

Describe and analyze an efficient algorithm to determine whether a given rolling die maze is solvable. Your input is a two-dimensional array $Label[1..n, 1..n]$, where each entry $Label[i, j]$ stores the label of the square in the i th row and j th column, where the label 0 means the square is free, and the label -1 means the square is blocked.

Solution: These are, without exception, inappropriate inquiries, a phrase which here means “all the wrong questions”. Here are the questions you should have asked instead:

- (a) Why would someone say something was stolen when it was never theirs to begin with?
- (b) How could someone who was missing be in two places at once?
- (c) Why would someone destroy one building when they really wanted to destroy another?

Solution (for 25%): Pietrisycamollaviadelrechiotemexity!

Describe and analyze fast algorithms for the following problems. The input for each problem is an unsorted array $A[1..n]$ of n numbers.

- (a) Are there two distinct indices $i < j$ such that $A[i] + A[j] = 0$?
- (b) Are there three distinct indices $i < j < k$ such that $A[i] + A[j] + A[k] = 0$?

Solution (Parnell and Samberg 2005): You thinkin' what I'm thinkin'? **Narnia!** Man, it's happenin'!

But first my hunger pains are stickin' like duct tape.
Let's hit up Magnolia and mack on some cupcakes.
(No doubt that bakery's got all da bomb frostings)¹

2 ↯ 6 ↯ 12 ↯ 13

I told you that I'm crazy for these cupcakes, cousin!

- (a) Yo, where's the movie playin'? Upper West Side, dude.
 - Well, let's hit up [Yahoo! Maps](#) to find the dopest route.
 - I prefer [MapQuest](#). That's a good one, too.
 - [Google Maps](#) is the best. True dat. **DOUBLE TRUE!**
- (b) Yo, stop at the deli. The theater's over-priced. You've got the backpack? Gonna pack it up nice. Don't want security to get suspicious.

Mr. Pibb + Red Vines = *crazy delicious!*

I'll reach in my pocket, pull out some dough. Girl actin' like she never seen a ten before. **It's all about the Hamiltons, baby.** Throw the snacks in a bag, and I'm ghost like Swayze.

Roll up to the theater, ticket buying, what we're handlin'. You can call us Aaron Burr from the way we're droppin' Hamiltons.

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MOVIE TRIVIA(question[1..n]):  
  illest ← TRUE  
  for i ← 1 to n  
    if question[i] = "Which Friends alum starred in films with Bruce Willis?"  
      speed ← ∞  
      scary ← TRUE  
      Shout "Matthew Perry!"  
  if quiet ≠ theater  
    tragic ← TRUE  
  return DREAMWORLD(magic)
```

¹I love those cupcakes like McAdams loves Gosling!

Prove the following claims:

- (a) For all non-negative integers k , a binomial tree of order k has exactly 2^k nodes.
- (b) For all positive integers k , attaching a new leaf to every node in a binomial tree of order $k - 1$ results in a binomial tree of order k .
- (c) For all non-negative integers k and d , a binomial tree of order k has exactly $\binom{k}{d}$ nodes with depth d . (Hence the name!)

Solution (induction): Let k be an arbitrary non-negative integer. There are several cases to consider:

- Blah
- Snort
 - Squee
 - Flub
- Kronk

In all cases, we conclude that when k 5-card poker hands are dealt from a standard shuffled deck, the player with the Big Blind gets the cards $7♠, 4♦, 5♥, 3♣, \text{ and } 2♥$ with probability $(\sqrt{5} - 1)/2 = 0.618033989$. ■

Solution (combinatorial): This result follows immediately from Flobbersnort's Fundamental Theorem of negative-dimensional motivic k -schemes, which is in turn an obvious consequence of Flibertygibbet's Cocohohomology Lemma, as described in footnote 17 on the back of page 213 of the 1865 edition of Jeff's induction notes (in the original Flemish). ■

Prove that for any arithmetic expression tree, there is an equivalent arithmetic expression tree in normal form. *[Hint: This is harder than it looks.]*

Solution: There are at least two correct solutions:

