Midterm 7-9pm   3 rooms in Eng. quad
Tue Feb 23

No class   Optional review session instead

Conflict Wed Feb 24

Cover HW 0-3.   HW4 due March 2

My old exams are all online

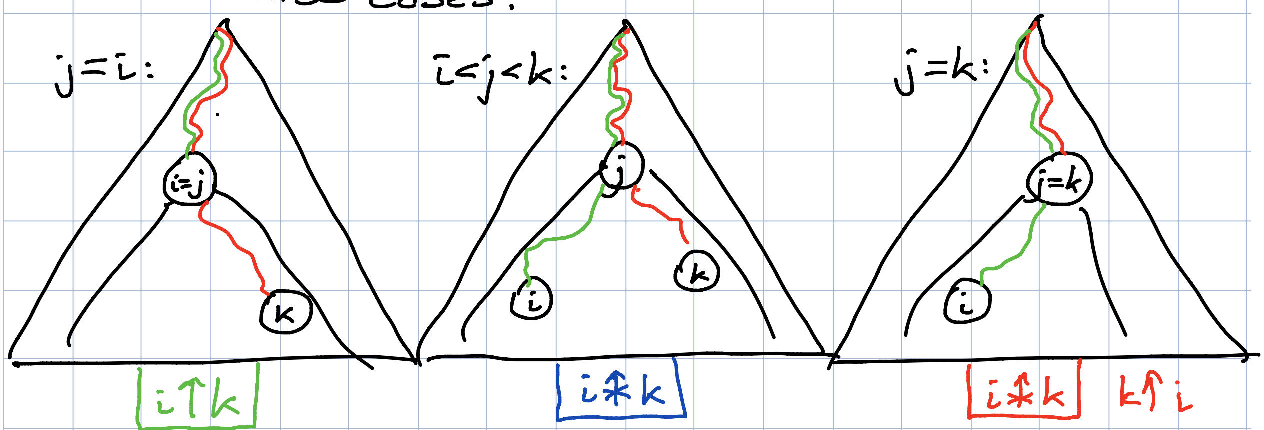
One cheat sheet

Dyn prog:
6 recurrence
3 data str.
1 time

Claim: Node i is a proper ancestor of node k if and only if node i has smallest priority among nodes $i, i+1, \ldots, k$. $i \uparrow k$

Proof: Suppose node j is the least common ancestor of node i and node k.

Three cases:



In all three cases, i and k are in the subtree rooted at j.

So every node $i, i+1, \ldots, k-1, k$ is in the subtree rooted at j. (BST)

So node j has smallest priority among nodes $i, i+1 \ldots, k-1, k$. (heap) $\square$

$E[T(x)] = ?$ $\quad Pr[T(x) > t] = ?$

$Pr[\text{max depth} \geq 10 \log n] = ?$

We know: $E[\text{depth}(k)] \leq H_k + H_{n-k} - 2 \leq 2 \ln n$

$$E[\max_k \text{depth}(k)] = ?$$

$$Pr(\max_k \text{depth}(k) > 10 \lg n]$$

$$\leq \sum_k Pr(\text{depth}(k) > 10 \lg n]$$

$\boxed{\text{Union bound}}$

Suppose we knew

$$Pr(\text{depth}(k) > 10 \lg n] \leq 1/n^4$$

Then $Pr[\text{max depth} > 10 \lg n] \leq \frac{1}{n^3}$

$$E[\text{max depth}] \leq Pr(\text{max depth} \leq 10 \lg n] \cdot 10 \lg n + Pr(\text{max depth} > 10 \lg n] \cdot n$$

$$\leq 10 \lg n + 1/n^2$$

Event happens <u>with high probability</u>

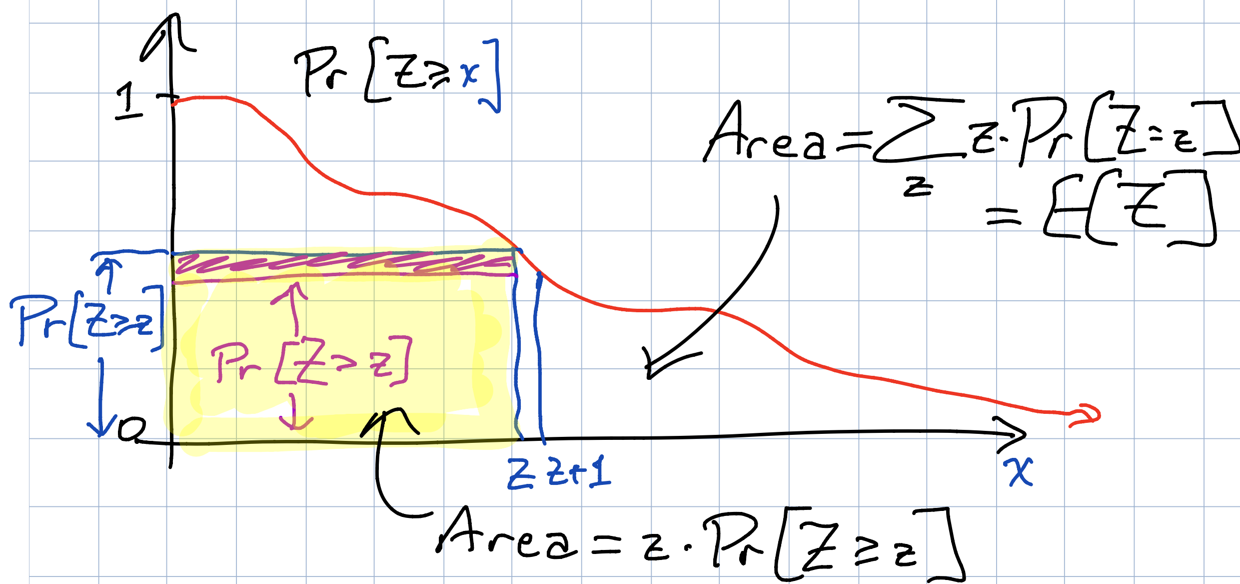
$= Pr[\text{event}] \geq 1 - 1/n^c$

Let Z be a non-neg. random variable

Markov's inequality: For any $z > 0$

$$\Pr[Z \geq z] \leq \frac{E[Z]}{z}$$

Proof:



We need stronger assumptions

- $Z = \sum_i X_i$ where $X_i \in \{0, 1\}$
- Some independence among X_i's.

X and Y are independent iff

$$\Pr[X=x \wedge Y=y] = \Pr[X=x]\cdot\Pr[Y=y]$$

$$\Downarrow$$

$$E[XY] = E[X]\cdot E[Y]$$

$\Rightarrow$ $f(X)$ and $f(Y)$ are independent

$X_1 \ldots X_n$ are <u>mutually</u> independent

$$\Pr\left[\bigwedge_i X_i = x_i\right] = \prod_i \Pr[X_i = x_i]$$

$$E\left[\prod X_i\right] = \prod E[X_i]$$

$X_1 \ldots X_n$ are <u>pairwise</u> indep if

X_i and X_j are indep for all $i \neq j$.

X = coin flip
Y = coin flip
$Z = X \oplus Y$

Chebyshev's inequality: $X = \sum_i X_i$

IF $X_1 \dots X_n$ are pairwise indep.

Then $\Pr\left[(X - E[X])^2 \geq z\right] \leq \frac{E[X]}{z}$

For all $z \geq 0$.

Proof: $p_i = \Pr[X_i = 1]$ $\quad \mu = E[X] = \sum_i p_i$

$Y_i = X_i - p_i \qquad Y = \sum_i Y_i = X - \mu$

$$
\begin{aligned}
E[Y^2] &= \\
&\vdots \\
&= \sum_i E[Y_i^2] \\
&\vdots \\
&= \sum_i p_i(1-p_i) \leq \sum_i p_i = \mu
\end{aligned}
$$

$$\Pr[Y^2 \geq z] \leq \frac{E[Y^2]}{z} \leq \frac{\mu}{z} \qquad \square$$

Chebyshev's inequality: $X = \sum_i X_i$

If $X_1 \dots X_n$ are pairwise indep.

Then $\Pr\left[(X - E[X])^2 \geq z\right] \leq \frac{E[X]}{z}$

For all $z \geq 0$.

$\Rightarrow \Pr\left[X \geq (1+\delta)E[X]\right] \leq \frac{1}{\delta^2 E[X]}$

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if X_i's are $2k$-independent

$\Pr\left[(X-\mu)^k \geq z\right] = O(\mu^k/z)$

$\Downarrow$

$\Pr\left[X \geq (1+\delta)\mu\right] = O\left(\left(\frac{1}{\delta^2\mu}\right)^k\right)$

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mutual independence

$E[\alpha^X] < e^{(\alpha-1)\mu}$

$$\Pr[X \geq (1+\delta)\mu] \leq \left(\frac{e^{\delta}}{(1+\delta)^{1+\delta}} \right)^{\mu}$$

for any k

$[1 \uparrow k]\ (2 \uparrow k]\ \cdots\ [k-1 \uparrow k]$

are mutually indep.