Treap with n vertices

node with ith smallest key

(a) $E[\#\text{leaves}] = \sum_{i=1}^{n} \Pr[\text{node } i \text{ is a leaf}]$

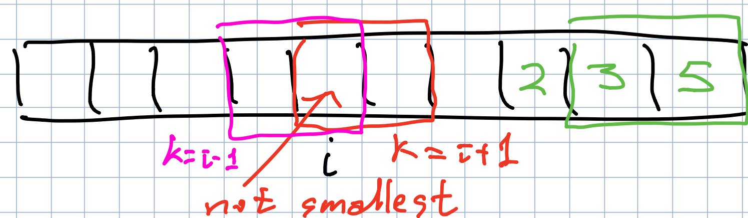
$= \sum_{i=1}^{n} \Pr[\text{node } i \text{ is a local max}]$

$= \sum_{i=2}^{n-1} 1/3 + 1/2 + 1/2 = \frac{n-2}{3} + 1 = \boxed{\frac{n+1}{3}}$

$[i \uparrow k] = [\text{node } i \text{ is prop ancestor of node } k]$

node i is a leaf $\iff \underline{[i \uparrow k] = 0 \text{ for all } k}$

$[i \uparrow k] \iff$ node i has smallest priority in $\{i \dots k\}$



i is a leaf $\iff$ priority$(i) >$ pri$(i-1)$ and pri$(i+1)$

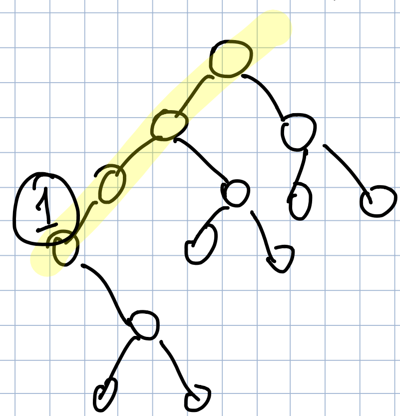
$\iff$ pri(i) is a local max

$$E[\text{length of left spine}] = \sum_i \Pr[i \text{ on left spine}]$$

$$= \sum_{i=1}^{n} \frac{1}{i} = H_n$$

$$\le \ln n + 1$$

$$O(\log n)$$



$[i \uparrow k]$

pri(i) is ~~larger~~ smaller than pr(1.. i-1)

$$\boxed{[i \uparrow 1]}$$

$$\Pr[i \uparrow 1] = \frac{1}{i}$$

$1 - \frac{1}{n^c}$

\# nodes on left spine = $O(\log n)$ whp

$$\mu = E[X] = H_n \approx \ln n$$

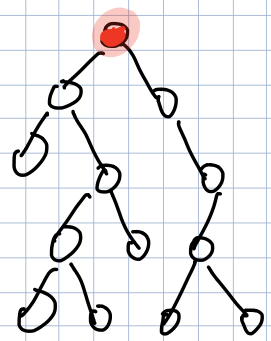
$$\Pr[X \ge (1+\delta)\mu] \le \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^{\mu}$$

$$\Pr[X \ge 10 \ln n] \le \left(\frac{e^9}{10^{10}}\right)^{\ln n} = n^{\ln(e^9/10^{10})}$$

$\boxed{\delta = 9}$ $\quad$ $\ll 1$

$$= n^{-\text{const}} \qquad \square$$

$$\boxed{a^{\log_b c} = c^{\log_b a}}$$



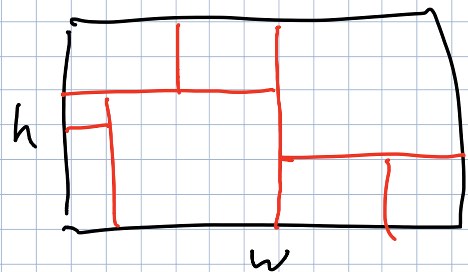
Each day
every marked node
marks one unmarked child

$\boxed{\text{MinDays(root)}}$

MinDays(v) = min # days to flood subtree rooted at v

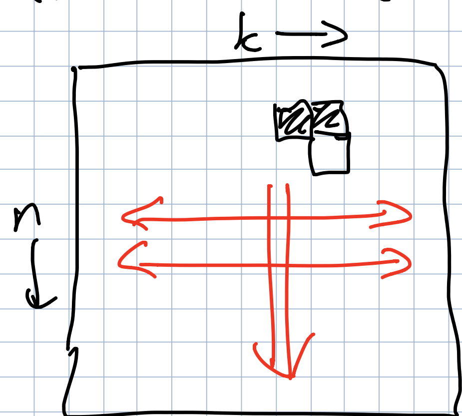
$$\text{MinDays}(v) = \begin{cases} 0 & \text{if } v \text{ is a leaf} \\ 1 + \text{MinDays}(w) & \text{if } v \text{ has one child } w \\ \min \begin{cases} \max\begin{Bmatrix} 1+\text{MinDays}(\ell) \\ 2+\text{MinDays}(r) \end{Bmatrix} \\ \max\begin{Bmatrix} 1+\text{MinDays}(r) \\ 2+\text{MinDays}(\ell) \end{Bmatrix} \end{cases} & \text{if } v \text{ has two kids } \ell \text{ and } r \end{cases}$$

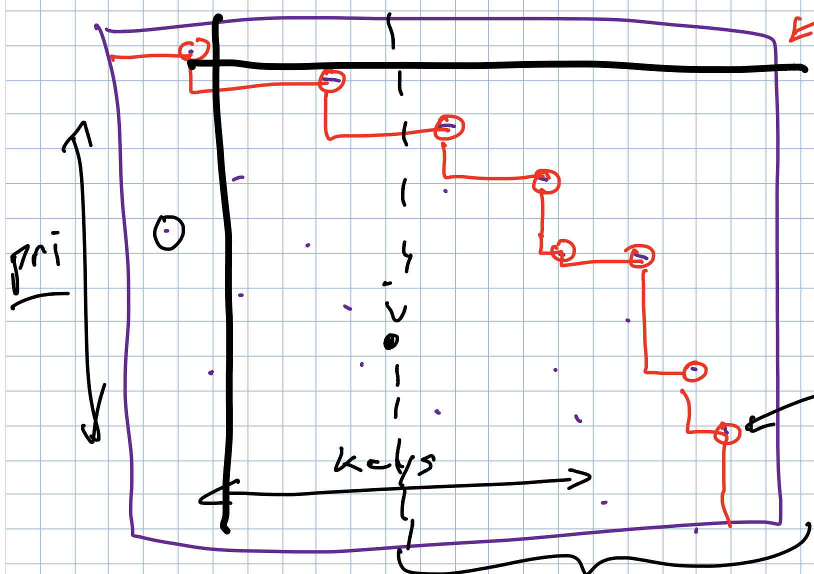
Memoize in T
Eval in postorder/DFS
$O(n)$ time



hw-1 moves

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$





$\leftarrow n$ random pts in $\square$

$E[\#\text{Pareto opt pts}] = H_n$

$\Pr[x_n \text{ is P.O.}] = 1$

$\Pr[x_1 \text{ is PO}] = \frac{1}{n}$

$x_1 \ldots x_n$ be pts in left to right order

$$\Pr[x_i \text{ is Pareto opt}] = \frac{1}{n-i+1}$$

Chernoff: $\Pr[X \geq 10 \ln n] \leq \left(\frac{e^9}{10^{10}}\right)^{\ln n} = n^{-\text{const}}$

$(1+\delta)\mu$

$\delta = 9 \quad \mu = H_n \approx \ln n$

$$\sum_{i=1}^{n} \frac{1}{i} = H_n \qquad \sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha} \quad \text{if } \alpha < 1$$

$$\sum_{i=1}^{n} i^c = O(n^{c+1}) \quad \text{if } c \neq -1$$