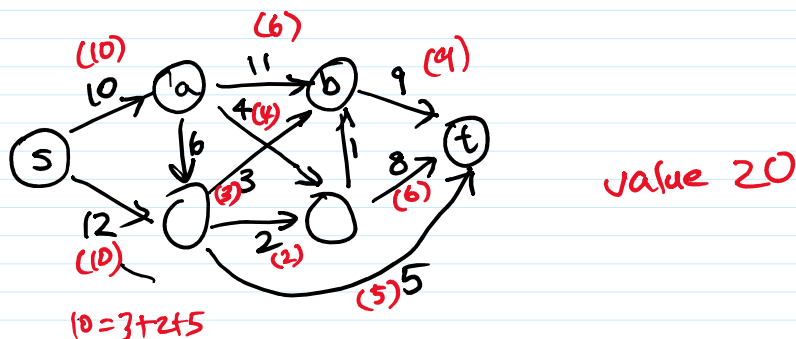


Problem Given directed graph $G=(V,E)$, $s,t \in V$
 where each $e \in E$ has a capacity $c(e) > 0$,
 find flow from s to t with max value



Def A flow is $f: E \rightarrow \mathbb{R}$ s.t.
 $\forall (u,v) \in E, 0 \leq f(u,v) \leq c(u,v)$

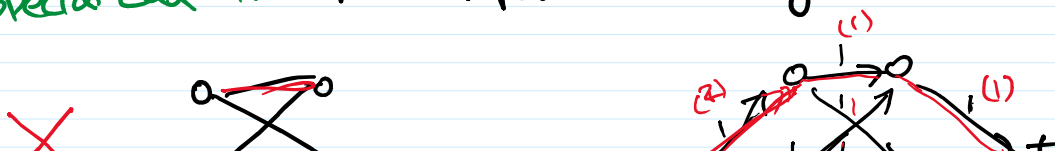
flow conservation $\forall v \in V - \{s,t\}, \sum_{u: (u,v) \in E} f(u,v) = \sum_{w: (v,w) \in E} f(v,w)$

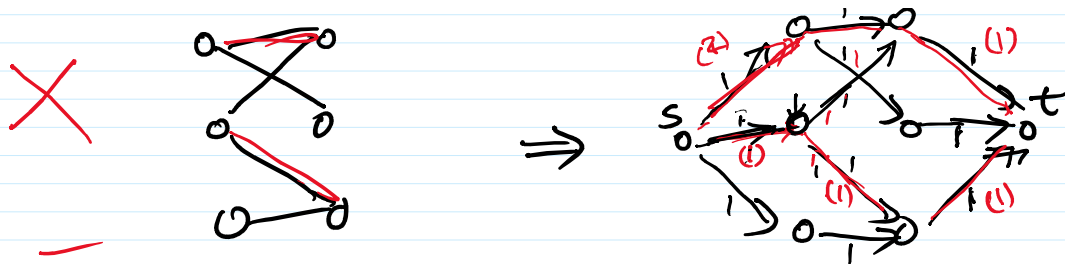
Value of $f = |f| = \sum_{v: (s,v) \in E} f(s,v)$
 $(= \sum_{v: (v,t) \in E} f(v,t))$

Convention: make $f(u,v) = -f(v,u)$ if $(v,u) \in E$
 & $f(u,v) \geq 0$ if $(u,v), (v,u) \notin E$
 flow conservation becomes $\forall v \in V - \{s,t\} \sum_{w \in V} f(v,w) = 0$

Appl's

Special case 1: max bipartite matching





(turns out: opt flow is integral)

Special case 2: assignment problems

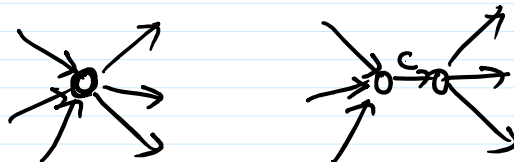
jobs $\leftrightarrow$ people

each person u can be $\leq c_u$ jobs

$\vdots$

Special case 3: find max # of edge-disjoint paths from s to t in dir. graphs

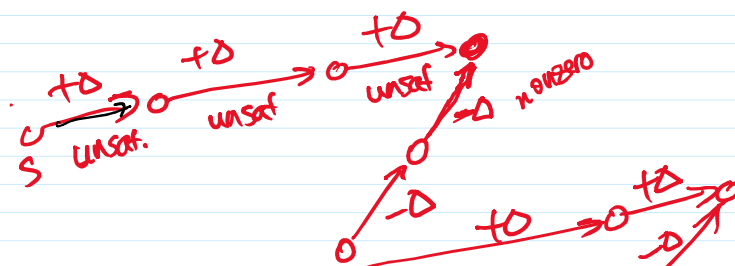
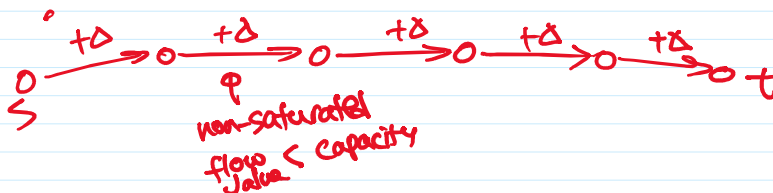
(can also do dir. vertex-disjoint)

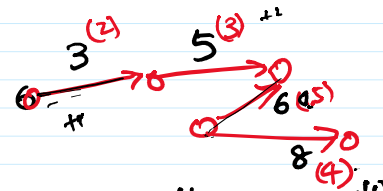
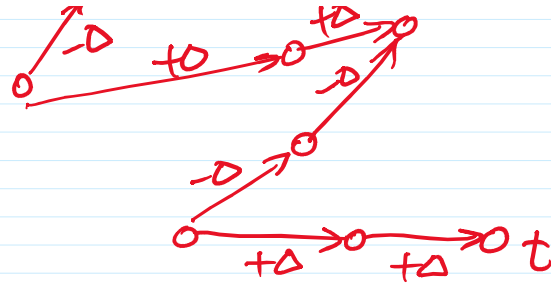


etc.

Ford-Fulkerson Alg'm ('56)

idea - local improvement again





Def Let f be a flow.

Define residual graph G_f :

if $(u, v) \in E$, $f(u, v) < c(u, v)$ ("not saturated")
 add (u, v) in G_f with capacity $c_f(u, v) = c(u, v) - f(u, v)$
 if $(v, u) \in E$, $f(v, u) > 0$ ("not zero")
 add (u, v) in G_f with capacity $c_f(u, v) = f(v, u)$

Alg'm:

$f \neq$ zero flow

repeat {

→ find an s-to-t path p in G_f ←

if not exist, return f

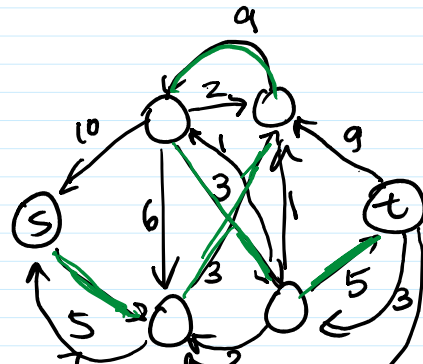
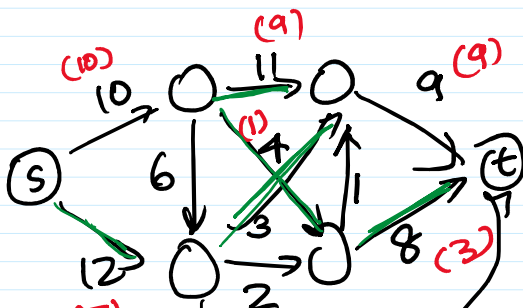
$\Delta =$ min edge capacity in p in G_f

← called bottleneck value

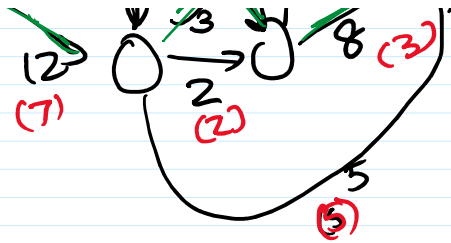
$\forall (u, v) \in p$, $f(u, v) \leftarrow f(u, v) + \Delta$
 $f(v, u) \leftarrow f(v, u) - \Delta$

}

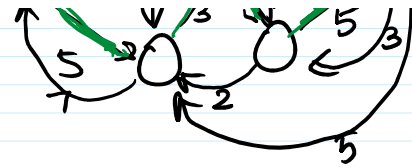
Ex



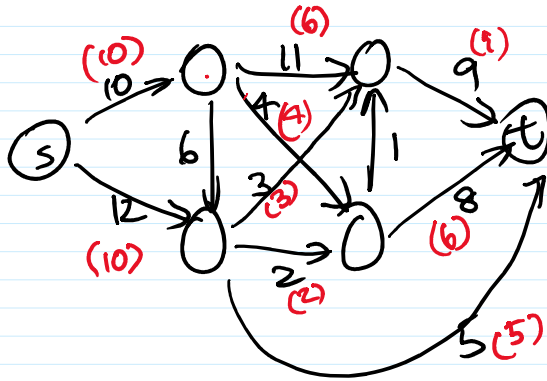
f_i :



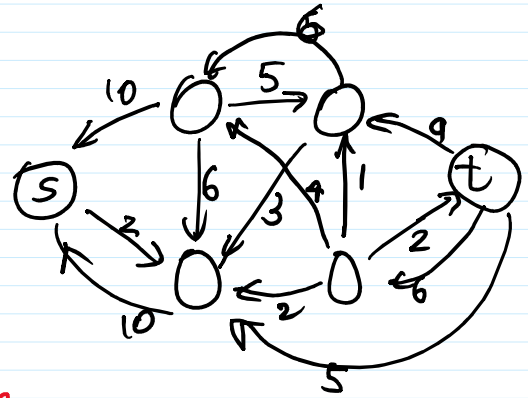
G_f :



set $\Delta = 3$



G_f



no path
done