CS 473: Undergraduate Algorithms, Spring 2009
Homework 7

Due Tuesday, April 14, 2009 at 11:59:59pm.

- Groups of up to three students may submit a single, common solution for this and all future homeworks. Please clearly write every group member's name and NetID on every page of your submission.

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1. A graph is *bipartite* if its vertices can be colored black or white such that every edge joins vertices of two different colors. A graph is *d-regular* if every vertex has degree d. A *matching* in a graph is a subset of the edges with no common endpoints; a matching is *perfect* if it touches every vertex.

   (a) Prove that every regular bipartite graph contains a perfect matching.

   (b) Prove that every d-regular bipartite graph is the union of d perfect matchings.

2. Let $G = (V, E)$ be a directed graph where for each vertex v, the in-degree of v and out-degree of v are equal. Let u and v be two vertices G, and suppose G contains k edge-disjoint paths from u to v. Under these conditions, must G also contain k edge-disjoint paths from v to u? Give a proof or a counterexample with explanation.

3. A flow f is called ***acyclic*** if the subgraph of directed edges with positive flow contains no directed cycles.

   (a) A *path flow* assigns positive values only to the edges of one simple directed path from s to t. Prove that every acyclic flow can be written as the sum of a finite number of path flows.

   (b) Describe a flow in a directed graph that *cannot* be written as the sum of path flows.

   (c) A *cycle flow* assigns positive values only to the edges of one simple directed cycle. Prove that every flow can be written as the sum of a finite number of path flows and cycle flows.

   (d) Prove that for any flow f, there is an acyclic flow with the same value as f. (In particular, this implies that some maximum flow is acyclic.)