CS 473 ✧ Spring 2016

☙ Homework 9 ❧

Due Tuesday, April 19, 2016, at 8pm

For problems that ask to prove that a given problem X is NP-hard, a full-credit solution requires the following components:

- Specify a known NP-hard problem Y, taken from the problems listed in the notes.
- Describe a polynomial-time algorithm for Y, using a black-box polynomial-time algorithm for X as a subroutine. Most NP-hardness reductions have the following form: Given an arbitrary instance of Y, describe how to transform it into an instance of X, pass this instance to a black-box algorithm for X, and finally, describe how to transform the output of the black-box subroutine to the final output. A cartoon with boxes may be helpful.
- Prove that your reduction is correct. As usual, correctness proofs for NP-hardness reductions usually have two components ("one for each f").

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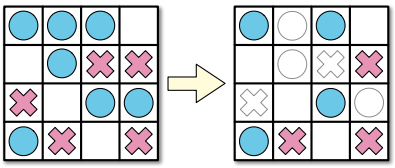
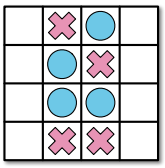
1. Consider the following solitaire game. The puzzle consists of an $n \times m$ grid of squares, where each square may be empty, occupied by a red stone, or occupied by a blue stone. The goal of the puzzle is to remove some of the given stones so that the remaining stones satisfy two conditions: (1) every row contains at least one stone, and (2) no column contains stones of both colors. For some initial configurations of stones, reaching this goal is impossible.



A solvable puzzle and one of its many solutions.



An unsolvable puzzle.

Prove that it is NP-hard to determine, given an initial configuration of red and blue stones, whether the puzzle can be solved.

2. Everyone's having a wonderful time at the party you're throwing, but now it's time to line up for ***The Algorithm March (***アルゴリズムこうしん)! This dance was originally developed by the Japanese comedy duo Itsumo Kokokara (いつもここから) for the children's television show PythagoraSwitch (ピタゴラスイッチ). The Algorithm March is performed by a line of people; each person in line starts a specific sequence of movements one measure later than the person directly in front of them. Thus, the march is the dance equivalent of a musical round or canon, like "Row Row Row Your Boat".[1] Proper etiquette dictates that each marcher must know the person directly in front of them in line, lest a minor mistake during lead to horrible embarrassment between strangers.

   Suppose you are given a complete list of which people at your party know each other. Prove that it is NP-hard to determine the largest number of party-goers that can participate in the Algorithm March. You may assume without loss of generality that there are no ninjas at your party.

[1] そろそろおわりかな。そろそろおわりかな。そろそろおわりかな。