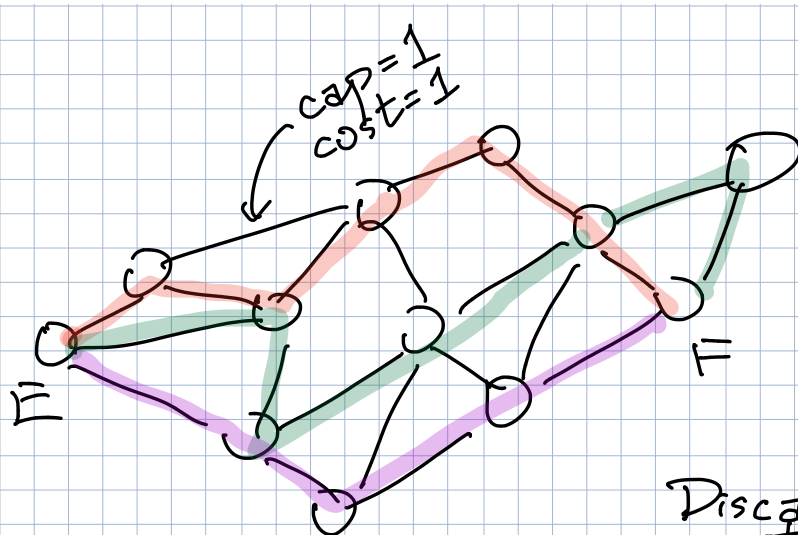


Discard any path len > h

$$\sum_i h - len(p_i) + 1$$

Find max collection of edge-disjoint path with min total length

NP-hardness

P: answer in poly time

NP: verify in poly time

X is NP-hard $\iff$ if $X \in P$, then ~~P=NP~~

$X \notin P$

Cook-Levin Theorem: CIRCUITSAT is NP-hard

$\Updownarrow$

SAT is NP-hard

$\Downarrow$

[Karp] 3SAT is NP-hard

$\Downarrow$

MIS is NP-hard

$\underbrace{(a \vee b \vee c)}_{\text{clause}} \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee b \vee \bar{d})$

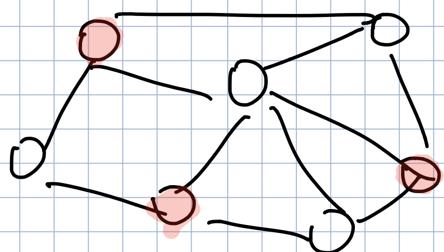
literals

variables

Reduction: To prove X is NP-hard

1. Choose known NP-hard Y
2. Build a poly-time algo for Y using a fictional poly-time algo X as a subroutine

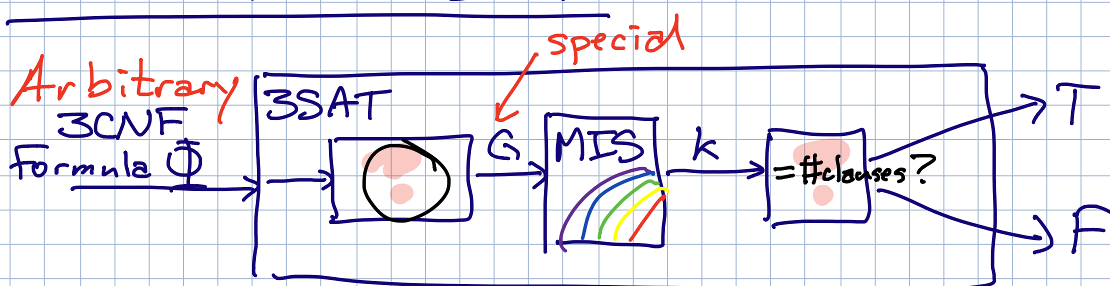
Maximum Independent Set



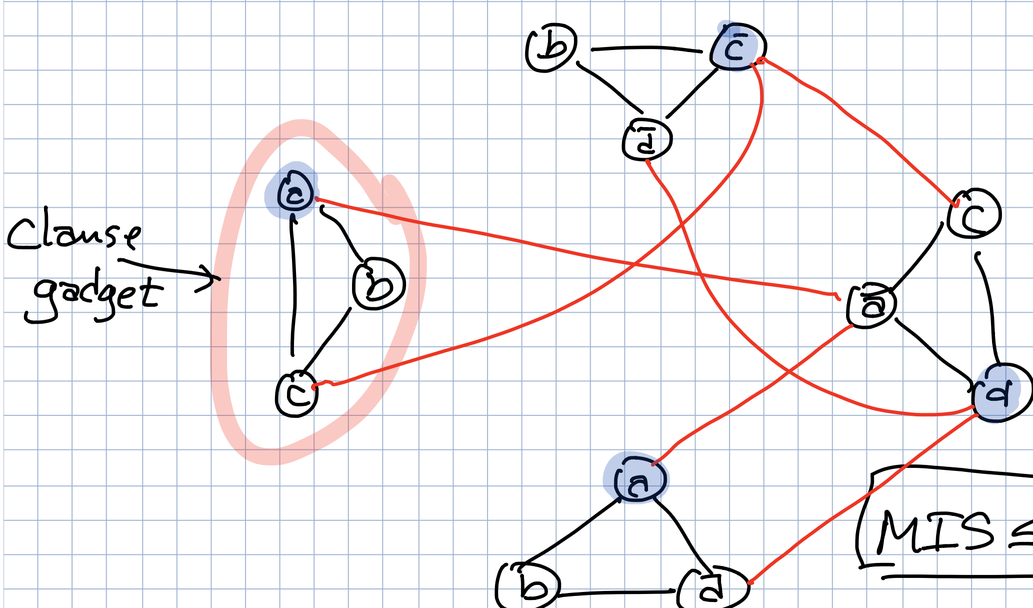
Input: undir Graph G

Output: max size of ind subset of vertices.

Reduction from 3SAT



$$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee b \vee \bar{d})$$



$\boxed{\text{MIS} \le \#\text{clauses}}$

Ind set in G of size $= \underline{\#\text{clauses}}$ $\Longleftrightarrow$ (partial) sat. assignment for Φ

G has an ind set of size k $\iff$ There is an assignment that satisfies k clauses in Φ

($\Rightarrow$) Spose G has an ind set of size k

$\Rightarrow$ Assign values to variables of Φ to make those k literals True

Consistent because of x —— $\bar{x}$

$\Rightarrow$ k clauses, each with ≥ 1 True literal $\square$

($\Leftarrow$) Spose some assgt makes k clauses in Φ True

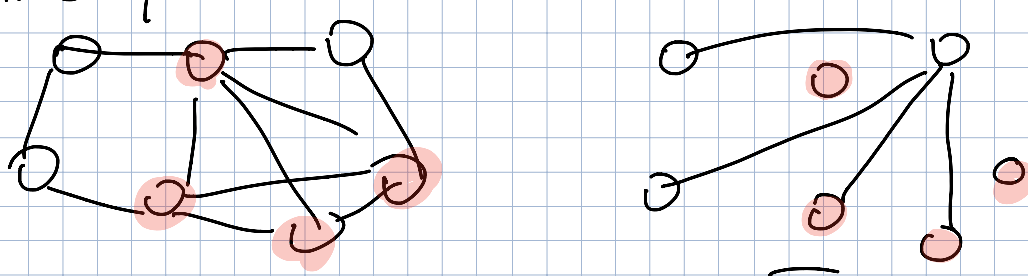
Choose one True literal from each of those clauses

Mark corr. verts in G

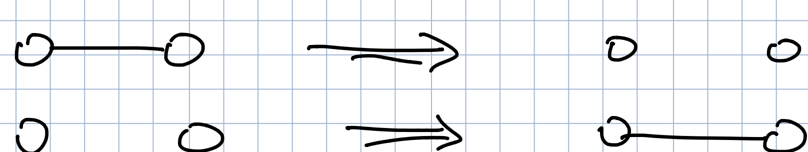
- Diff Δs
- Ind x —— $\bar{x}$

$\Big\}$ Ind set of k verts $\square$

Max Clique

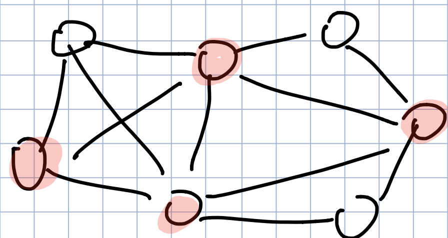


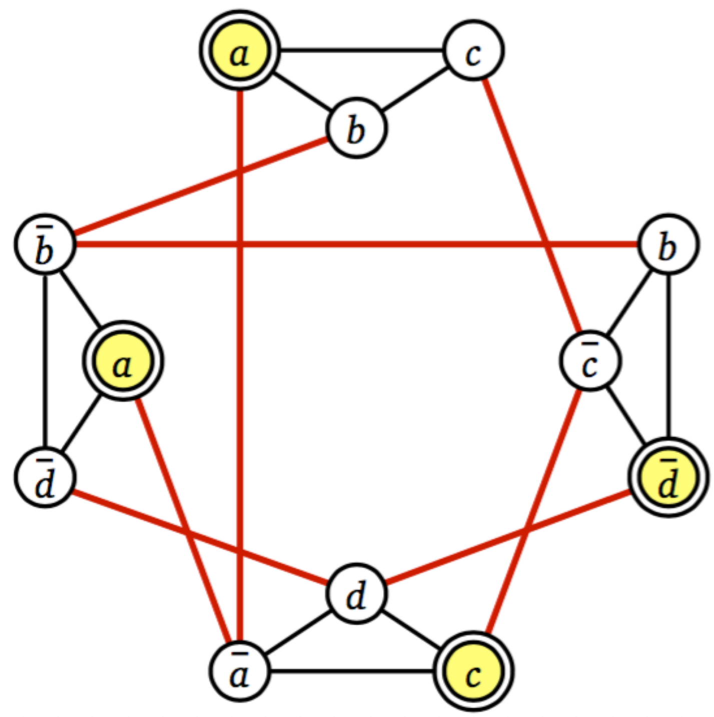
G $\qquad\qquad$ $\overline{G}$



clique $\iff$ ind set

Min Vertex Cover = V - Max Ind Set

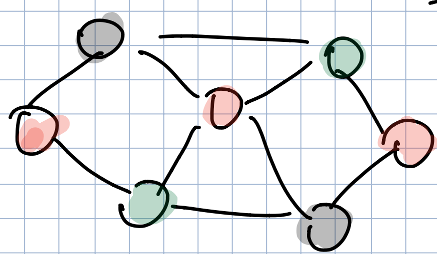



a
c
b
$\bar{b}$
b
a
$\bar{c}$
$\bar{d}$
$\bar{d}$
d
$\bar{a}$
c

3 COLOR

Given G

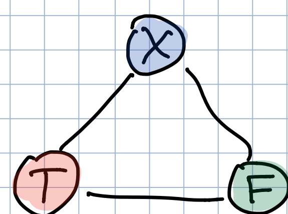
Color verts with 3 colors
s.t. every edge touches 2 colors



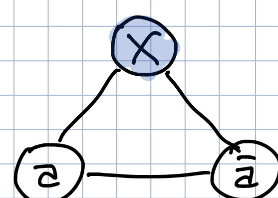
Reduction from 3SAT
because 3.

Given ARBITRARY 3CNF Φ
Build graph G

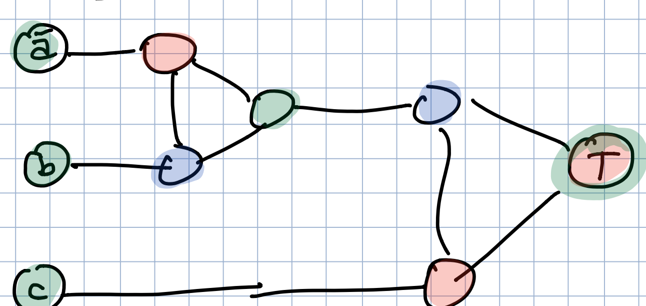
① Truth gadget

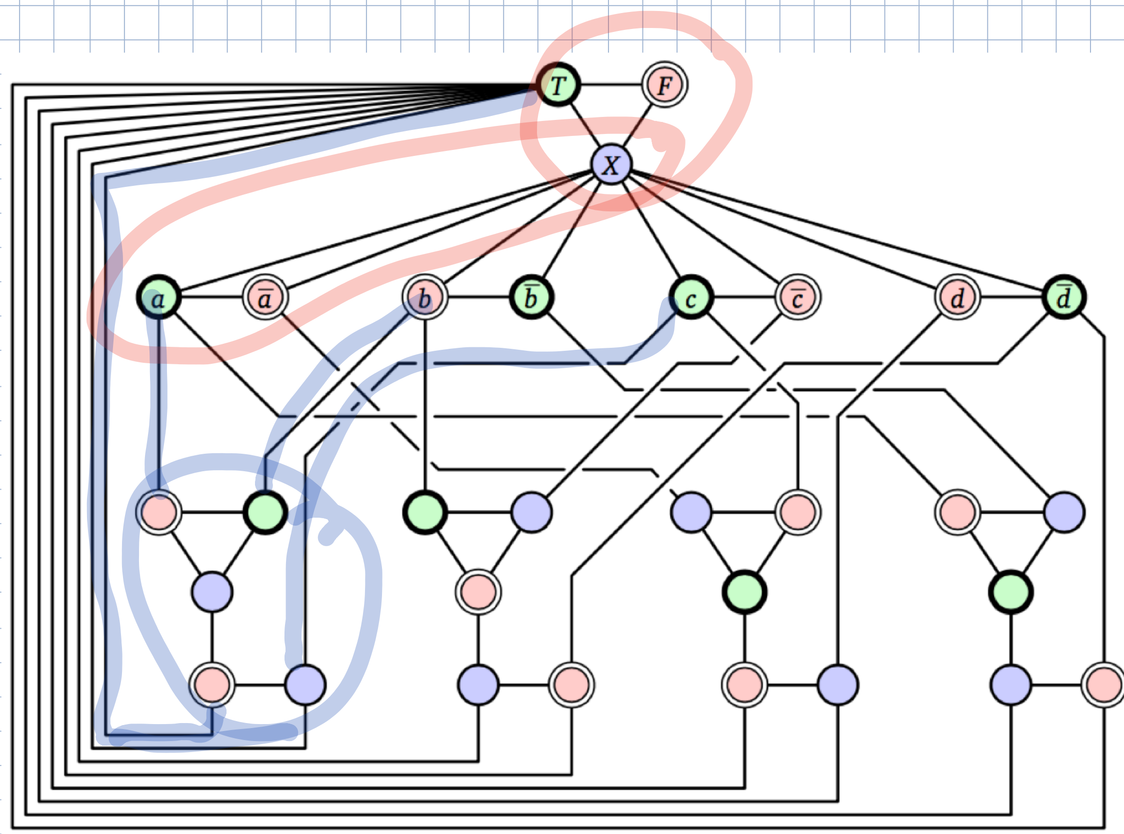


② Variable gadget



③ Clause gadget $(\bar{a} \vee b \vee c)$





A 3-colorable graph derived from the satisfiable 3CNF formula
$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$