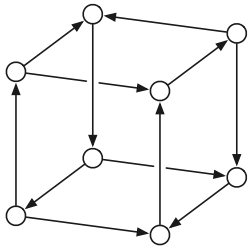
CS 473 ✧ Fall 2024

Homework 9

Due Tuesday, November 19, 2024 at 9pm Central Time

---

1. A ***k*-orientation** of an undirected graph G is an assignment of directions to the edges of G so that every vertex of G has at most k incoming edges. For example, the figure below shows a 2-orientation of the graph of the cube.



Describe and analyze an algorithm that determines the smallest value of k such that G has a k-orientation, given the undirected graph G as input. Equivalently, your algorithm should find an orientation of the edges of G such that the maximum in-degree is as small as possible. For example, given the cube graph as input, your algorithm should return the integer 2.

2. The Faculty Senate at Sham-Poobanana University has decided to convene a committee to determine once and for all whether Alma Otter, Pinto Bean, or the Belted Kingfisher should be officially adopted as the new official ~~mascot~~ *symbol* of SPU's athletic teams, The Fighting Pooh-Bahs. (After all, it has been almost two decades since the previous ~~mascot~~ *symbol* Baron Factotum was officially retired.) Exactly one faculty member must be chosen from each academic department to serve on this committee. Some faculty members have appointments in multiple departments, but each committee member can represent only one department. For example, if Prof. Blagojevich is affiliated with both the Department of Corruption and the Department of Stupidity, and he is chosen as the Stupidity representative, then someone else must represent Corruption. Finally, University policy requires that every faculty committee must contain exactly the same number of assistant professors, associate professors, and full professors. Fortunately, the number of departments is a multiple of 3.

   Describe and analyze an algorithm to choose a subset of the SPU faculty to fill the Post-Factotum ~~Mascot~~ *Symbol* Committee, or correctly report that no valid committee is possible. Your input is a bipartite graph indicating which professors belong to which departments; each professor vertex is labeled with that professor's rank (assistant, associate, or full). Assume that there are n professors and $3k$ departments.

3. Given points $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ in the plane, the ***linear regression problem*** asks for real numbers a and b such that the line $y = ax + b$ fits the points as closely as possible, according to some criterion. The most common fit criterion is minimizing the ***L_2 error***, defined as follows:[1]

$$\varepsilon_2(a, b) = \sum_{i=1}^{n} (y_i - ax_i - b)^2.$$

But there are many other ways of measuring a line's fit to a set of points, some of which can be optimized via linear programming.

(a) The ***L_1 error*** (or *total absolute deviation*) of the line $y = ax + b$ is defined as follows:

$$\varepsilon_1(a, b) = \sum_{i=1}^{n} \left| y_i - ax_i - b \right|.$$

Describe a linear program whose solution (a, b) describes the line with minimum L_1 error.

(b) The ***L_∞ error*** (or *maximum absolute deviation*) of the line $y = ax + b$ is defined as follows:

$$\varepsilon_\infty(a, b) = \max_{i=1}^{n} \left| y_i - ax_i - b \right|.$$

Describe a linear program whose solution (a, b) describes the line with minimum L_∞ error.

[1] This measure is also known as *sum of squared residuals*, and the algorithm to compute the best fit is normally called *(ordinary/linear) least squares*.