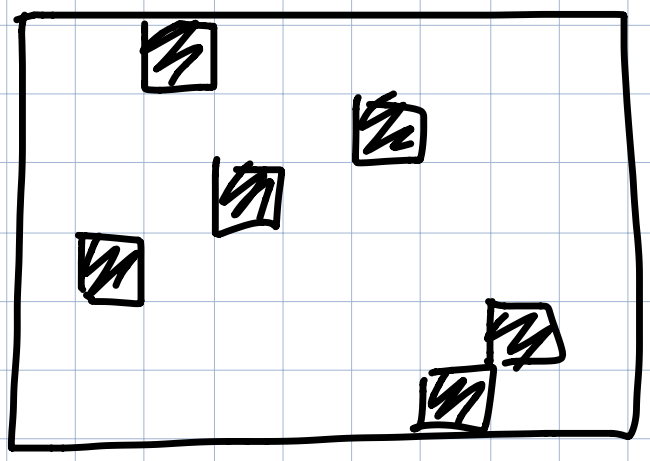
HW1 due 8pm

HW2 due next Tue 8pm

- Two problems

+ ✕ c

Core of many DP algorithms



Find min of every row.

$$M[i,j] = OPT(j, k-1) + D[i,j]$$

Brute Force: $O(mn)$ time

Monotone:

Row minima indices are nondecreasing

			n		
	12	21	~~38~~	~~76~~	~~27~~
	74	14	~~14~~	~~29~~	~~60~~
m	21	8	25	10	71
	~~68~~	45	29	15	76
	~~97~~	8	12	2	6

Find min of middle row

recurse in UL and LR quadrants

$$T(m,n) = O(n) + T(\tfrac{m}{2}, h) + T(\tfrac{m}{2}, n-h+1)$$

$$= O(n + n \log m)$$

M is totally monotone if

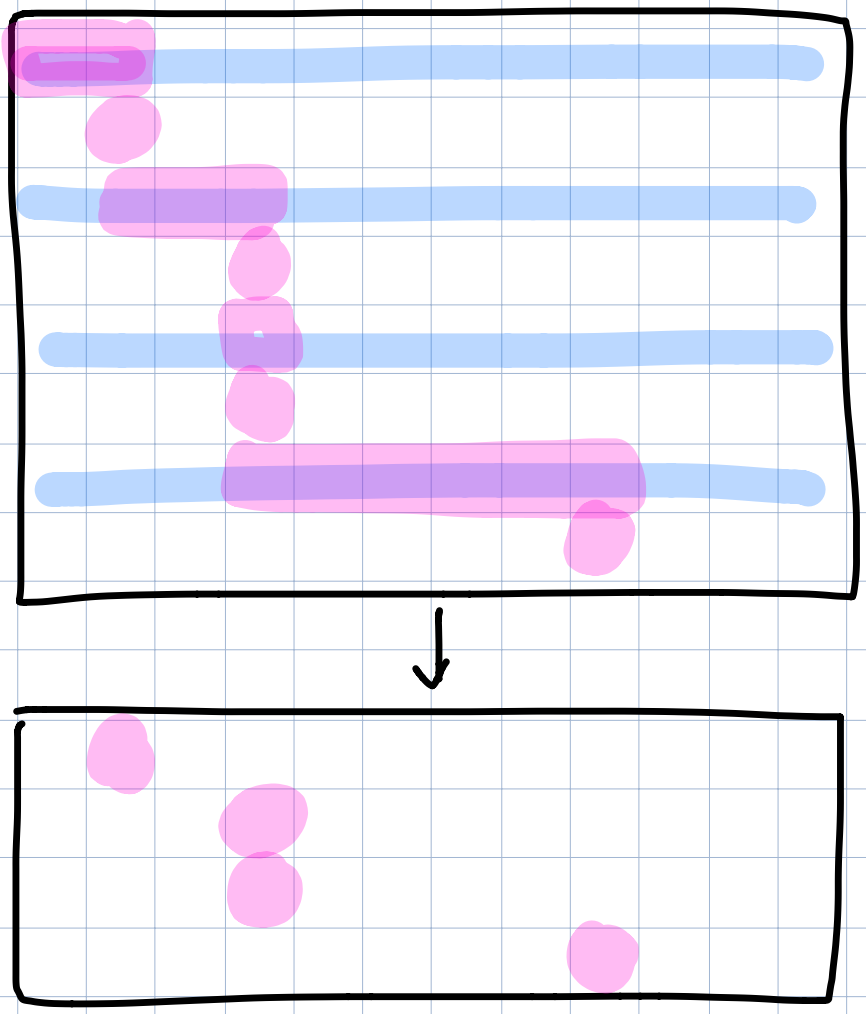
every 2x2 submatrix is monotone

monotone but not totally

$$\begin{bmatrix} 12 & 21 & 38 & 76 & 27 \\ 74 & 14 & 14 & 29 & 60 \\ 21 & 8 & 25 & 10 & 71 \\ 68 & 45 & 29 & 15 & 76 \\ 97 & 8 & 12 & 2 & 6 \end{bmatrix}$$

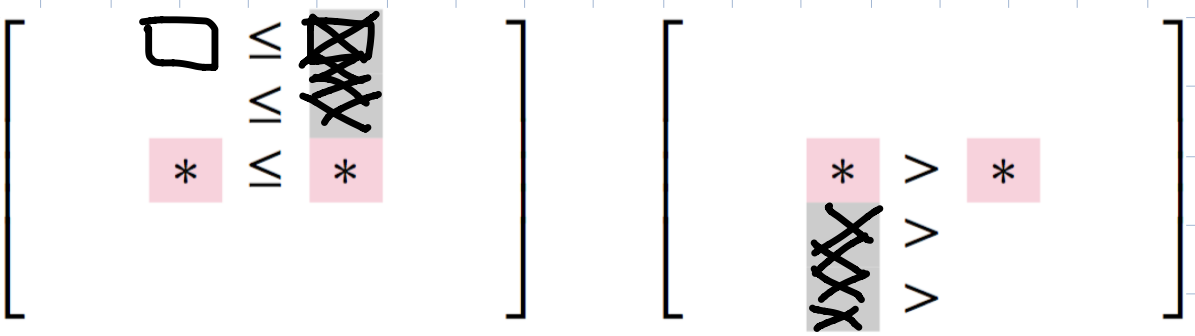
totally mono:

$$\begin{bmatrix} 12 & 21 & 38 & 76 & 89 \\ 47 & 14 & 14 & 29 & 60 \\ 21 & 8 & 20 & 10 & 71 \\ 68 & 16 & 29 & 15 & 76 \\ 97 & 8 & 12 & 2 & 6 \end{bmatrix}$$

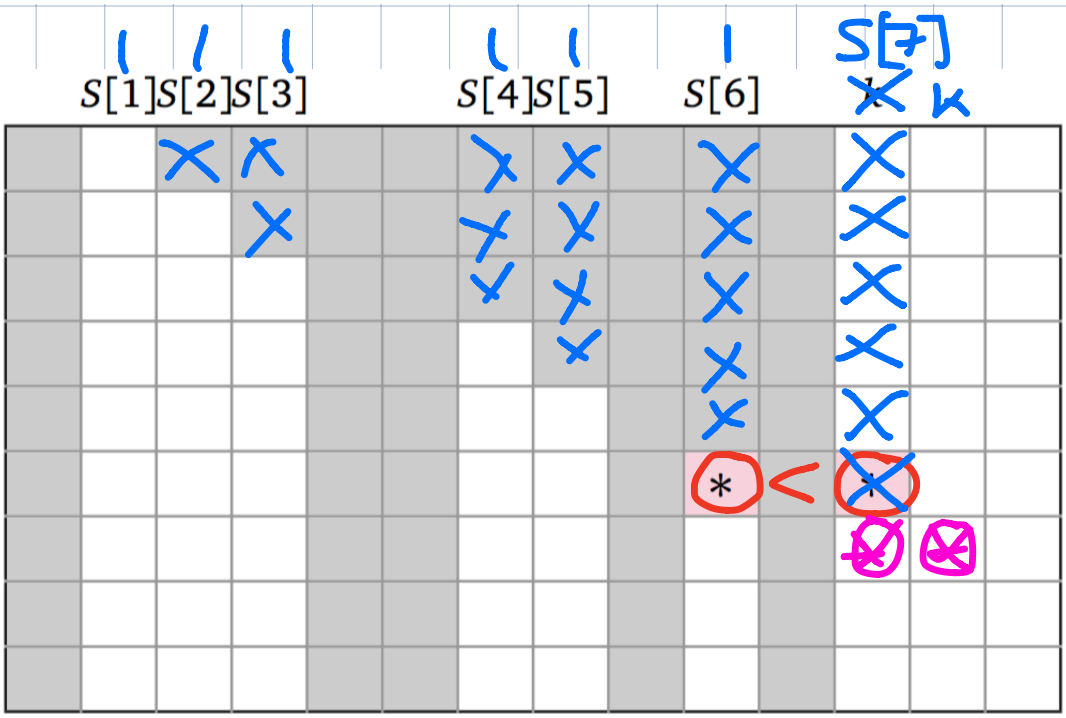


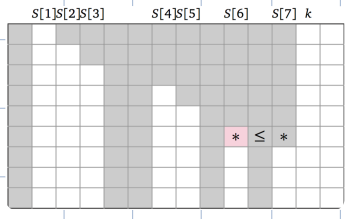
$$T(m,n) = T\left(\frac{m}{2}, n\right) + O(m+n)$$

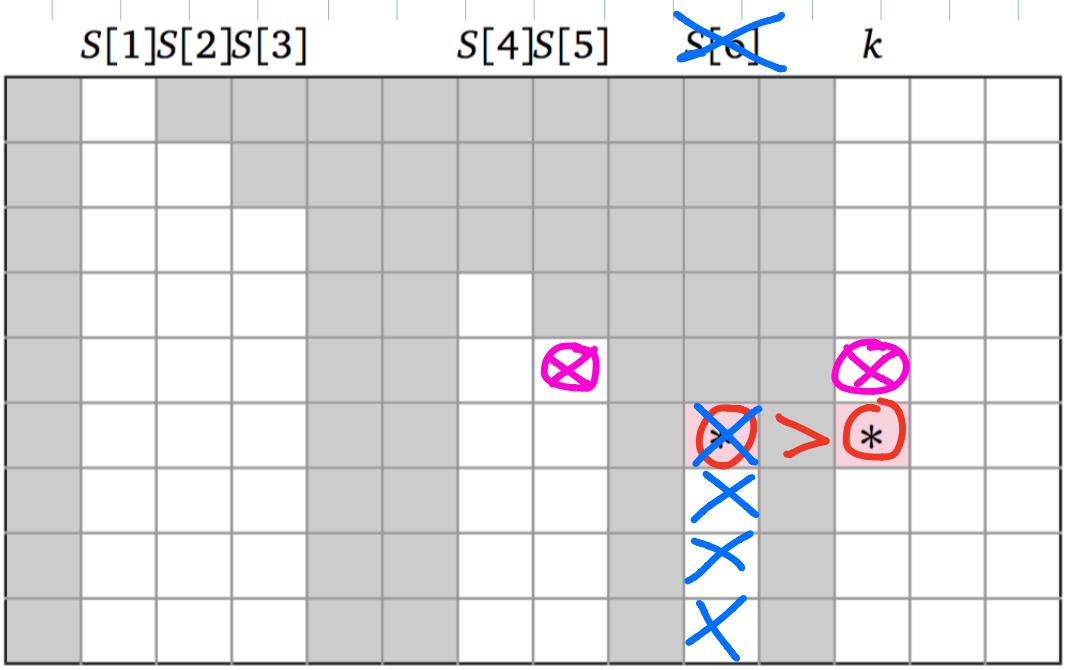
$$= O(m + n \log m)$$



```
REDUCE(M[1..m, 1..n]):
  t ← 1
  S[t] ← 1
  for k ← 1 to n
      while t > 0 and M[t, S[t]] ≥ M[t, k]
          t ← t − 1        ⟨⟨pop⟩⟩
      if t < m
          t ← t + 1
          S[t] ← k         ⟨⟨push k⟩⟩
  return S[1..t]
```


S[1]S[2]S[3]
S[4]S[5]
S[6]
S[7]
k
*
<
*


S[1]S[2]S[3]
S[4]S[5]
S[6]
S[7]
k
*
≤
*

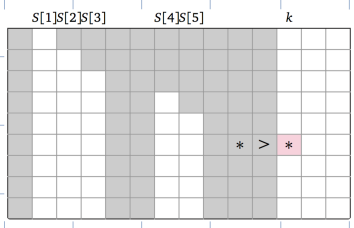


In every iteration
either kill a column
or push a column

≤ n

≤ n+m

O(n+m) time



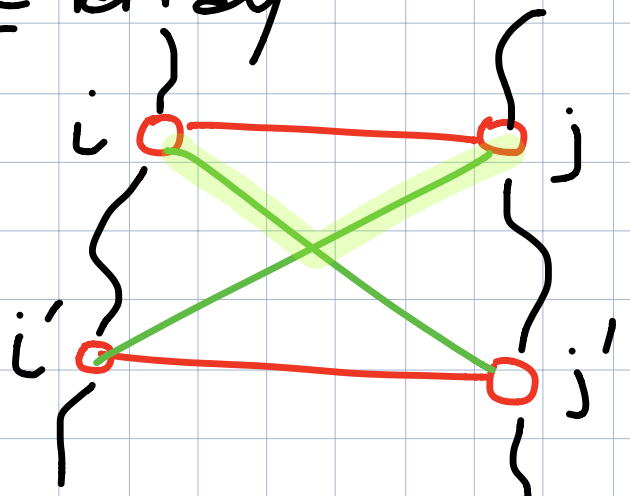
SMAWK(M)

if $n > m$
FILTER(M)
EvenRow recursion

$$T(m,n) \leq \begin{cases} O(m) + T(\frac{m}{2}, n) & m \geq n \\ O(n) + T(\frac{m}{2}, m) & m < n \end{cases}$$

$$= O(n+m)$$

Monge array



$$d(i,j) + d(i',j') \leq d(i,j') + d(i',j)$$

for all $i < i'$ and $j < j'$

Monge $\Rightarrow$ total mono

Suffices consider $i' = i+1$ $j' = j+1$

1	1	1	1
3	3	3	3
-5	-5	-5	-5
7	7	7	7

1	3	5	7
1	3	5	7
1	-3	5	7
1	-3	5	7

0	0	1	1	1
0	0	1	1	1
0	0	1	1	1
0	0	0	0	0
0	0	0	0	0

$\alpha \cdot$ Monge = Monge

Monge + Monge = Monge