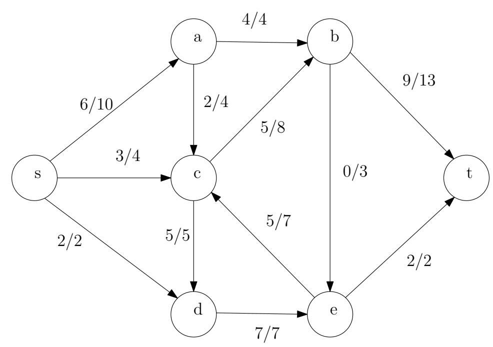
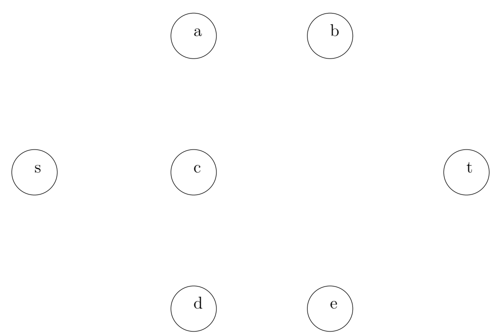
CS 473: Algorithms, Fall 2010
HBS 8

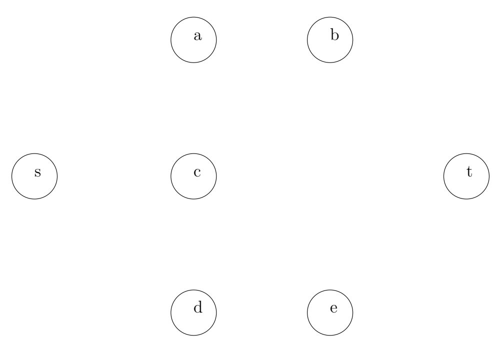
Problem 1. **[Go With the Flow]**
The figure below shows a flow network along with a flow. In the figure, the notation a/b for an edge means that the flow on the edge is a and the capacity of the edge is b.



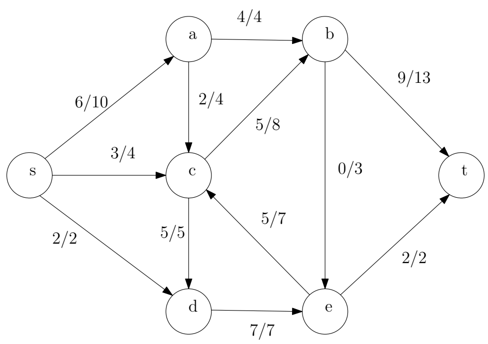
(a) What is the value of the given flow? Is it maximal? Show the residual graph for the above graph and flow in the figure below.

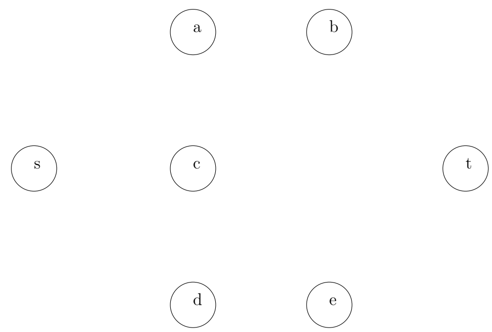


(b) Show an $s-t$ path in the residual graph and state its bottleneck capacity. You only need to draw the path from the graph you showed in (a).



(c) Show the new flow on the *original* graph after augmenting on the path you found in (b). Use the notation a/b to indicate the flow on an edge and its capacity.





(d) What is the capacity of a minimum-cut in the given graph? Find a cut with that capacity.

Problem 2. **[Residual Graph Properties]**
Prove the following property about residual graphs:

Let f be a flow in G and G_f be the residual graph. If f' is a flow in G_f, then $f + f'$ is a flow in G of value $v(f) + v(f')$.

Problem 3. **[Capacities on Nodes]**
In a standard $s - t$ maximum flow problem, we assume that edges have capacities, and there is no limit on how much flow is allowed to pass through a node. In this problem, we consider the variant where nodes have capacities.

Let $G = (V, E)$ be a directed graph with source s and sink t. Let $c : V \to \mathbb{R}^+$ be a capacity function. Recall that a flow f assigns a flow value $f(e)$ to each *edge* e. A flow f is *feasible* if the total flow into every vertex v is at most $c(v)$:

$$f^{\text{in}}(v) \leq c(v) \qquad \text{for every vertex } v$$

Design a polynomial time algorithm that finds a feasible $s - t$ flow of maximum value in G.