CS 473: Algorithms, Fall 2010
HW 9 (due Tuesday, November 16)

This homework contains four problems. **Read the instructions for submitting homework on the course webpage**. In particular, *make sure* that you write the solutions for the problems on separate sheets of paper; the sheets for each problem should be stapled together. Write your name and netid on each sheet.

Collaboration Policy: For this home work, Problems 1-3 can be worked in groups of up to 3 students each.

Problem 0 should be answered in Compass as part of the assessment HW9-Online and should be done individually.

0. (10 pts) HW9-Online on Compass.

1. (25 pts) Given an undirected graph $G = (V, E)$ and two distinct nodes s, t describe an algorithm that decides if there are k internally node-disjoint paths between s and t. Two s-t paths P and Q are internally node-disjoint if they do not share any nodes other than s and t.

2. (35 pts) Let $G = (V, E)$ be a flow network with source s and sink t. Describe a polynomial time algorithm to check if G has a *unique* minimum cut.

3. (30 pts) The Computer Science Department at UIUC has n professors. They handle department duties by taking part in various committees. There are m committees and the j'th committee requires k_j professors. The head of the department asked each professor to volunteer for a set of committees. Let $S_i \subseteq \{1, 2, \ldots, m\}$ be the set of committees that professor i has volunteered for. A committee assignment consists of sets $S'_1, S'_2, \ldots, S'_n$ where $S'_i \subseteq \{1, 2, \ldots, m\}$ is the set of committees that professor i will participate in. A *valid* committee assignment has to satisfy two constraints: (i) for each professor i, $S'_i \subseteq S_i$, that is each professor is only given committees that he/she has volunteered for, and (ii) each committee j has k_j professors assigned to it, or in other words j occurs in at least k_j of the sets $S'_1, S'_2, \ldots, S'_n$.

   (a) Describe a polynomial time algorithm that the head of the department can employ to check if there is a valid committee assignment given the lists $S_1, S_2, \ldots, S_n$. The algorithm should output a valid assignment if there is one.

   (b) The head of the department notices that often there is no valid committee assignment because professors naturally are inclined to volunteer for as few committees as possible. To overcome this, the definition of a valid assignment is relaxed as follows. Let ℓ be some integer. An assignment $S'_1, S'_2, \ldots, S'_n$ is now said to be valid if (i) $|S'_i - S_i| \leq \ell$ and (ii) each committee j has k_j professors assigned to it. The new condition (i) means that a professor i may be assigned up to ℓ committees not on the list S_i that he/she volunteered for. Describe an algorithm to check if there is a valid committee assignment with the relaxed definition.

   (b) is a more general problem than (a) so you can do (b) for full points or (a) only for 25 pts.