CS 473 (Fall 2026)
Homework 8 (due Nov 12 Thu 10am)

Instructions: Carefully read `http://courses.grainger.illinois.edu/cs473/fa2026/policies.html#hw` and `http://courses.grainger.illinois.edu/cs473/fa2026/integrity.html` (in particular, remember to include the 3 disclosure statements, on collaborators, outside sources, and AI use, for each problem).

Problem 8.1: We are given two sets P and S of points in 2D, with $|P| + |S| = n$. The points in S represent light sources. If a light source $s \in S$ has intensity x_s, then a point $p \in P$ receives $\frac{x_s}{d(p,s)^2}$ amount of light from s, where $d(p, s)$ denotes the Euclidean distance between p and s. (Intensities fade quadratically as a function of distance.)

(a) (25 pts) Consider the problem of setting the intensities at the light sources in S, so that every point of P receives a total amount of light at least 1, while minimizing the sum of the intensities at all the light sources. Formulate this problem as a linear program. Let $f^*(P, S)$ be the optimal value of this minimization problem.

(b) (25 pts) Consider the problem of setting the intensities at the light sources in S, so that every point P receives a total amount of light at most 1, while maximizing the sum of the intensities at all the light sources. Formulate this problem as a linear program. Let $g^*(P, S)$ be the optimal value of this maximization problem.

(c) (25 pts) Prove that $f^*(P, S) = g^*(S, P)$.

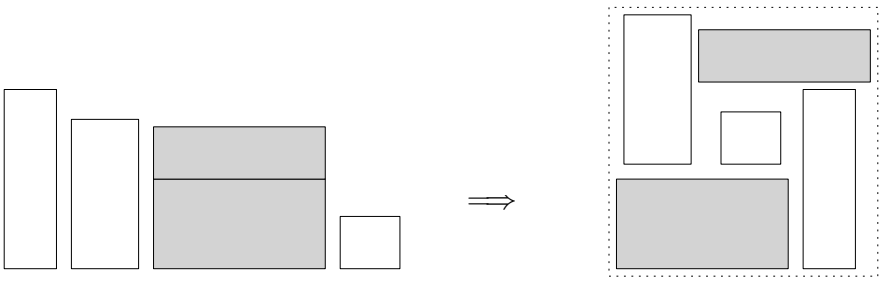
(d) (25 pts) Suppose we are also given a value B. Consider the problem of setting the intensities at the light sources in S, so that the sum of the intensities at all the light sources is at most B, while maximizing $\sum_{p \in P} \min\{w(p), 1\}$, where $w(p)$ denotes the total amount of light received by p. Formulate this problem as a linear program.

Problem 8.2: We are given n rectangles $r_1, \ldots, r_n$ where r_i has width w_i and height h_i. We want to determine whether it is possible to pack the rectangles inside a rectangular container of width W and H. We are not allowed to rotate the rectangles, but we are allowed to cut each rectangle *horizontally* (with no limits on how many horizontal cuts per rectangles).

Assuming that $\min_i w_i > 0.1W$, solve this problem using linear programming. The number of variables and constraints should be polynomial in n.

(For example: In the figure in the next page, the gray rectangle is cut horizontally into two subrectangles.)

(Hint: use the $\min_i w_i > 0.1W$ condition to bound the number of possible "configurations" at any horizontal line in the container.)



Optional Problem 8.3 (ungraded):

(a) (80 pts) Run the simplex method on the following linear program:

$$\begin{array}{lrcl} \text{maximize} & x_1 + x_2 & & \\ \text{s.t.} & 3x_1 + 2x_2 & \leq & 27 \\ & x_2 & \leq & 8 \\ & x_1 + 5x_2 & \leq & 35 \\ & 2x_1 + x_2 & \leq & 17 \\ & x_1, x_2 & \geq & 0. \end{array}$$

Start with the initial basic solution $(\overline{x}_1, \overline{x}_2) = (0, 0)$, and *choose x_1 as the entering variable in the first iteration*. Show the new slack form after every iteration.

(b) (20 pts) Write down the dual of the linear program from (a). What is the optimal dual solution?