This homework is optional. Any problem that you do not submit will be automatically forgiven.

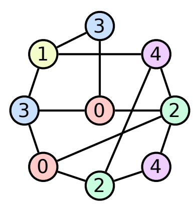
A useful list of NP-hard problems appears on the next page.

1. In the task scheduling problem, we are given n tasks, identified by the integers 1 through n, and a set of precedence constraints of the form "Task i must be executed before task j." A ***feasible schedule*** is an ordering of the n tasks that satisfies all the given precedence constraints.

   (a) Given a set of tasks and precedence constraints, describe and analyze a polynomial-time algorithm to determine whether a feasible schedule exists.

   (b) Suppose we are given a set of precedence constraints for which there is no feasible schedule. In this case, we would like a schedule that violates the minimum number of precedence constraints. Prove that finding such a schedule is NP-hard.

2. Recall that a 5-coloring of a graph G is a function that assigns each vertex of G a 'color' from the set $\{0, 1, 2, 3, 4\}$, such that for any edge uv, vertices u and v are assigned different 'colors'. A 5-coloring is *careful* if the colors assigned to adjacent vertices are not only distinct, but differ by more than 1 (mod 5). Prove that deciding whether a given graph has a careful 5-coloring is NP-hard. *[Hint: Reduce from the standard* 5COLOR *problem.]*



A careful 5-coloring.

3. (a) A ***tonian path*** in a graph G is a path that goes through at least half of the vertices of G. Show that determining whether a given graph has a tonian path is NP-hard.

   (b) A ***tonian cycle*** in a graph G is a cycle that goes through at least half of the vertices of G. Show that determining whether a given graph has a tonian cycle is NP-hard. *[Hint: Use part (a).]*

You may assume the following problems are NP-hard:

CircuitSat: Given a boolean circuit, are there any input values that make the circuit output True?

PlanarCircuitSat: Given a boolean circuit drawn in the plane so that no two wires cross, are there any input values that make the circuit output True?

3Sat: Given a boolean formula in conjunctive normal form, with exactly three literals per clause, does the formula have a satisfying assignment?

Max2Sat: Given a boolean formula in conjunctive normal form, with exactly two literals per clause, what is the largest number of clauses that can be satisfied by an assignment?

MaxIndependentSet: Given an undirected graph G, what is the size of the largest subset of vertices in G that have no edges among them?

MaxClique: Given an undirected graph G, what is the size of the largest complete subgraph of G?

MinVertexCover: Given an undirected graph G, what is the size of the smallest subset of vertices that touch every edge in G?

MinSetCover: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S, what is the size of the smallest subcollection whose union is S?

MinHittingSet: Given a collection of subsets $S_1, S_2, \ldots, S_m$ of a set S, what is the size of the smallest subset of S that intersects every subset S_i?

3Color: Given an undirected graph G, can its vertices be colored with three colors, so that every edge touches vertices with two different colors?

MaxCut: Given a graph G, what is the size (number of edges) of the largest bipartite subgraph of G?

HamiltonianCycle: Given a graph G, is there a cycle in G that visits every vertex exactly once?

HamiltonianPath: Given a graph G, is there a path in G that visits every vertex exactly once?

TravelingSalesman: Given a graph G with weighted edges, what is the minimum total weight of any Hamiltonian path/cycle in G?

SubsetSum: Given a set X of positive integers and an integer k, does X have a subset whose elements sum to k?

Partition: Given a set X of positive integers, can X be partitioned into two subsets with the same sum?

3Partition: Given a set X of n positive integers, can X be partitioned into $n/3$ three-element subsets, all with the same sum?