1. Let $G = (V, E)$ by a directed graph. A *half-of-a-Hamiltonian-path* is a path P that visits $n/2$ vertices, where $n = |V|$ is the total number of vertices in G. Prove that deciding whether G contains a half-of-a-Hamiltonian-path is NP-complete.

2. Let G be a directed, weighted graph. Given such a G, the zero-length cycle problem asks us to check if G has a simple cycle C such that the sum of the weights on the edges in C is exactly 0. Prove that this problem is NP-Complete.

3. In the *Bounded Degree Spanning Tree* problem we are given a graph G and a number k. It should return "yes" exactly when the graph has a spanning tree where each vertex has degree at most k. Prove that this problem is NP-complete.

4. In the *subset sum problem*, you are given n positive integers $X = \{x_1, ..., x_k\}$ and a positive integer S, and you want to know whether there exists a subset Y of X that sums to S.

   Assume you have a black box that answers the decision version of this problem. Use a polynomial number of calls to the black box to construct such a Y, if one exists.

5. In the 2-partition problem, we are given positive integers $A = \{a_1, ..., a_n\}$, and we want to know whether there is a partition of the numbers into two sets such that the sum of the numbers in each set is exactly $(\Sigma_i a_i)/2$. Prove that this problem is NP-Complete.

6. A *Canadian graph* $G = (V, E)$ is a directed graph such that each edge is colored red or white. A *Canadian path* in a Canadian graph G is a path P whose edges alternate between red and white; that is, no two consecutive edges in P are both red or both white. A *Canadian Hamiltonian path* in G is a path that is both Canadian and Hamiltonian path.

   (a) Prove that the problem of deciding whether G contains a Canadian Hamiltonian path is NP-complete.

   (b) (Harder) In the opposite direction, reduce the Canadian Hamiltonian path decision problem to the Hamiltonian path problem.

7. Two graphs are *isomorphic* if one can be transformed into the other by relabeling the vertices. Consider the following decision problems:

   - Graph Isomorphism: Given two graphs G and H, determine whether G and H are isomorphic.
   - Even Graph Isomorphism: Given two graphs G and H, such that every vertex in G and H has even degree, determine whether G and H are isomorphic.
   - Subgraph Isomorphism: Given two graphs G and H, determine whether G is isomorphic to a subgraph of H.

   (a) Describe a polynomial-time reduction from Graph Isomorphism to Even Graph Isomorphism.

(b) Describe a polynomial-time reduction from Graph Isomorphism to Subgraph Isomorphism.

(c) Prove that Subgraph Isomorphism is NP-Complete by reducing from Clique.