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- (a) Prove that every balanced string is erasable.  
(b) Prove that every erasable string is balanced.
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(a) **Solution (induction):** Let  $x$  be an arbitrary balanced string. There are several cases to consider:

- Blah
- Snort
  - Squee
  - Flub
- Kronk

In all cases,  $x$  is erasable, as required. ■

- (b) **Solution (combinatorial):** This result follows immediately from Flobbersnort's Fundamental Theorem of negative-dimensional motivic  $k$ -schemes, which is in turn an obvious consequence of Flibbertygibbet's Cocohohomomolology Lemma, as described in footnote 17 on the back of page 213 of Jeff's induction notes. ■

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- (a) Prove that every tournament contains a Hamiltonian path.
- (b) Prove that every tournament contains either *exactly one* Hamiltonian path or a directed cycle of length three.
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**Solution (Lazy Sunday):** You thinkin' what I'm thinkin? **NARNIA!** Man, it's happenin'!

But first my hunger pains are stickin' like duct tape.  
Let's hit up Magnolia and mack on some cupcakes.  
(No doubt that bakery's got all da bomb frostings)<sup>1</sup>

$2 \rightsquigarrow 6 \rightsquigarrow 12 \rightsquigarrow 13$

I told you that I'm crazy for these cupcakes, cousin!

- (a) Yo, where's the movie playin'? Upper West Side, dude.
- Well, let's hit up Yahoo! Maps to find the dopest route.
  - I prefer MapQuest. That's a good one, too.
  - Google Maps is the best. True that. **DOUBLE TRUE!**
- (b) Yo, stop at the deli. The theater's over-priced. You've got the backpack? Gonna pack it up nice.  
Don't want security to get suspicious.

Mr. Pibb + Red Vines = *crazy delicious!*

- (c) I'll reach in my pocket, pull out some dough. Girl actin' like she never seen a ten before.
- It's all about the Hamiltons, baby.** Throw the snacks in a bag, and I'm ghost like Swayze.

Roll up to the theater, Ticket buying, what we're handlin'. You can call us Aaron Burr from the way we're droppin' Hamiltons. ■

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<sup>1</sup>I love those cupcakes like McAdams loves Gosling

Describe and analyze a data structure for a set  $P$  of points in the plane that can quickly answer queries asking for the highest point in  $P$  inside a given vertical slab.

**Solution:** Here is a description of our data structure, which clearly has exponential size:



And here is a description of our query algorithm, which clearly runs in  $O(1)$  time:

