CS 473: Algorithms, Fall 2009
HBS 13

Problem 1

You step in a party with a camera in your hand. Each person attending the party has some friends there. You want to have exactly one picture of each person in your camera. You want to use the following protocol to collect photos. At each step, the person that has the camera in his hand takes a picture of one of his/her friends and pass the camera to him/her. Of course, you only like the solution if it finishes when the camera is in your hand. Given the friendship matrix of the people in the party, design a polynomial algorithm that decides whether this is possible, or prove that this decision problem is NP-hard.

Problem 2 (Self-Reductions)

In each case below, assume that you are given a black box which can answer the decision version of the indicated problem. Use a polynomial number of calls to the black box to construct the desired set.

- Independent set: Given a graph G and an integer k, does G have a subset of k vertices that are pairwise nonadjacent?
- Subset sum: Given a multiset (elements can appear more than once) $X = x_1, \ldots, x_k$ of positive integers, and a positive integer S, does there exist a subset of X with sum exactly S?

Problem 3

- Recall from lecture the Subset Sum Problem: Given n integers $a_1, a_2, \ldots, a_n$ and a target B, is there a subset S of $\{a_1, \ldots, a_n\}$ such that the numbers in S add up precisely to B?

  And Knapsack: Given n items with item i having size s_i and profit p_i, a knapsack of capacity B, and a target profit P, is there a subset S of items that can be packed in the knapsack and the profit of S is at least P?

  Show that Knapsack is NP-Complete via a reduction from Subset Sum.

- Consider the following problem, called Partition-into-3-Cliques: Given a graph $G = (V, E)$, is it possible to partition V into 3 sets C_1, C_2, C_3 such that each C_i is a clique in G?

  Show that Partition-into-3-Cliques is NP-Complete via a reduction from 3-Colorability.

Problem 4 (Harder. For thinking outside HBS)

Let G be a graph. A set S of vertices of G is a dominating set if every vertex in G is either in S or adjacent to a vertex in S. Show that, given G and an integer k, deciding if G contains a dominating set of size at most k is NP-complete.

- Hint: Reduce from the Vertex Cover problem. Recall that the Vertex Cover problem asks, given a graph G and an integer k, whether there is a vertex cover of G containing at most k vertices.
- Stronger Hint: Let (G, k) be an instance of the vertex cover problem. Construct a new graph G' by adding new vertices and edges to the graph G as follows: For each edge (v, w) of G, add a vertex vw and the edges (v, vw) and (w, vw) to G'. Consider the instance (G', k) of the dominating set problem.