CS 473, Fall 2017
Homework 8 (due Nov 15 Wednesday at 8pm)

You may work in a group of at most 3 students. Carefully read `http://engr.course.illinois.edu/cs473/policies.html` and `http://engr.course.illinois.edu/cs473/integrity.html`. One member of each group should submit via Gradescope.

1. [*13 pts*]

   (a) [*10 pts*] Run the simplex method on the following linear program:

$$\begin{array}{lrcl} \text{maximize} & x_1 + x_2 & & \\ \text{s.t.} & 3x_1 + 2x_2 & \leq & 27 \\ & x_2 & \leq & 8 \\ & x_1 + 5x_2 & \leq & 35 \\ & 2x_1 + x_2 & \leq & 17 \\ & x_1, x_2 & \geq & 0. \end{array}$$

   Start with the initial basic solution $(\overline{x}_1, \overline{x}_2) = (0, 0)$, and *choose x_1 as the entering variable in the first iteration.* Show the new slack form after every iteration.

   (b) [*3 pts*] Write down the dual of the linear program from (a). What is the optimal dual solution?

2. [*20 pts*] For two points $p = (p_1, \ldots, p_d) \in \mathbb{R}^d$ and $q = (q_1, \ldots, q_d) \in \mathbb{R}^d$, their *rectilinear distance* is defined as

$$D(p, q) = |p_1 - q_1| + \cdots + |p_d - q_d|.$$

   In the *rectilinear 1-center* problem in d dimensions, we are given a set P of n points in $\mathbb{R}^d$, we want to find a point $q \in \mathbb{R}^d$ (not necessarily in P) that minimizes $\max_{p \in P} D(p, q)$.

   (a) [*8 pts*] First show that the problem can be solved in $O(n)$ time for any constant dimension d. How does the running time of your algorithm grow as a function of d?

   (Hint: find a small number of candidate points in P that could be the farthest point from any q...)

   (b) [*12 pts*] Show how to solve this problem for large (nonconstant) dimensions d by using linear programming. The number of variables and constraints should be polynomial in n and d. (Remember to justify correctness of your reduction.)

3. [*12 pts*] We are given n tasks to perform. Task i requires p_i units of power consumption for a duration of h_i hours. At any moment in time, we can perform at most 3 different tasks, and at any moment in time, the total power consumption must be at most P. A task may be preempted (possibly multiple times) at no extra cost. The problem is to devise a schedule to perform all n tasks with the minimum total number of hours.

   For example: for $n = 5$ with $p_1 = 10$, $h_1 = 8.5$, $p_2 = 20$, $h_2 = 9$, $p_3 = 60$, $h_3 = 4$, $p_4 = 80$, $h_4 = 3.5$, $p_5 = 90$, $h_5 = 2$, and $P = 100$, one feasible solution is to do tasks 1 and 5 for 2 hours, then tasks 2 and 4 for 3.5 hours, then tasks 1, 2, and 3 for 4 hours, then tasks 1 and 2 for 1.5 hours, and finally task 1 for 1 hour; the total number of hours is 12. (I did not check if this is optimal. Also, for this small example, the constraint that we can do at most 3 tasks at any time is not important; but it could make a difference on larger instances.)

Describe how to solve this problem using linear programming. The number of variables and constraints should be polynomial in n.