$\alpha$- approximation algorithm

computes a solution $A(X)$ for input $X$ such that

$$\frac{A(X)}{OPT(X)} \le \alpha \quad \text{and} \quad \frac{OPT(X)}{A(X)} \le \alpha$$

$\alpha = 1.5$

$\alpha$- approx computes solution $\le 50\%$ bigger than optimal

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Minimum-makespan scheduling

Greedy : 2-approx

Sort + greedy : $\frac{3}{2}$-approx

actually $\frac{4}{3}$-approx

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Vertex Cover

Greedy : $O(\log n)$-approx $\qquad n = |V|$

Stupid : 2-approx

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for all edges uv
    if u and v both unmarked
        mark u and v
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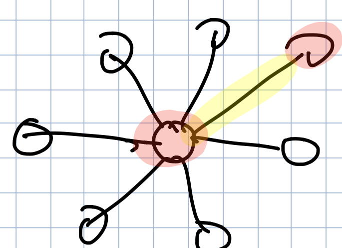

GreedyVC(G):

$C \leftarrow \emptyset$
$G_0 \leftarrow G$
$i \leftarrow 0$

while $G_i$ has an edge
  $i \leftarrow i+1$
  $d_i \leftarrow$ max degree in $G_{i-1}$
  $v_i \leftarrow$ any vertex with degree $d_i$
  $G_i \leftarrow G_{i-1} \setminus v_i$
  $C \leftarrow C + v_i$
return $C$

$A(x) \le F(x) \le \alpha \cdot OPT(x)$

$C^*$ = optimal vertex cover

$$\sum_{v \in C^*} \deg(v) \ge E(G) \implies \underset{\substack{\text{over all } v \in C^* \\ \wedge| \\ \text{max degree}}}{\text{average } \deg(v)} \ge \frac{E(G)}{OPT}$$

$$d_i \ge \frac{E(G_{i-1})}{OPT} \ge \frac{E(G_{OPT})}{OPT} \quad \text{for all } i \le OPT$$

$$\sum_{i=1}^{OPT} d_i \ge \sum_{i=1}^{OPT} \frac{E(G_{OPT})}{OPT} = E(G_{OPT})$$

$$= E(G) - \sum_{i=1}^{OPT} d_i$$

$$\boxed{\sum_{i=1}^{OPT} d_i \ge \frac{E(G)}{2}}$$

After $\boxed{2 \lg n \cdot OPT}$ iterations

$$\#\text{edges left} \le \frac{E(G)}{2^{\lg n}} < 1$$

$|C| \leq 2\lg n \cdot OPT$ $\square$

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TSP

undir.

Given Graph $G=(V,E)$

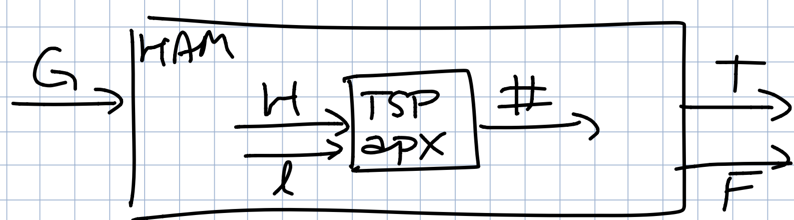
$\ell : E \to \mathbb{R}^+$

Find length of shortest Ham. cycle.

BAD News: NO approx algo (unless P=NP)

Claim: $f(n)$-approx TSP is NP-hard

Proof: Reduce from Ham. cycle

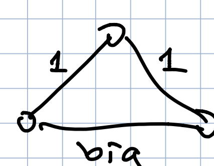



$H$ = complete graph with $n$ vertices

$$\ell(uv) = \begin{cases} 1 & \text{if } uv \in E(G) \\ f(n)\cdot n+1 & \text{o/w} \end{cases}$$

1. $OPT\ TSP(H) = n \iff G$ is Hamiltonian
2. $OPT\ TSP(H) \geq f(n)\cdot n^{+1} \iff G$ is not Hamiltonian

Concorde TSP solver



Triangle ≠ $\ell(uw) \le \ell(uv) + \ell(vw)$

<u>Metric</u> TSP $\Rightarrow$ for all u,v,w

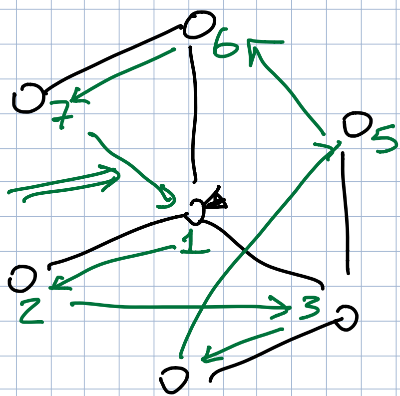
2-approximation algo:

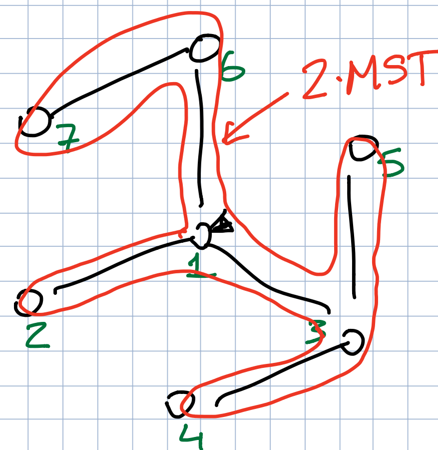
① Compute MST(G)

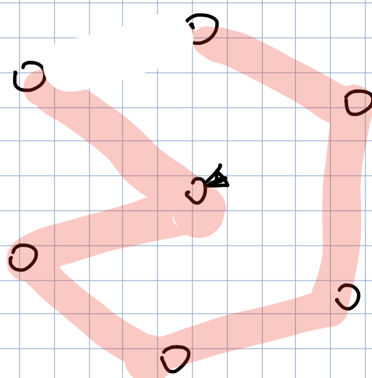
② DFS <u>numbering</u>

<u>Done</u>

$\boxed{A(G) \le 2 \cdot MST}$



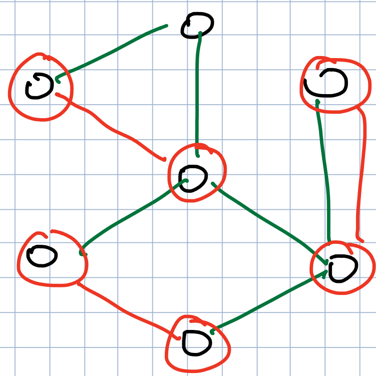



$\ge MST$

$< OPT$

$\boxed{MST \le OPT}$

Christofedes ('79) = 3/2-approximation



1. MST $\leq$ OPT

2. Min cost matching $M$ of odd vertices in MST

$M \leq OPT/2$

Best known in 2016

$\boxed{\text{Conjecture: } \frac{4}{3}\text{-approx}}$