10/25/18

More on matchings

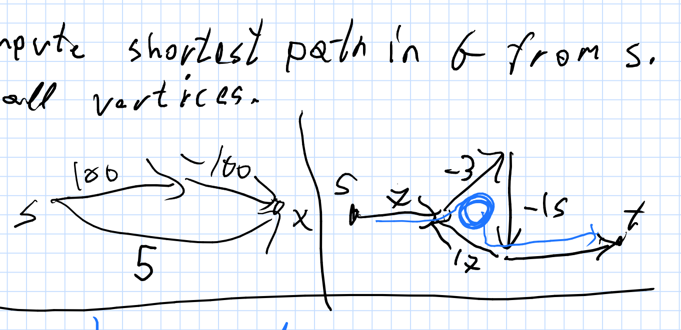
- Bipartite case $O(m)$
  - $O(nm)$
  - $O(m\sqrt{n})$ Hopcroft Karp

- Weighted matching in Bipartite graph
- Unweighted matching in regular graphs.

Weighted bipartite matching

Belman-Ford Algorithm

G: Directed graph n, m
weights on the edges (weights can be negative)
s: source
Q: Compute shortest path in G from s. To all vertices.

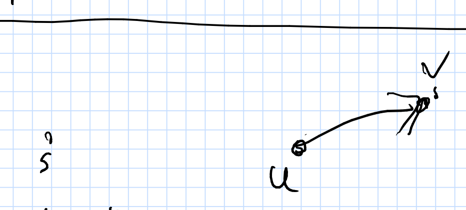


No negative cycles

$d[v]$: shortest distance found so far from s to v

$d[s] \leftarrow 0$

$\forall v \in V,\ v \neq s \quad d[v] = +\infty$



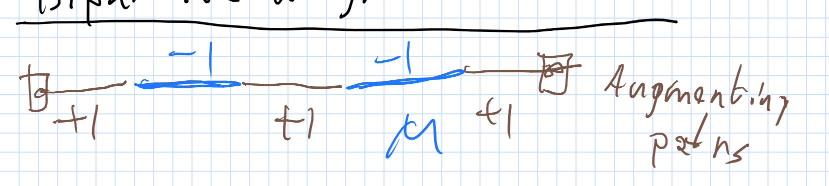
If $d[u] + w(u \to v) < d[v]$ then
  $d[v] \leftarrow d[u] + w(u \to v)$
] Relax$(u \to v)$

tense edge

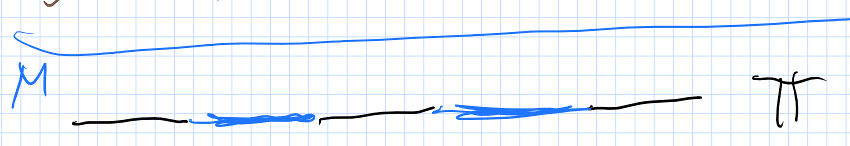
For i=1 to n+1 do ←
  For $e \in E(G)$ do
    relax(e)

$O(nm)$ Running time

Bipartite weighted matching

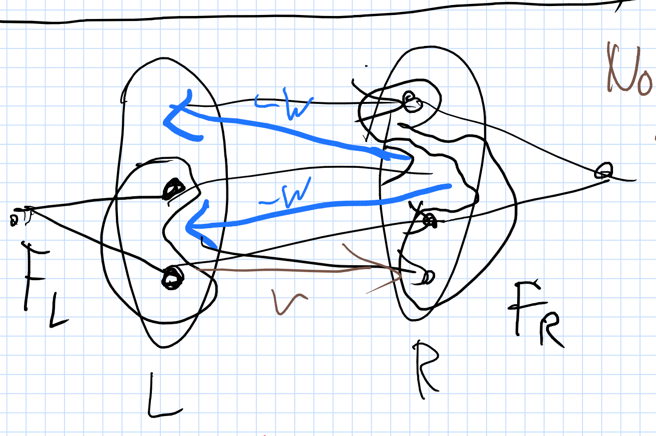


$3-2=1$



$$\partial(\pi, M) = \sum_{e \in \pi \setminus M} w(e) - \sum_{e \in \pi \cap M} w(e)$$

$$w(M \oplus \pi) = w(M) + \partial(\pi, M)$$



No positive cycles

Negate all weights
Use Belman-Ford

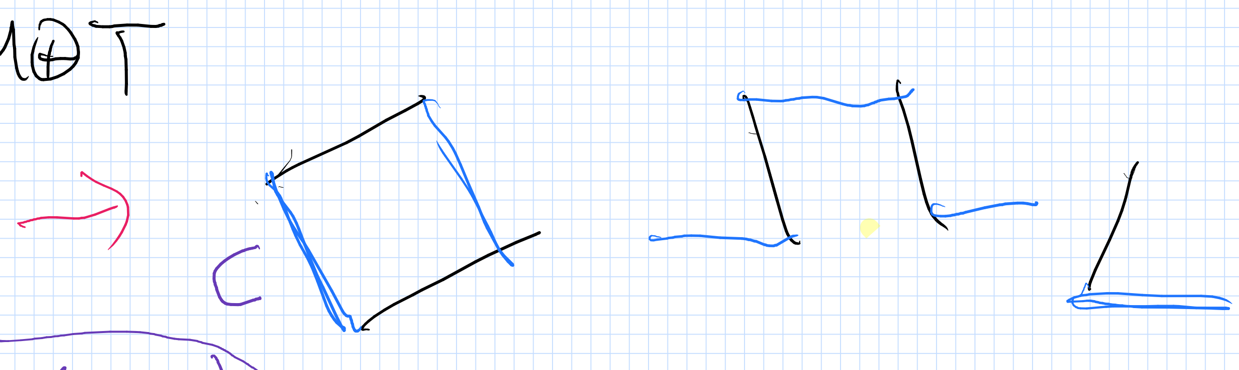
$O(nm)$ n

Alg compute max weight matching in bipartite graph in $O(n^2 m)$ time

M maximum weight matching with k edges

T maximum weight matching with k+1 edges

$M \oplus T$



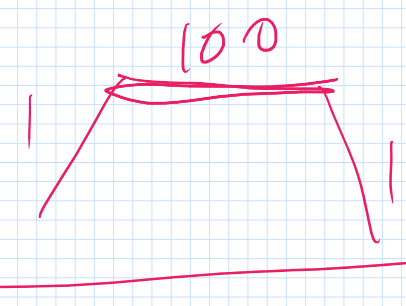
$\partial(C, M) + \partial(C, T) = 0$

$\forall C \in \overline{M \oplus T}$ cycles $\partial(C, M) = 0$

π is an augmenting path from M to T.

π is the heaviest relative aug path.

$O(nm)$

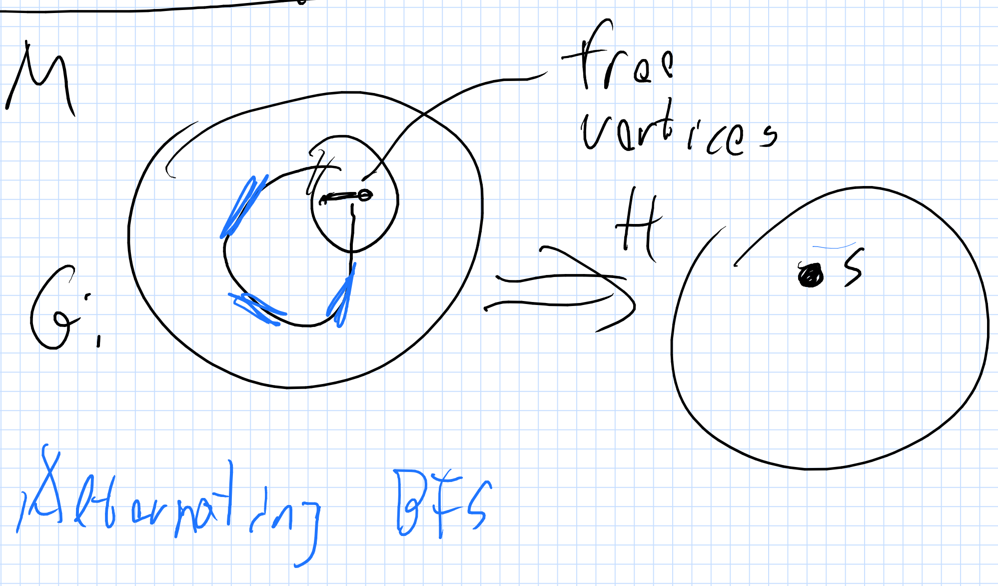


$n \times O(m + nm)$ inner iteration

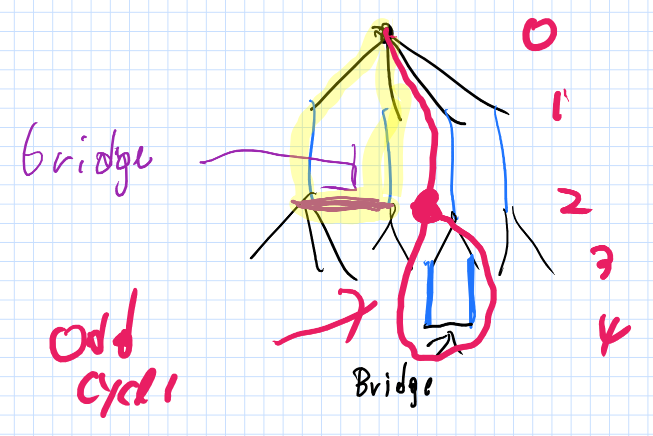
$O(n^2 m)$

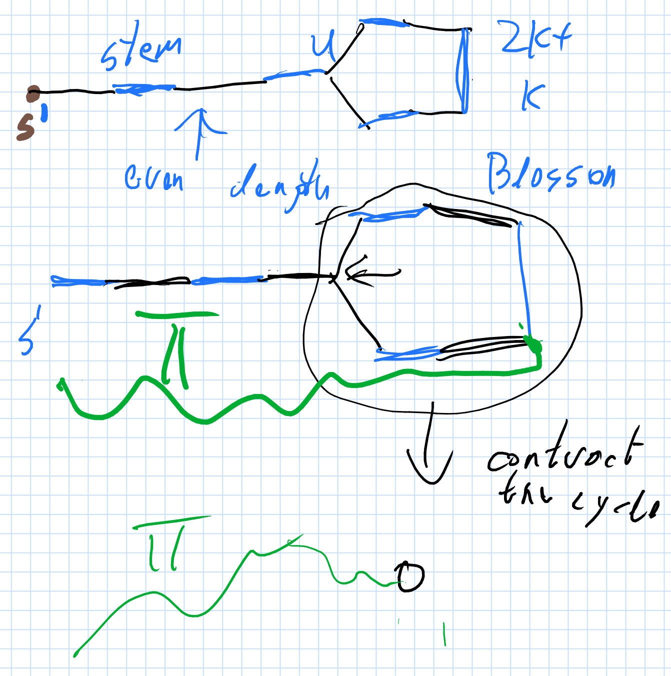
Unweighted matching in regular graph

Augmenting path



Alternating BFS





$O(mn)$ time

$O(mn^2)$

Hungarian method