CS 473: Fundamental Algorithms, Fall 2014

Discussion 14

December 9 / 10, 2014

14.1. Paths, Paths, Paths

Let $G = (V, E)$ be an unweighted, undirected graph and let u and v be two vertices of G. Describe a linear time algorithm to find the number of shortest paths from u to v. Note that we only want the number of paths as there may be an exponential number of them. Give an example graph with an exponential number of $(u, v)-$paths.

14.2. Self-Reductions

In each case below, assume that you are given a black box which can answer the decision version of the indicated problem. Use a polynomial number of calls to the black box to construct the desired set.

- Independent set: Given a graph G and an integer k, does G have a subset of k vertices that are pairwise nonadjacent?
- Subset sum: Given a multiset (elements can appear more than once) $X = x_1, \ldots, x_k$ of positive integers, and a positive integer S, does there exist a subset of X with sum exactly S?

14.3. Multiple Interval Scheduling

In the Multiple Interval Scheduling problem, each job requires a set of intervals of time during which it needs to use the processor. For example, a single job could require the processor from 10 AM to 11 AM, and again from 2 PM to 3 PM. If you accept this job, it ties up your processor during those two hours, but you could still accept jobs that need any other time periods, including the hours from 11 AM to 2 PM.

For a given number k, we want to know if it is possible to accept at least k of the jobs so that no two of the accepted jobs have any overlap in time. Prove that this problem is *NP-complete*.

14.4. Zero Length Cycle

Let $G = (V, E)$ be a directed graph that has weights on its edges; $w(e)$ represents the weight of edge e and it can be positive or negative. Given G the Zero-Length-Cycle problem is to check if G has a (simple) cycle C such that the sum of the weights on the edges in C is exactly equal to 0. Show that this problem is *NP-complete*.