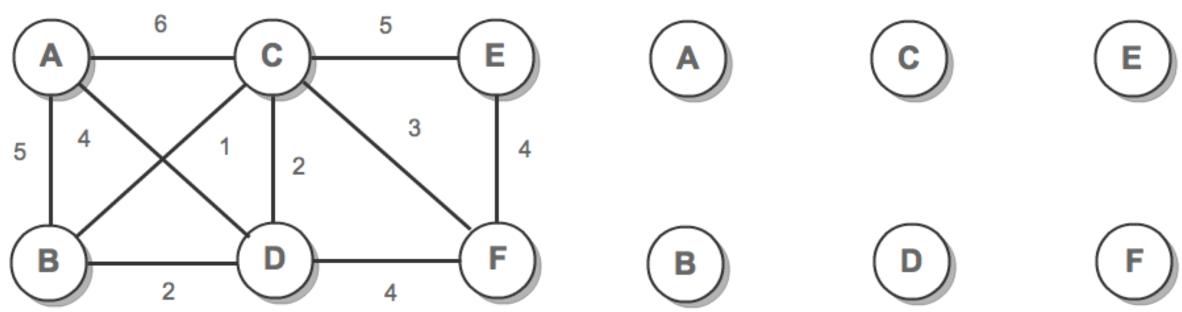
CS 473: Algorithms, Fall 2010
HBS 6

Problem 1. [Minimum Spanning Tree]



- Draw the edges in the Minimum Spanning Tree for the following graph.
- Given G and MST T, suppose you d ecrease the weight of an edge e not in T. Give an algorithm to recompute the MST in $O(n)$ time.

Problem 2. [Stock Picking]
You have a group of investor friends who are looking at n consecutive days of a given stock at some point in the past. The days are numbered. $i = 1, 2, \ldots, n$. For each day i, they have a price $p(i)$ per share for the stock on that day.

For certain (possibly large) values of k, they want to study what they call *k-shot strategies*. A k-shot strategy is a collection of m pairs of days $(b_1, s_1), \ldots, (b_m, s_m)$, where $0 \leq m \leq k$ and

$1 \leq b_1 < s_1 < b_2 < s_2 \cdots < b_m < s_m \leq n$

We view these as a set of up to k nonoverlapping intervals, during each of which the investors buy 1,000 shares of the stock (on day b_i and then sell it (on day s_i. The *return* of a given k-shot strategy is simply the profit obtained from the m buy-sell transactions, namely,

$$1000 \cdot \sum_{i=1}^{m} p(s_i) - p(b_i)$$

- Design an efficient algorithm that determines, given the sequence of prices, the k-shot strategy with the maximum possible return. Since k may be relatively large, your running time should be polynomial in both n and k.

- Now, modify your algorithm to only use $O(n)$ space.

Problem 3. [Weighted Scheduling]
We have n jobs $J_1, J_2, \dots, J_n$ which we need to schedule on a machine. Each job J_i has a processing time t_i and a weight w_i. A schedule for the machine is an ordering of the jobs. Given a schedule, let C_i denote the finishing time of job J_i. For example, if job J_j is the first job in the schedule, its finishing time C_j is equal to t_j; if job J_j follows job J_i in the schedule, its finishing time C_j is equal to $C_i + t_j$. The weighted completion time of the schedule is $\sum_{i=1}^{n} w_i C_i$.

- Given an efficient algorithm that finds a minimum weighted schedule when $w_i = 1$ for all i.
- Give an efficient algorithm that finds a schedule with minimum weighted completion time given arbitrary weights.