

CS 473: Fundamental Algorithms, Fall 2014

Discussion 13

December 2 / 3, 2014

13.1. Solve the following recurrences:

(a) $T(n) = T(\lceil n/15 \rceil) + T(\lceil n/10 \rceil) + 2T(\lceil n/6 \rceil) + n.$

(b) $T(n) = T(n - 1) + 2n - 1, T(0) = 0.$

(c) $T(n) = 5T(n/2) + n^2, T(1) = 1.$

(d) $T(n) = 4T(n/2) + n^2, T(1) = 1.$

(e) $T(n) = 3T(n/2) + n^2, T(1) = 1.$

13.2. The ranking officer of the ROTC decides to postpone a picnic and must notify everyone else.

Each ROTC person except the ranking officer reports to a single superior officer. If u is the superior officer of v , then v is the direct subordinate of u .

To notify everyone of the postponement, the ranking officer first calls each of her direct subordinates, one at a time. As soon as each subordinate gets the phone call, he or she immediately notifies each of his or her direct subordinates, one at a time. The process continues this way until everyone has been notified. Note that in this process each person can only call direct subordinates.

We can picture this process as being divided into rounds. At each round, each person who has already learned of the postponement can call one of his or her direct subordinates on the phone.

The number of rounds it takes for everyone to be notified depends on the sequence in which each person calls their direct subordinates. Give an efficient algorithm that determines the minimum number of rounds needed for everyone to be notified.

13.3. You are organizing a three-day event where several 30-minute sets of music will be played. You need to hire DJs according to the following constraints:

- Exactly k sets of music must be played each day, and thus $3k$ sets altogether.
- Each set must be played by a single DJ in a single music genre.
- Each genre must be played at most once per day.
- Each candidate DJ has given you a list of genres they are willing to play.
- Each DJ can play at most three sets during the entire event.

Suppose there are n DJs and g genres. Describe and analyze an efficient algorithm that either assigns a DJ and a genre to each of the $3k$ sets, or correctly reports that no such

assignment is possible.

13.4. Pebbling is a solitaire game played on an undirected graph G , where each vertex has zero or more pebbles. A single pebbling move consists of removing two pebbles from a vertex v and adding one pebble to an arbitrary neighbor of v . (Obviously, the vertex v must have at least two pebbles before the move.)

Given a graph $G = (V, E)$ and a pebble count $p(v)$ for each vertex v , the Pebble Destruction asks whether there is a sequence of pebbling moves that removes all but one pebble. Prove that this problem is NP-complete.