

1. An *extendable array* is a data structure that stores a sequence of items and supports the following operations.
  - `ADDTOFRONT( $x$ )` adds  $x$  to the *beginning* of the sequence.
  - `ADDTOEND( $x$ )` adds  $x$  to the *end* of the sequence.
  - `LOOKUP( $k$ )` returns the  $k$ th item in the sequence, or `NULL` if the current length of the sequence is less than  $k$ .

Describe a *simple* data structure that implements an extendable array. Your `ADDTOFRONT` and `ADDTOBACK` algorithms should take  $O(1)$  *amortized* time, and your `LOOKUP` algorithm should take  $O(1)$  *worst-case* time. The data structure should use  $O(n)$  space, where  $n$  is the *current* length of the sequence.

2. An *ordered stack* is a data structure that stores a sequence of items and supports the following operations.
  - `ORDEREDPUSH( $x$ )` removes all items smaller than  $x$  from the beginning of the sequence and then adds  $x$  to the beginning of the sequence.
  - `POP` deletes and returns the first item in the sequence (or `NULL` if the sequence is empty).

Suppose we implement an ordered stack with a simple linked list, using the obvious `ORDEREDPUSH` and `POP` algorithms. Prove that if we start with an empty data structure, the amortized cost of each `ORDEREDPUSH` or `POP` operation is  $O(1)$ .

3. Chicago has many tall buildings, but only some of them have a clear view of Lake Michigan. Suppose we are given an array  $A[1..n]$  that stores the height of  $n$  buildings on a city block, indexed from west to east. Building  $i$  has a good view of Lake Michigan if and only if every building to the east of  $i$  is shorter than  $i$ .

Here is an algorithm that computes which buildings have a good view of Lake Michigan. What is the running time of this algorithm?

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GOODVIEW( $A[1..n]$ ):
  initialize a stack  $S$ 
  for  $i \leftarrow 1$  to  $n$ 
    while ( $S$  not empty and  $A[i] > A[\text{TOP}(S)]$ )
      POP( $S$ )
    PUSH( $S, i$ )
  return  $S$ 

```