

1. Let $G = (V, E)$ be a directed graph. If the indegree of each vertex is at least 1 prove that G contains a cycle. Give an algorithm to find a cycle in such a graph. How fast is your algorithm?
2. At a party with n people P_1, \dots, P_n , certain pairs of individuals cannot stand each other. Given a list of such pairs, determine if we can divide the n people into two groups such that all the people in both group are amicable, that is, they can stand each other.
3. Given a connected graph $G = (V, E)$ prove that one can orient each edge so that after the orientation, the indegree and outdegree of each vertex differ by at most 1. How fast can you compute this orientation? [**Hint: Can you change the graph so that it has an Euler tour?**]