Ford-Fulkerson]
  - Max Flow = Σ paths
  - Any Flow = $\underbrace{\Sigma \text{ paths} + \Sigma \text{ cycles}}_{\# \leq |E|}$
  - Integer cap $\Longrightarrow$ integer flow
- Which path?
  - Arbitrary — guarantees weak $\boxed{O(E \cdot F)}$
  - Fat pipes — $O(E^2 \log E \log F)$
  - Short pipes — $O(E^2 V)$
  - $\boxed{\text{Orlin} — O(EV)}$

when all $c=1$
$F \leq V-1$

So what? Applications of flows + cuts

vertex
~~Edge~~-disjoint paths problem

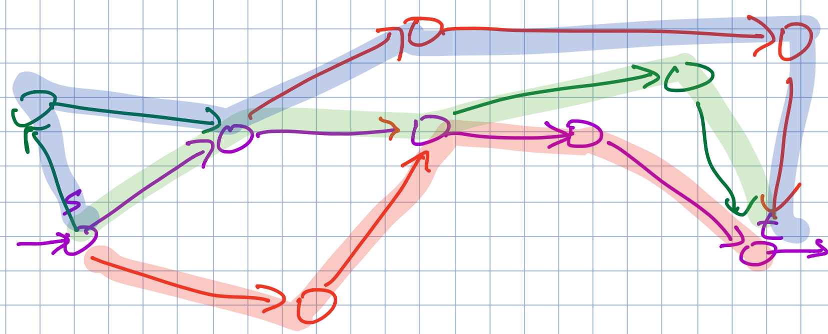
Given directed graph $G=(V,E)$
vertices s, t

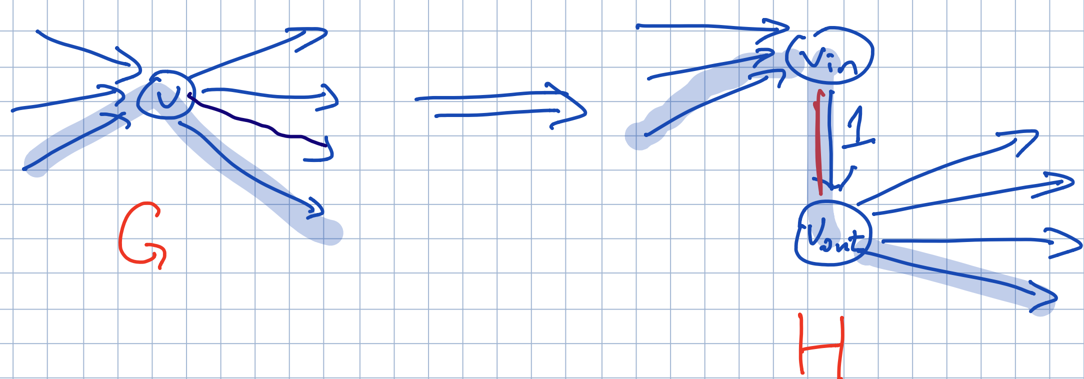
Find max # paths from s to t
that contain each ~~edge~~ vertices at most once.

Solution: Set $c(e) \leftarrow 1$
Run FF $\quad O(VE)$

Compute Flow decomposition

$\boxed{O(VE)}$





Claim: # vertex-disjoint paths in G

= # edge-disjoint paths in H

Proof: Two things to prove.

① Suppose there are k vertex-dis path in G.

then there are $\geq k$ edge-dis paths in H

② Suppose k edge-disj paths in H

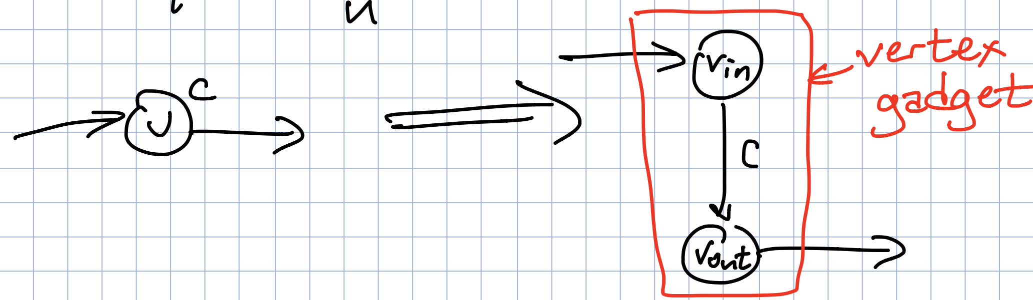
$s \to v_{in} \to v_{out} \to w_{in} \to w_{out} \to \cdots \to z_{out} \to t$

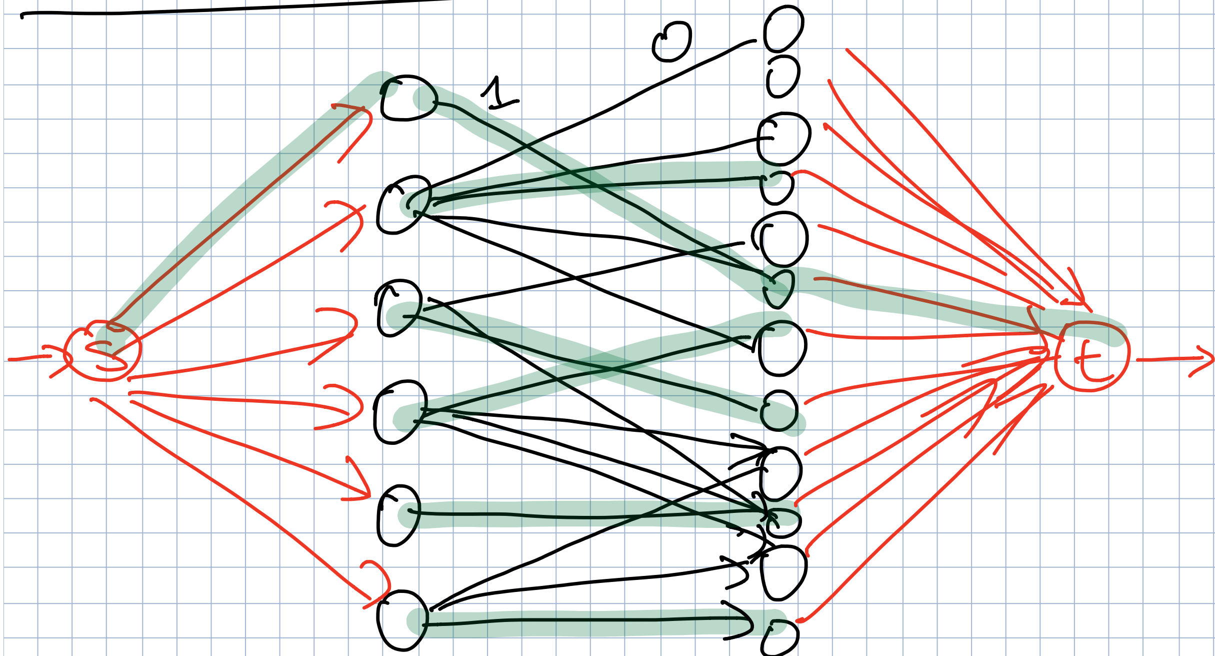
$s \to v \to w \to \cdots \to z \to t$

So $\geq k$ vert-dis paths in G $\square$

Vertex capacities $\quad c: V \to \mathbb{R}^+$

Require $\sum_u f(u\to v) \leq c(v)$



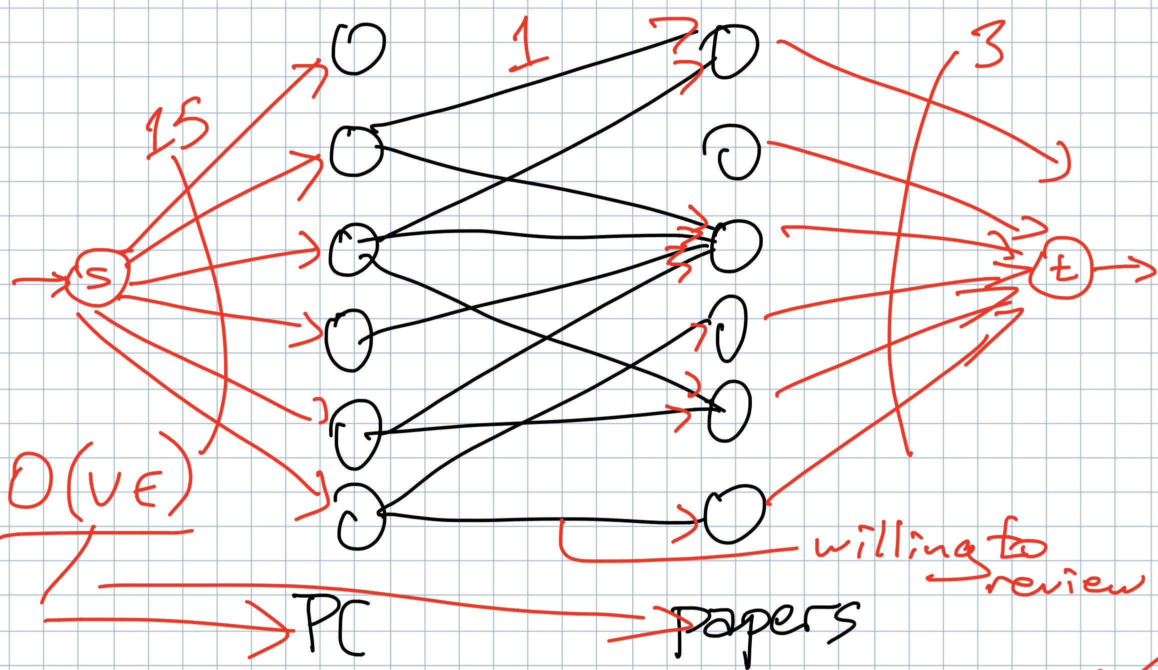


CS depts $\qquad$ CS PhDs

Maximum matching in a bipartite graph.

Add s, t
$c(e) \leftarrow 1$
Compute integral max flow
$f(u\to v)=1 \iff uv \in M$

$\boxed{O(VE)}$



$O(VE)$

- Each PC member can review ≤ 15 papers
- Each paper needs ≥ 3 reviews.

Assignment Problem

Is maxflow = 3#papers?