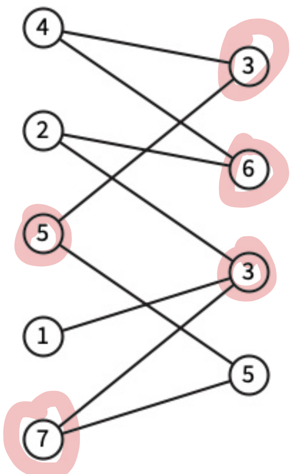
Next Tuesday — university holiday — GO VOTE!

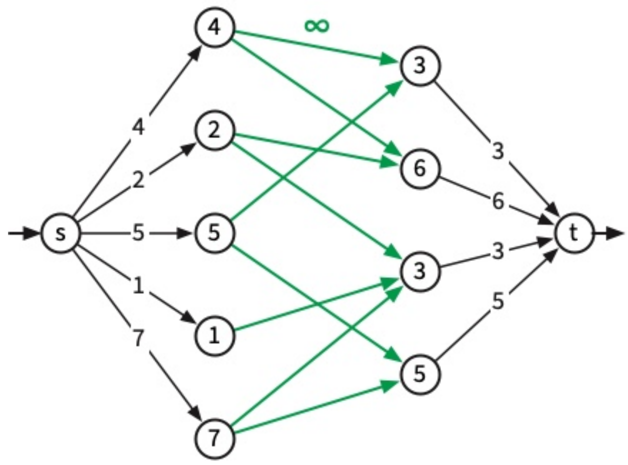
HW 9 will be last graded HW

Bipartite Min Vertex Cover

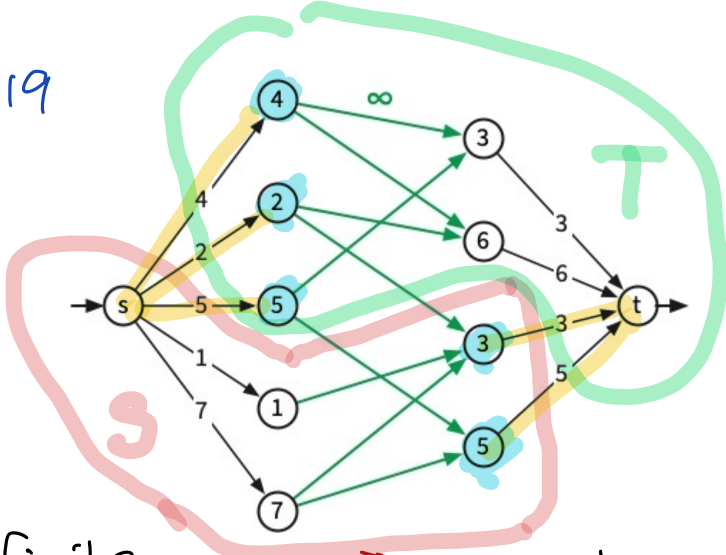
Vertex cover = subset of vertices that touch every edge

NP-hard for non-bipartite graphs





$\|S,T\| = 4+2+5+3+5 = 19$



Cuts with finite capacity $\|S,T\|$ $\rightleftarrows$ Vertex covers

$$\$(C) = \sum_{v \in C} \$(v)$$

$L \cap T$ $\quad$ $R \cap T$

$L \cap S$ $\quad$ $R \cap S$

$C = (L \cap T) \cup (R \cap S)$

$S = (L \setminus C) \cup (R \cap C) \cup \{s\}$

$T = V \setminus S$

Compute min cut $\|S,T\|$ in $\boxed{O(VE)\text{ time}}$ [Orlin]

Project selection

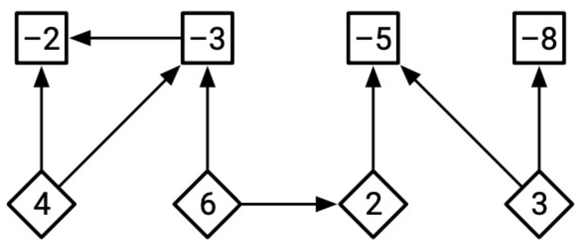
Input is DAG

V = projects
E = dependencies

value $\$(v)$ for every vertex v

$u \to v$
u can only be done after v.

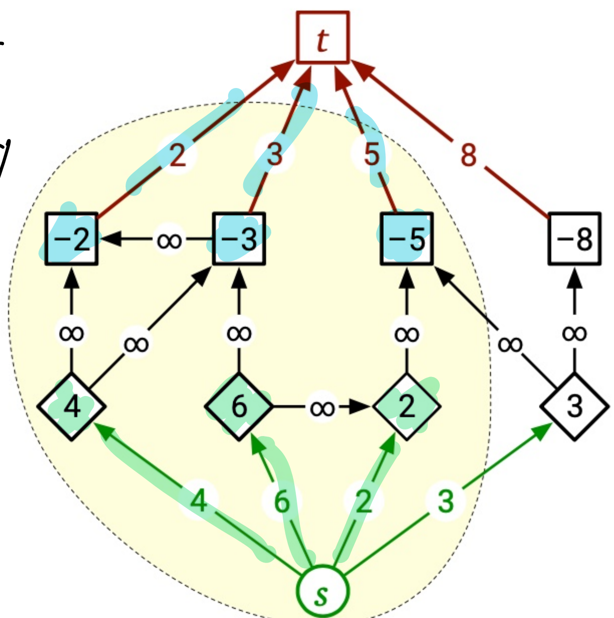
Output: $S \subseteq V$ max $\$(S)$



Any (s,t)-cut S,T with finite capacity

$\Updownarrow$

Valid selection $S \setminus \{s\}$



Claim: $\$(S-s) = P - \|S,T\|$ where $P = \sum_{\$(v)>0} \$(v)$

$P = \$(S-s) + \|S,T\|$

For any $X \subseteq V$: $\quad \text{cost}(X) = \sum_{\substack{\$(v)<0 \\ v\in X}} -\$(v) = \sum_{v \in X} c(v \to t)$

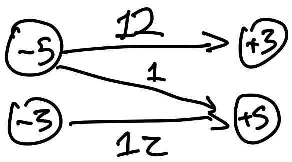
$\text{income}(X) = \sum_{\substack{v\in X \\ \$(v)>0}} \$(v) = \sum_{v\in X} c(s \to v)$

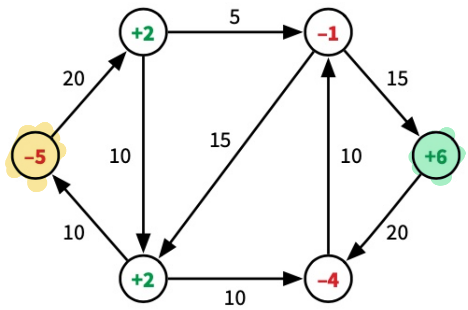
$$\text{profit}(X) = \text{income}(X) - \text{cost}(X) = \sum_{v \in X} \$(v)$$

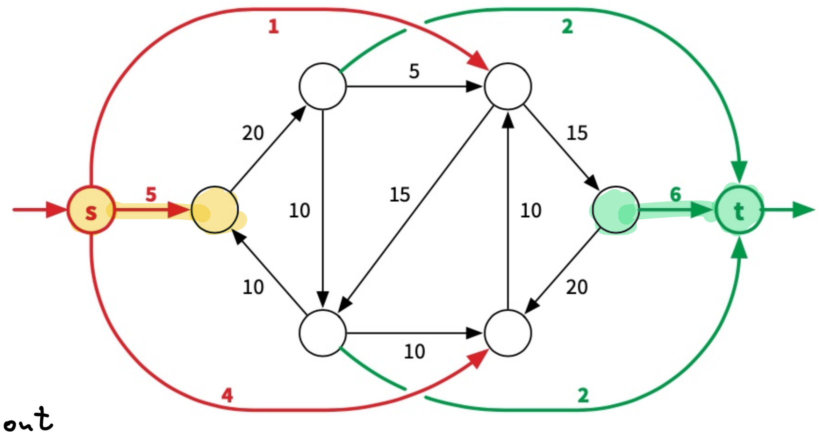
$$P = \text{income}(V) = \text{income}(S) + \text{income}(T)$$

$$\|S,T\| = \text{cost}(S) + \text{income}(T)$$

$$P - \|S,T\| = \text{income}(S) - \text{cost}(S)$$







b(v) – balance = Flow in – Flow out

Is there a feasible flow?

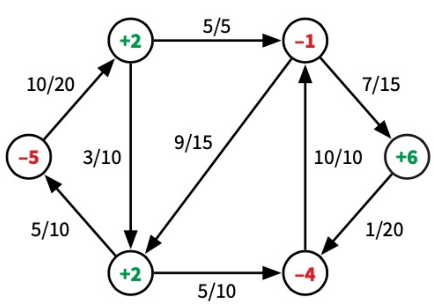
Necessary: $\sum_v b(v) = 0$

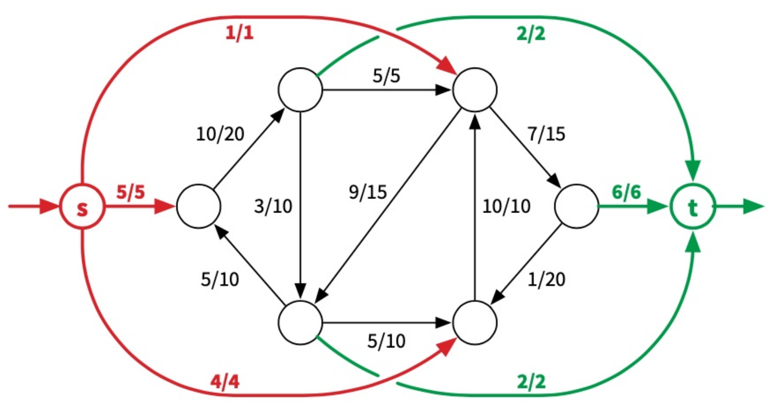
Feasible flow f in G

$\downarrow\uparrow$

Feasible flow f' in G'
with value

$\sum_{b(v)>0} b(v)$ — saturates all edges from s



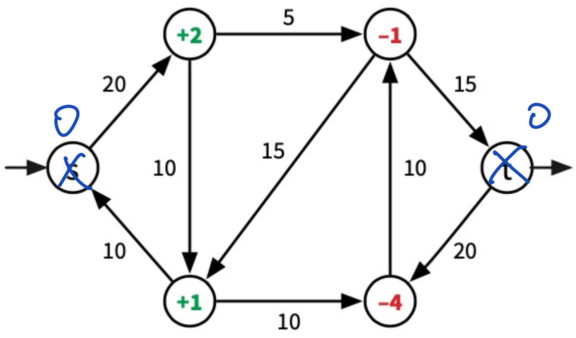


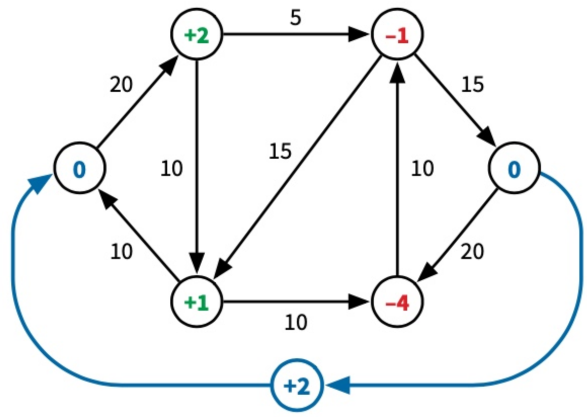
Maximum flow in network with nonzero balances

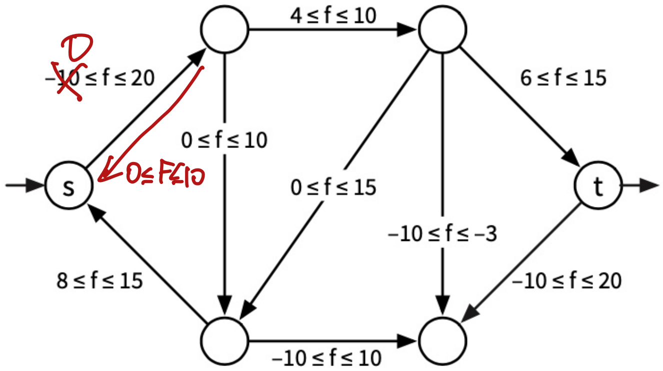
① Feasible flow $F \rightarrow$ max flow F' in G'

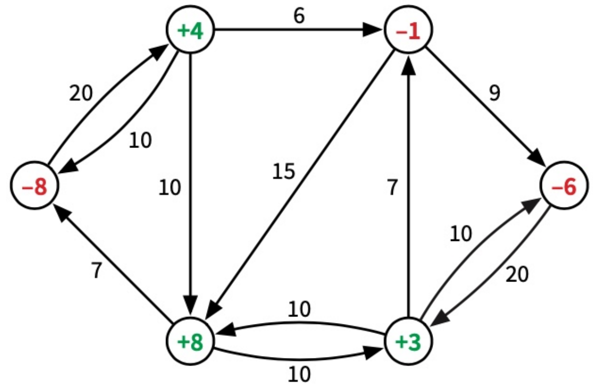
② Maximize it $\rightarrow$ max flow F'' in G_F

return $F' + F''$






4 ≤ f ≤ 10
−10 ≤ f ≤ 20
0
0≤f≤10
0 ≤ f ≤ 10
0 ≤ f ≤ 15
6 ≤ f ≤ 15
s
t
−10 ≤ f ≤ −3
8 ≤ f ≤ 15
−10 ≤ f ≤ 20
−10 ≤ f ≤ 10


+4
−1
−8
−6
+8
+3
6
20
10
9
10
15
7
10
20
7
10
10