

rand. analysis:

rewrite algm:

$\min(S)$: // backwards version

0. if $S = \emptyset$ return ∞
1. pick $x \in S$ randomly
2. $\text{ans} = \min(S - \{x\})$
3. if $x < \text{ans}$ then
4. $\text{ans} = x$ (*)
5. return ans

for any fixed S ,

$$\begin{aligned} \Pr[(*) \text{ is done}] &= \Pr[x = \min(S)] \\ &= \frac{1}{n} \end{aligned} \quad \leftarrow$$

5, 3, 1, 2, 4
 ↑
 no

5, 2, 3, 4, 1
 ↑
 yes

Expected total # of times (*) is done

$$T(n) = T(n-1) + \frac{1}{n} \cdot 1 + \left(1 - \frac{1}{n}\right) \cdot 0$$

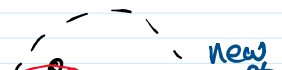
$$\Rightarrow T(n) = T(n-1) + \frac{1}{n}$$

$$\begin{aligned} \Rightarrow T(n) &= \frac{1}{n} + \frac{1}{n-1} + \frac{1}{n-2} + \dots + \frac{1}{2} + \frac{1}{1} \\ &= \boxed{O(\log n)} \quad \leftarrow \text{Harmonic number} \end{aligned}$$

back to 2D min enclos. circle ...

Seidel-Welzl's Las Vegas algm ('91)

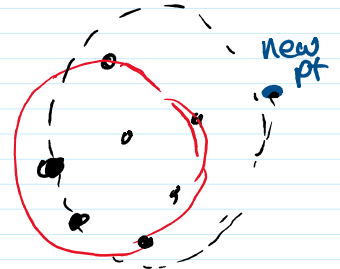
idea - incrementally add pts



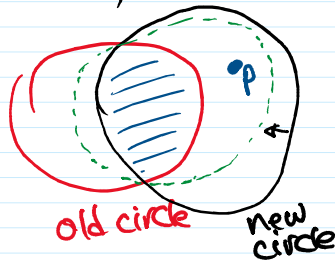
idea - incrementally add pts

If p is inside current circle, unchanged ✓

else new circle must have p on boundary



Pf Sketch: by contradiction.



idea - insert in RAND ORDER

circle(S, B): // backwards version

// find min circle enclosing S with extra constraint that boundary passes thru pts in B

0. if $S = \emptyset$ or $|B|=3$ return ...

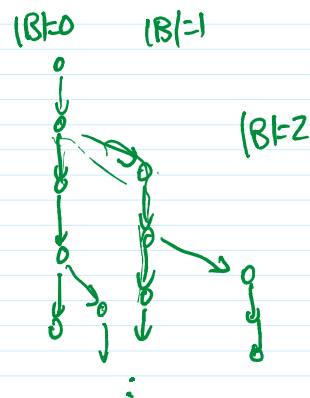
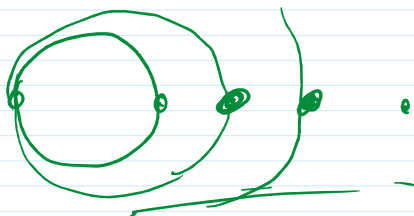
1. pick $p \in S$ randomly

2. $C = \text{circle}(S - \{p\}, B)$

3. if p outside C then

4. $C = \text{circle}(S - \{p\}, B \cup \{p\})$ (*)

5. return C



Rand Analysis:

... $|B|=n$ $|B|=h$

Rand Analysis:

let $T_b(n)$ = expected runtime for $|S|=n$, $|B|=b$.
 $b \in \{0, 1, 2, 3\}$

for any fixed S, B ,

$$\Pr\{(*) \text{ is done}\} = \Pr\{p \text{ is on boundary of circle}(S, B)\}.$$

$\nearrow$ outside circle($S - \{p\}, B$)



$$\leq \frac{3}{n}$$

$$T_b(n) \leq T_b(n-1) + O(1) + \frac{3}{n} \cdot T_{b+1}(n-1)$$

base case: $T_3(n) = O(1)$.

$$\Rightarrow T_2(n) \leq T_2(n-1) + O(1) \Rightarrow T_2(n) = O(n)$$

$$\Rightarrow T_1(n) \leq T_1(n-1) + O(1) + \frac{3}{n} \cdot O(n)$$

$O(1)$

$$\Rightarrow T_1(n) = O(n)$$

$$\Rightarrow T_0(n) \leq T_0(n-1) + O(1) + \frac{3}{n} \cdot O(n)$$

$$\Rightarrow T_0(n) = O(n)$$

expected time

Rmk.: in d dims, $O((d+1)!n)$ expected time

det: $2^d n$
 $? d^2 n$
 Megiddo

rand:
 $d^2 n + 2^{O(\sqrt{\log d})}$

$$2^n n \text{ megados}$$

$$3d^2 n$$

$$ddiz_n$$

$$d^2 n + 2^{O(\log n)}$$

Problem

Given n numbers,

find median $((n/2)^{\text{th}}$ smallest)

more generally, select k^{th} smallest

81, 42, 61, 50, 83, 4, 21, 95, 25

Sorting $1 n \log n$ comps.

Known deterministic alg's:

Blum, Floyd, Pratt, Rivest, Tarjan '73

$O(n)$ time

"median-of-medians-of-5"

$\sim 16n$ comparisons

$\sim 5.4n$ "

Schonhage, Paterson Pippenger '76 $\sim 3n$

Dor, Zwick '95 $\sim 2.95n$

Lowerbd $> (2+\epsilon)n$

Randomized alg's?

Method 1: ^{rand.} quickselect (variant of quicksort)

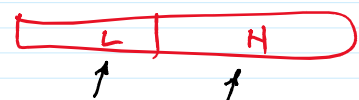
select($\{a_1, \dots, a_n\}, k$): // return k^{th} smallest
($k \in \{1, \dots, n\}$)

1. if $n=1$ return a_1

2. pick "pivot" x AT RANDOM from $\{a_1, \dots, a_n\}$

3. $L = \{a_i: a_i \leq x\}$

$H = \{a_i: a_i > x\}$



4. if $k \leq |L|$ then select(L, k)

also select($H, k - |L|$)

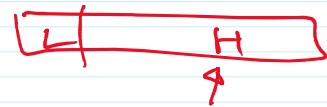
4. if $k \leq |L|$ then select (L, k)
 else select $(H, k - |L|)$

Worst-case:

$|L| = 1$ or $n-1$

$$\Rightarrow T(n) \leq T(n-1) + O(n)$$

$$\Rightarrow O(n^2)$$

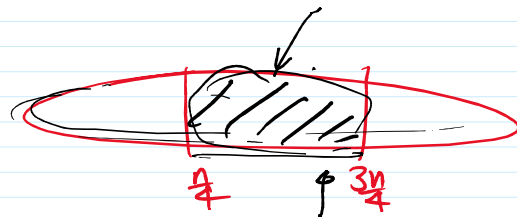


rand. analysis:

$|L|$ is equally likely to be $1, 2, \dots, n$

$$\Pr\left[\frac{n}{4} \leq |L| \leq \frac{3n}{4}\right] \geq \frac{1}{2}$$

ideally
 $T(n) = T(\frac{n}{2}) + O(n)$
 $\Rightarrow O(n)$



$$\Rightarrow T(n) \leq T\left(\frac{3}{4}n\right) + 2 \times O(n)$$

$$\Rightarrow T(n) \leq O\left(2\left(1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \dots\right)n\right)$$

$$= \boxed{O(n)} \text{ expected}$$

with ^{more} careful analysis,
 (expected # comps $\leq 3.4n$).