CS 473: Fundamental Algorithms, Fall 2014

Discussion 11

November 11 / 12, 2014

11.1. Reductions

Define the decision version of the Interval Scheduling Problem as follows: Given a collection of intervals on a time-line, and a bound k, does the collection contain a subset of nonoverlapping intervals of size at least k?

(a) Is it the case that Interval Scheduling $\leq_P$ Vertex Cover?

(b) Is it the case that Independent Set $\leq_P$ Interval Scheduling? (Hint: This is a trick question.)

11.2. Dominating Set

Let $G = (V, E)$ be an undirected graph. A *dominating set* $S \subset V$ is a subset of vertices such that every vertex $v \in V$ is either in S or adjacent to a vertex in S. The *minimum dominating set* is a dominating set of the smallest cardinality among all dominating sets in G.

Reduce minimum dominating set and minimum set cover to one another, in both directions.

11.3. Triangle Freedom

A subset S of vertices in an undirected graph G is called *triangle-free* if the induced graph G_S has no 3-cycles (aka triangles). That is, for every three vertices $u, v, w \in V$, at least one of the three edges uv, uw, vw is absent from G.

Reduce Independent Set to this Triangle-Free problem.

11.4. Graph Isomorphisms

Two graphs are *isomorphic* if one can be transformed into the other by relabeling the vertices. Consider the following decision problems:

- Graph Isomorphism: Given two graphs G and H, determine whether G and H are isomorphic.
- Even Graph Isomorphism: Given two graphs G and H, such that every vertex in G and H has even degree, determine whether G and H are isomorphic.
- Subgraph Isomorphism: Given two graphs G and H, determine whether G is isomorphic to a subgraph of H.

(a) Describe a polynomial-time reduction from Graph Isomorphism to Even Graph Isomorphism.

(b) Describe a polynomial-time reduction from Graph Isomorphism to Subgraph Isomorphism.

(c) Prove that Subgraph Isomorphism is NP-Complete by reducing from Clique.