cs473: Algorithms	Assigned: *Tue., Aug. 27, 2019*
	Problem Set #0
Prof. Michael A. Forbes Prof. Chandra Chekuri	Due: *Wed., Sep. 4, 2019 (10:00am)*

Some reminders about logistics.

- **Submission Policy:** See the course webpage for how to submit your pset via gradescope.
- **Collaboration Policy:** For *this* problem set, each student must work independently and submit their *own* solutions. For the *other* problem sets, this rule will not apply. See the course webpage for more details.
- **Late Policy:** Late psets are not accepted. Instead, we will drop several of your lowest pset problem scores; see the course webpage for more details.

All problems are of equal value.

1. Solve the following recurrences in the sense of giving an asymptotically tight bound of the form $\Theta(f(n))$ where $f(n)$ is a standard and well-known function. No proof necessary for the first five parts; simply state the bound.

   (a) $A(n) = n^{1/4}A(n^{3/4}) + n$, $A(n) = 1$ for $1 \leq n \leq 16$.

   (b) $B(n) = B(n/4) + \sqrt{n}$, $B(1) = 1$.

   (c) $C(n) = 5C(n-1) + 1$, $C(1) = 0$.

   (d) $D(n) = 3D(n/3) + 4D(n/4) + n^3$, $D(n) = 1$ for $n \leq 4$.

   (e) $E(n) = 2E(\sqrt{n}) + \log n$, $E(n) = 1$ for $n < 4$.

   (f) *Prove* by induction that the $T(n)$ defined by the recurrence

   $$T(n) = n + \frac{1}{n}\sum_{i=1}^{n-1}(T(i) + T(n-i-1))$$

   with $T(n) = 1$ for $n \leq 2$ satisfies the bound $T(n) = O(n \log n)$.

2. Problem 18 from Jeff Erickson's algorithms book. See page 215 in http://jeffe.cs.illinois.edu/teaching/algorithms/book/05-graphs.pdf.

3. Consider the standard balls and bins process. A collection of m identical balls are thrown into n bins: each ball is thrown independently into a bin chosen uniformly at random.

   (a) What is the (precise) probability that a particular bin i contains exactly k balls at the end of the experiment?

   (b) Suppose $m = n$. Let Y be the number of bins that are empty. What is the expectation of Y?

   (c) What is the variance of Y?

   Explain your calculations when you derive the bounds.