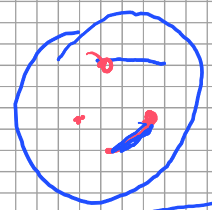
11 473 | 10/11/18

Approximation algorithm

Vertex Cover

G,



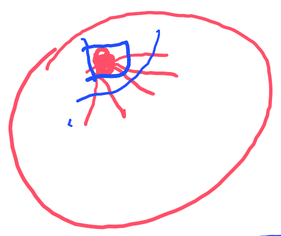
G, opt = size of optimal solution

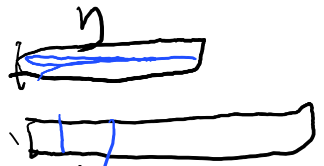
- Poly time.
- Solution is not too bad.

I: input instance

$$1 \le \frac{alg(I)}{OPT(I)} \le O(f(n))$$

$n = |I|$





n vertices

n

$n-2$ $n \ge i$ $i = 1$ $n-i$ $\lfloor \frac{n}{i} \rfloor$ vertices vertices $i{:}\ n-2$

$\lfloor \frac{n}{n-i} \rfloor$ vertices in ith class

$n - i = \frac{n}{3}$

$i = 0$

$\Theta(n \log n)$

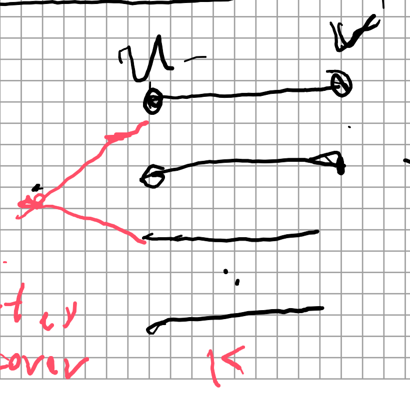
$$\sum_{i=0}^{n-2} \left\lfloor \frac{n}{n-i} \right\rfloor = \Omega(n \log n)$$

$$\left(\sum_{i=}^{n} \frac{n}{i} \right) - n$$

Greedy Vertex Cover $O(\log n)$ approximation

prove this later

1 for the price of two



matching

maximal matching

M

2k vertices VC

$opt \geq k$

$$approx \leq \frac{2k}{opt} \leq \frac{2k}{k} = 2$$

Theorem: VC can not be approx better than $2-\delta$ unless $P = NP$, $\delta \in (0,1)$

$$\frac{alg(G)}{opt(G)} \quad \begin{matrix} \leftarrow 2k \\ \geq k \end{matrix}$$

2 7 alg

1.4 opt

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Set Cover

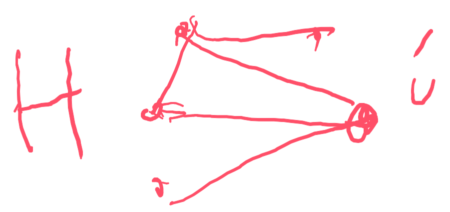
$(G, \{f_1, f_2, \dots, f_m\})$

F ??

Q: Compute min number of sets of F that their union covers G.

NP-Hard

VC



$G = V(H)$

$F = \{N(i) \mid i \in V(H)\}$

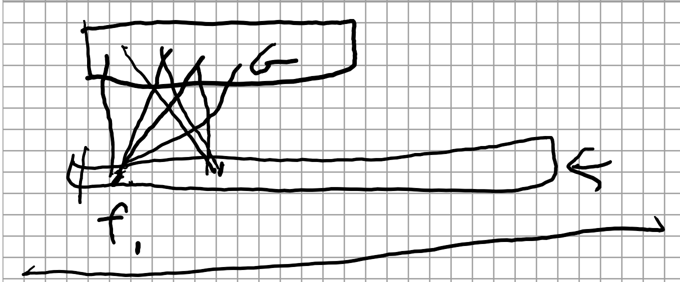
Greedy alg set cover

Pick set that covers the largest number of elements not yet cover

Repeat

Thm Greedy set cover is $O(\log n)$ approximation

Thm One can not do better than $\Omega(\log n)$ unless $P = NP$.

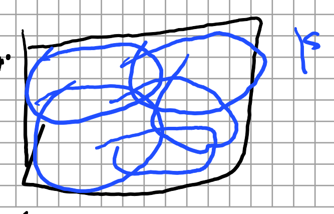


k = size of optimal solution

n_i: number of elements not yet covered in the beginning of the ith iteration

$n_1 = n$

n elements



$< \frac{n}{k}$

$k\left(\frac{n}{k}-1\right) < n$

$\frac{n}{k}$

The ith set picked covers at least $\frac{n_i}{k}$ elements.

$n_{i+1} \le n_i -$ elements covered in ith iteration

$\le n_i - \frac{n_i}{k}$

$= \left(1-\frac{1}{k}\right) n_i$

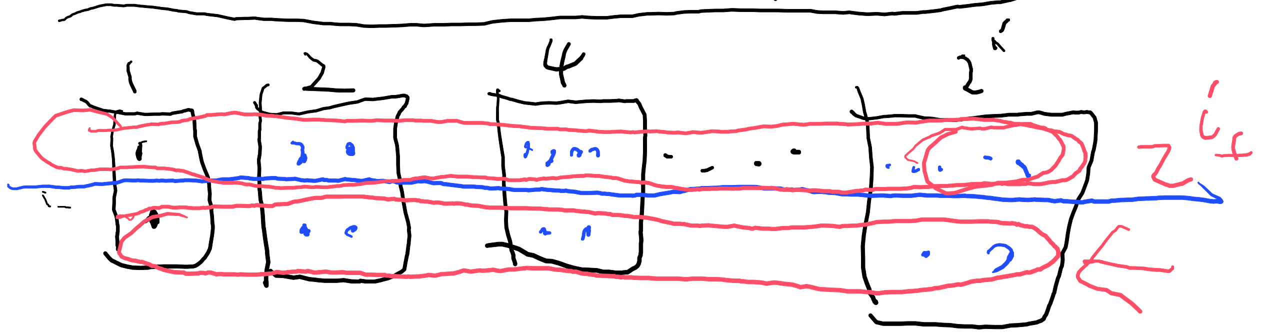
$\le \left(1-\frac{1}{k}\right)^{i-1} n$

k iterations $\le \left(1-\frac{1}{k}\right)^{k} n$

$\le \left(\exp\left(-\frac{1}{k}\right)\right)^{k} n = e^{-1} n = \frac{n}{e}$

$1 - x \le e^{-x}$ $\quad \le \frac{n}{2}$

$k \ln(n) \Rightarrow \frac{alg}{opt} \le \frac{k \ln n}{k} = \ln n$



$2^i + 2^{i-1} + 2^{i-2} + \dots + 1 = 2^{i+1} - 1$

$2 \cdot 2^i$

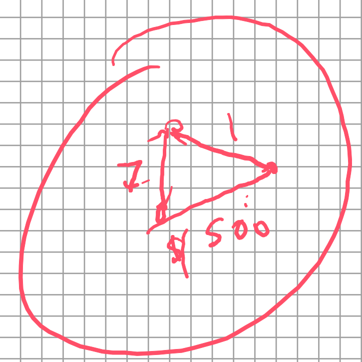
i sets Greedy

2 optimal.

Traveling Salesperson Problem

K_n

find a simple tour that visits all vertices.



$n-1$ $\quad$ 1

c

n $\quad$ $n-1+c$

n

Triangle inequality

$w(xz) \le w(xy) + w(yz)$

$\boxed{\text{2-approx}}$