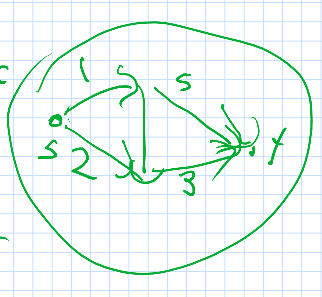
0/30/18 | Network flow

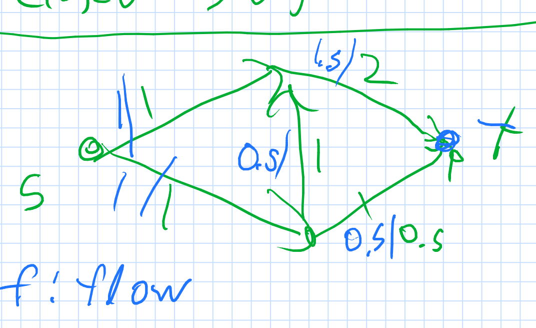
Network

G: directed graph
s: source
t: target/sink

Electric flow
water flow

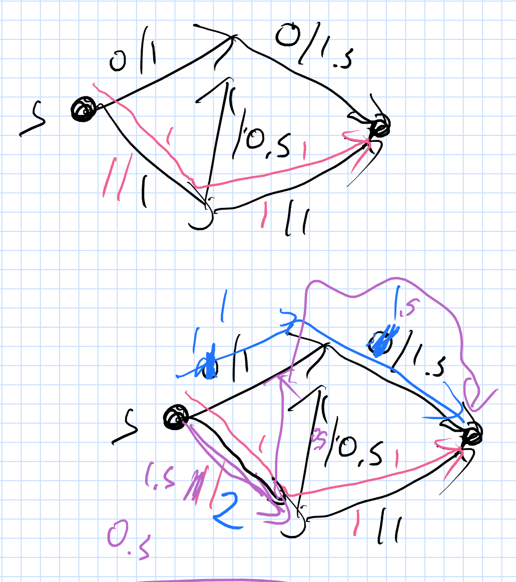


$\forall e \in E(G)$
$c(e) > 0$
$c(e) = 0 \Rightarrow$ edge is not in the graph



f: flow
$f: E \to R$
$\forall e \in E \quad f(e) \le c(e)$
- Preservation of flow
  $\forall v \in V \setminus \{s,t\}$
  all flow entering v leaves v.
- anti symmetry
  $u \to v \in E \Rightarrow$
  $f(u \to v) = -f(v \to u)$
  $u \xrightarrow{1/7} v$

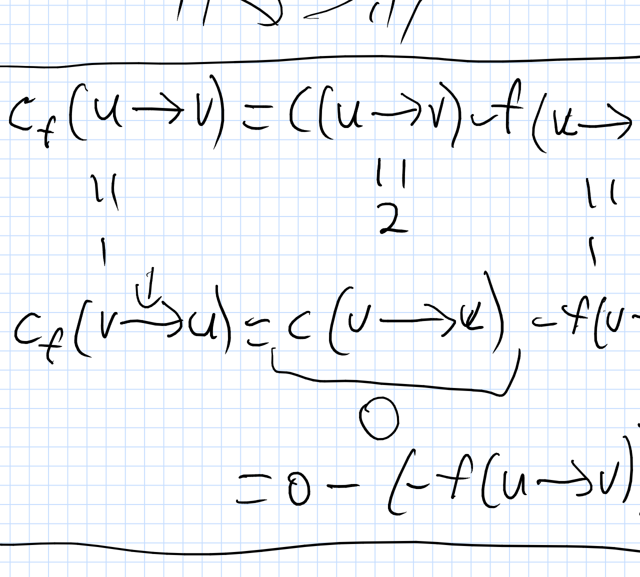
Compute the maximum flow



Ford Fulkerson method

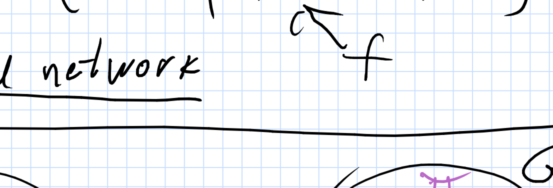
Residual capacity — f current flow
e: edge

$c_f(e) = c(e) - f(e)$



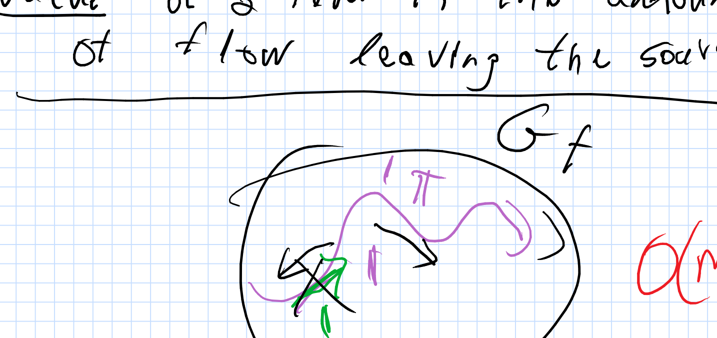
$c_f(u \to v) = c(u \to v) - f(u \to v)$

$c_f(v \to u) = c(v \to u) - f(v \to u)$
$= 0 - (-f(u \to v)) = f(u,v)$



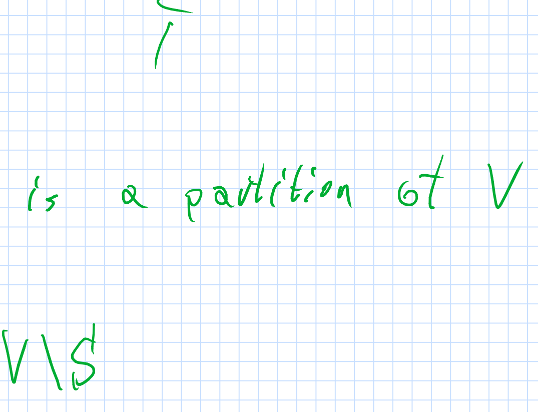
Residual network
$G = (V, E)$, s, t, c, f (flow)

$G_f = (V, \{u \to v \mid c_f(u \to v) > 0\})$
Residual network

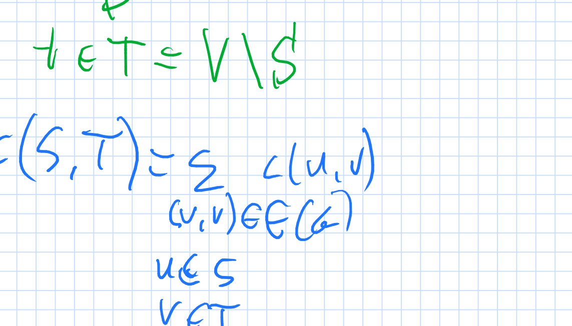


$c_f(e)$
Augmenting path

Value of a flow is the amount of flow leaving the source



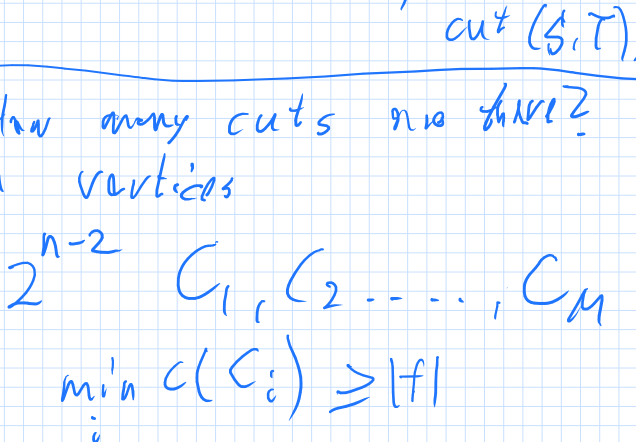
$O(m)$



Cut (S,T) is a partition of V
$s \in S$
$t \in T = V \setminus S$

$c(S,T) = \sum_{\substack{(u,v) \in E(G) \\ u \in S \\ v \in T}} c(u,v)$

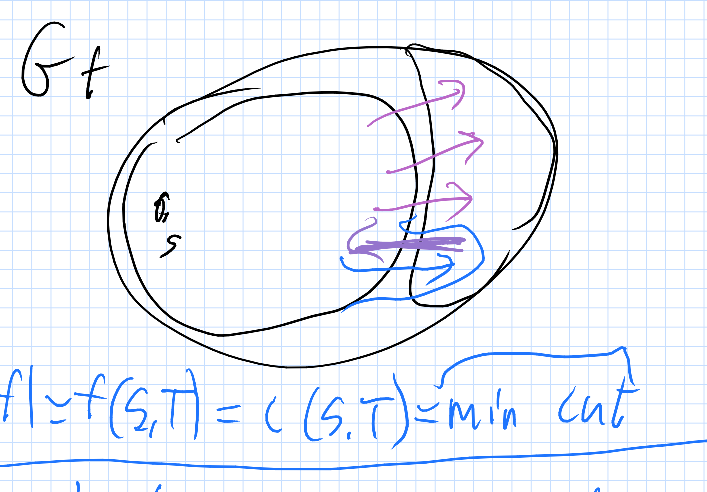
All edges of (S,T) are fully utilized.

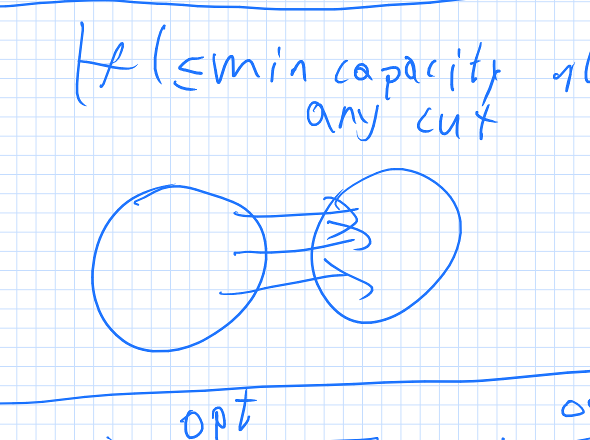


value of flow — 2

Claim $|f| \le c(S,T)$ for any cut (S,T).

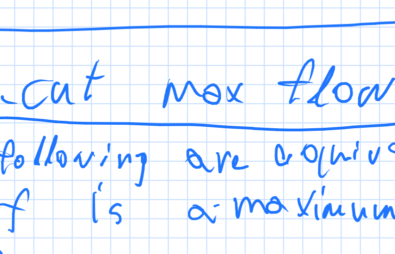
How many cuts we have?
n vertices
2^{n-2} $\quad C_1, C_2, \dots, C_M$
$\min_i c(C_i) \ge |f|$





$|f| = f(S,T) = c(S,T) = \text{min cut}$

$|f| \le$ min capacity of any cut



opt Max flow	=	opt Min cut

Min cut max flow theorem

The following are equivalent
(A) f is a maximum flow
(B) G_f contains no aug paths.
(C) $|f| = c(S,T)$ for some cut (S,T) which is the minimum cut in G.

Duality in Linear Programming

Integrality theorem

If all capacities are integers then there is a maximum flow that is integral.

The running time of Ford F is $O(|f| m)$

Example

max flow
sink
integer