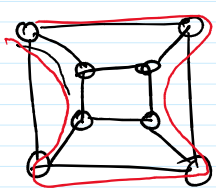
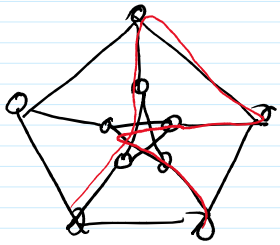
ex 6 Directed-Hamiltonian-Cycle

Given dir graph $G=(V,E)$,
decide ∃ cycle visiting every vertex once.



Yes



no

∃ ... st. ...

$\leq_p$

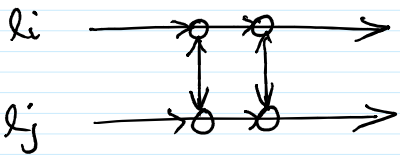
Vertex Cover polytime-reduces to Dir.-Hamiltonian-Cycle

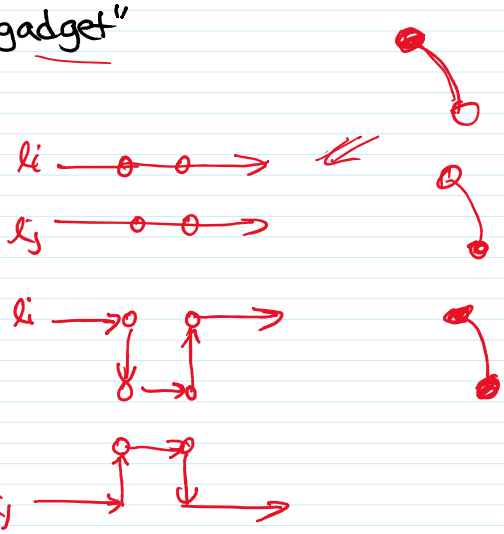
Given undir. graph $G=(V,E)$, K, $V=\{v_1,..,v_n\}$

construct new dir. graph G':

for each $v_i \in V$, create "line" ℓ_i

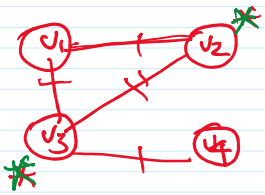
for each $v_i v_j \in E$, add "gadget"





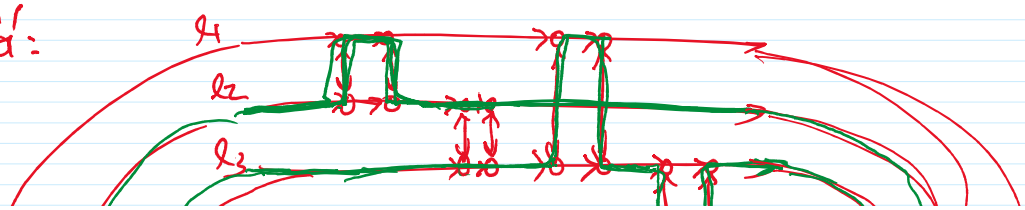
add K extra vertices $z_1, .., z_K$
connecting to start & end of each line ℓ

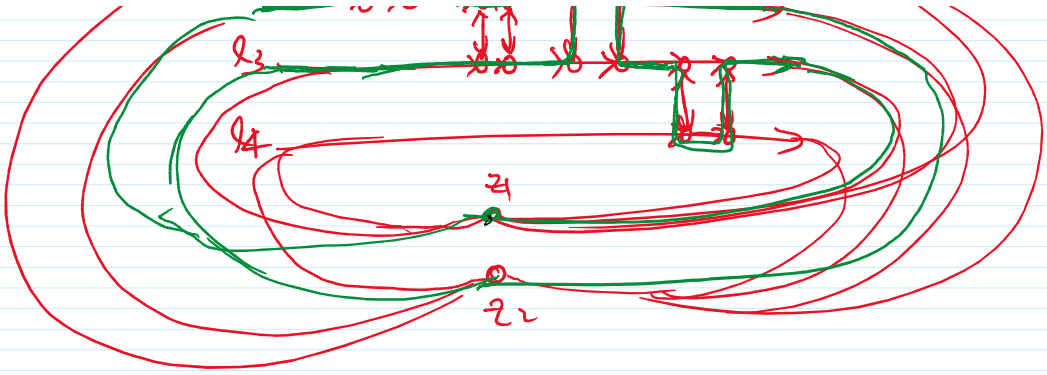
e.g. G:



$K=2$

G':





Correctness: $\exists$ vertex cover of size k in G
$\iff$ $\exists$ Ham cycle in G'.

⋮

reductions can also prove polynomial conditional LBs!
↳ "fine-grained complexity"

Ex diameter
Given unweighted undir graph $G=(V,E)$,
compute $D = \max_{u,v \in E} \delta_G(u,v)$
where $\delta_G(u,v)$ = shortest path length between u & v.

Known: $O(mn)$ time by n BFSs
$O(n^{2.81})$ time ...
for sparse graphs $m = O(n)$; $\Rightarrow$ $O(n^2)$ time

Thm (Roditty-Vassilevska W. '13)
If diameter could be solved in $O(mn^{0.99})$ time for sparse graphs,
then k-SAT could be solved in $O(1.999^n)$ time for any const k.
↑ "Strong Exponential-Time Hypothesis"

"Strong Exponential-Time Hypothesis"

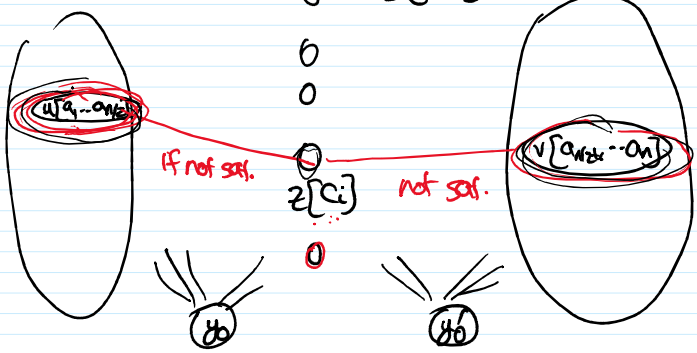
Pf: Reduce k-SAT to diameter:

Given k-CNF formula F with vars $x_1, \ldots, x_n$,
& clauses $C_1, \ldots, C_m$, $m = O(n^k)$

Construct graph G:

vertices $O(2^{n/2})$

- for each $a_1, \ldots, a_{n/2} \in \{0,1\}$, create vertex $u[a_1 \cdots a_{n/2}]$
- for each $a_{n/2+1}, \ldots, a_n \in \{0,1\}$, create vertex $v[a_{n/2+1} \cdots a_n]$
- for each clause C_i, create vertex $z[C_i]$
- if C_i is not satisfied by $x_1 = a_1, \ldots, x_{n/2} = a_{n/2}$
  add edge $u[a_1 \ldots a_{n/2}]\ z[C_i]$
- if C_i is not satisfied by $x_{n/2+1} = a_{n/2+1}, \ldots, x_n = a_n$
  add edge $z[C_i]\ v[a_{n/2+1} \cdots a_n]$



- add extra vertices y_0, y_0'

Correctness: diam of $G > 2$

iff $\exists\ a_1, \ldots, a_{n/2}, a_{n/2+1}, \ldots, a_n$ s.t.
$\forall C_i$, $u[a_1, \ldots, a_{n/2}]\ z[C_i]$ not edge
or $z[C_i]\ v[a_{n/2} \cdots a_n]$ not edge

C_i satisfied by $x_1 = a_1, \ldots, x_{n/2} = a_{n/2}$
or C_i satisfied by $x_{n/2+1} = a_{n/2+1}, \ldots, x_n = a_n$

$\Rightarrow$ C_i sat.

iff $\exists$ sat. assignment.

So runtime to $O(M N^{0.99})$

$N = O(2^{n/2})$
$M = O(2^{n/2} n^k)$

So runtime to $O(MN^{0.99})$

$N = O(2^{n/2})$

$M = O(2^{n/2} n^k)$

$$= O(2^{n/2} n^k (2^{n/2})^{0.99})$$

$$= O(2^{0.995n} n^k)$$

$$\leq O(1.999^n).$$

□