CS 473 ✧ Spring 2016

Homework 10

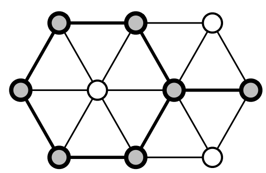
Due Tuesday, April 26, 2016, at 8pm

This is the last graded homework of the semester.

---

1. A *double-Hamiltonian circuit* in an undirected graph G is a closed walk that visits every vertex in G exactly twice. Prove that determining whether whether a given undirected graph contains a double-Hamiltonian circuit is NP-hard.

2. A subset S of vertices in an undirected graph G is called ***triangle-free*** if, for every triple of vertices $u, v, w \in S$, at least one of the three edges uv, uw, vw is *absent* from G. Prove that finding the size of the largest triangle-free subset of vertices in a given undirected graph is NP-hard.



A triangle-free subset of 7 vertices.
This is ***not*** the largest triangle-free subset in this graph.

3. Suppose you are given a magic black box that can determine **in polynomial time**, given an arbitrary graph G, whether G is 3-colorable. Describe and analyze a **polynomial-time** algorithm that either computes a proper 3-coloring of a given graph or correctly reports that no such coloring exists, using the magic black box as a subroutine. *[Hint: The input to the magic black box is a graph. Just a graph. Vertices and edges. Nothing else.]*