CS 473: Algorithms, Fall 2009

HW 0 (due Tuesday, September 1st, in class)

This homework contains four problems. **Read the instruction for submitting homework on the course webpage**. In particular, *make sure* that you write the solutions for the problems on separate sheets of paper and do not staple the sheets for different problems. Write your name and netid on each sheet. HW 0 is due in class.

Collaboration Policy: For this home work, each student should work independently and write up their own solutions and submit them.

Read the course policies before starting the homework.

1. (25pts) Sort the following functions in asymptotic order. No proofs are needed or should be given but you may want to work them out to make sure your answers are correct. More precisely, if $f(n)$ and $g(n)$ are two functions and $f(n) = o(g(n))$ then $f(n)$ should come earlier than $g(n)$ in the ordering. In case of ties, that is $f(n) = \Theta(g(n))$, you can put them in either order but indicate this. For simplicity of notation, if $f(n) = o(g(n))$ write $f(n) \ll g(n)$ and if $f(n) = \Theta(g(n))$ write $f(n) \equiv g(n)$. As an example $n \equiv 10n + \sqrt{n} \ll n^{10} \ll n^{10} \lg n$.

n	$(\sqrt{3})^{\lg n}$	$n \lg \lg \lg n$	$n^{1+1/\lg n}$	$(1-1/n)^n$	$\lg n$
$\lg \lg n$	H_n	3^{H_n}	$(\sqrt{2})^{\lg n}$	$\lg F_{\sqrt{n}}$	n^2
$2^{\sqrt{\ln n}}$	$\lg^{\lg n} n$	n^n	$n!$	2^n	$(1+1/n)^n$

   To jog your memory:

   - $\lg n = \log_2 n$ and $\ln n = \log_e n$.
   - $\lg^2 n = (\lg n)^2$ and $\lg \lg n = \lg(\lg n)$.
   - H_n is the n'th harmonic number and $H_n = \sum_{i=1}^{n} 1/i \simeq \ln n + 0.577215\ldots$.
   - F_n is the n'th Fibonacci number and satisfies the recurrence $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0, F_1 = 1$. It can be verified by induction (try it!) that $F_n = (\phi^n - (-1/\phi)^n)/\sqrt{5}$ where $\phi = (1+\sqrt{5})/2$ is the golden ratio.

2. (25 pts) Solve the following recurrences and give a tight asymptotic bound for each function in terms of simple known functions. That is, you should state your solution as $f(n) = \Theta(g(n))$ where $f(n)$ is the recurrence in question and $g(n)$ is expressed in terms of known functions. No proofs needed but do them to convince yourself of the correctness of your answers.

   - $A(n) = A(n/5) + n$ for $n > 5$ and $A(n) = 1$ for $n < 5$.
   - $B(n) = B(n-4) + 6n$ for $n > 5$ and $B(n) = 1$ for $1 \leq n \leq 4$.
   - $C(n) = C(n/2) + 1/n$ for $n \geq 2$ and $C(n) = 1$ for $n = 1$.
   - $D(n) = D(n-2) + n^3$ for $n > 2$ and $D(n) = 1$ for $n = 1, 2$.

- $E(n) = 4E(n/2) + n^2$ for $n \geq 2$ and $E(n) = 1$ for $n = 0, 1$.

3. (25 pts) You are given the height and weight of n individuals; h_i denotes the height and w_i denotes the weight of the ith person. Assume that the heights are all distinct. Your goal is design a data structure that can answer queries of the form $\text{MinWt}(a, b)$ where a, b are two parameters such that $a \leq b$: $\text{MinWt}(a, b)$ should report the minimum weight amongst all individuals with height in the range $[a, b]$. Give a linear space data structure that can support these queries in $O(\log n)$ time per query. You need to describe an algorithm to build the data structure and an algorithm to answer a given query using the data structure. How much time does your algorithm take to build the data structure? Note that your structure is static and does not need to support insertions or deletions. (*Hint:* Adapt the standard binary search tree.)

4. (25 pts) A set of n tennis players $P_1, \ldots, P_n$ play in a tournament in which every player plays a match with every other player (a total of $\binom{n}{2}$ matches are played). There are no ties so for each pair of players (P_i, P_j) either P_i wins over P_j or vice-versa. We write $P_i \prec P_j$ if P_j wins against P_i in their match. We wish to rank the players from 1 to n with 1 being the best and n being the worst and justify the ranking. Let $P_{i_1}, P_{i_2}, \ldots, P_{i_n}$ be a ranking of the players from 1 to n. A ranking is justified if $P_{i_n} \prec P_{i_{n-1}} \prec \ldots \prec P_{i_1}$. Show the following: for any given outcomes of the $\binom{n}{2}$ matches, there is a ranking that is justified. *Hint:* use induction.