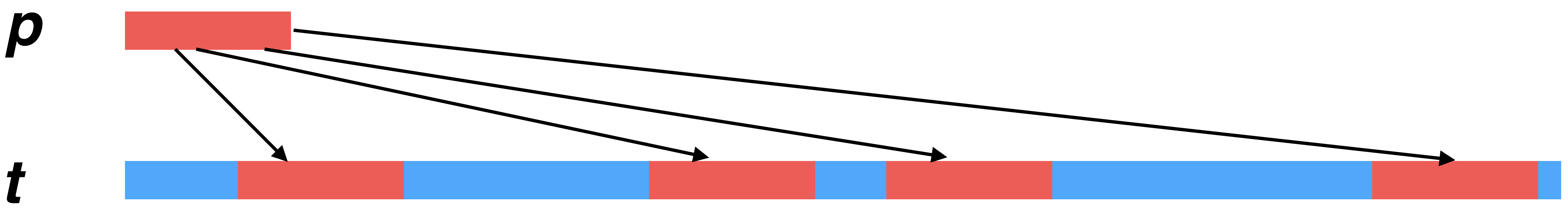
Exact Pattern Matching



Goal: *Find all occurrences of a pattern in a text*

Input: Pattern $\boldsymbol{p} = p_1 \ldots p_n$ and text $\boldsymbol{t} = t_1 \ldots t_m$

Output: All positions $1 \leq i \leq (m - n + 1)$ such that the n-letter substring of $\boldsymbol{t}$ starting at i matches $\boldsymbol{p}$

Motivation: Searching database for a known pattern

Pattern Matching: Running Time

- Naïve runtime: O(nm)
  - How?
- On average, it should be close to O(m)
  - Why?
- Can solve problem in O(m) time ?
  - Yes, we'll see how (in a later lecture)

Naive algorithm is inefficient

As we saw, our alignment algorithms scale as O(nm). When $n \approx 10^9$ and $m \approx 10^2$ this becomes intractable (especially when we 10 of millions of strings of length ~m)

Even ignoring, e.g memory access, say filling in each matrix cell takes C = 10 CPU cycles.

$N = 10^9$	$M = 10^2$	$R = 10^7$
order of genome	order of read length	order of # of reads

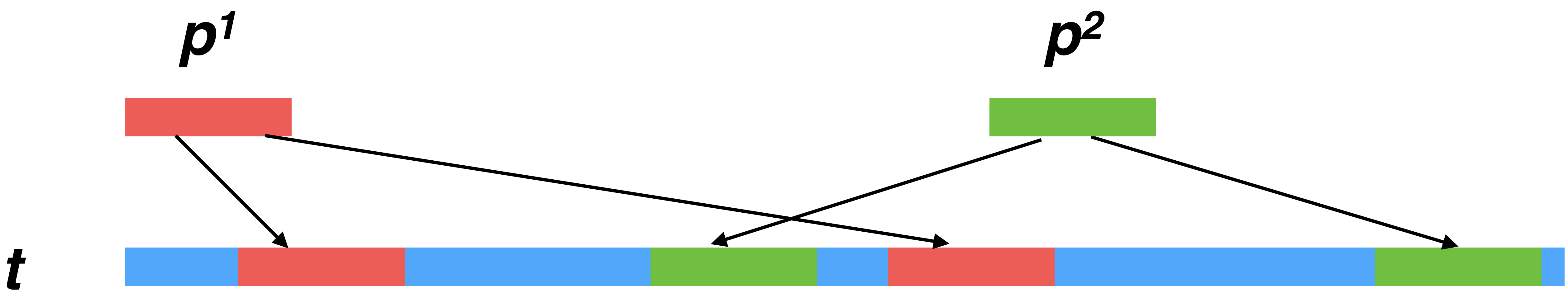
$$\# \text{ of ops} \approx N * M * R * C = 10^{19}$$

$$\text{ops/sec} \approx 3*10^9 \text{ (3GHz CPU)}$$

$$\# \text{ ops} / (\text{ops/sec}) = \text{secs} \approx 10^{19} / (3*10^9) = (1/3) * 10^{10}$$

~106 Years! (for a relatively small 10M read dataset)

Multiple Pattern Matching



<u>Goal</u>: Given a set of patterns and a text, find all occurrences of any of patterns in text

<u>Input</u>: k patterns $\mathbf{p}^1,\ldots,\mathbf{p}^k$, and text $\mathbf{t} = t_1\ldots t_m$

<u>Output</u>: Positions $1 \leq i \leq m$ where substring of $\mathbf{t}$ starting at i matches $\mathbf{p}_j$ for $1 \leq j \leq k$

Motivation: Searching database for known multiple patterns

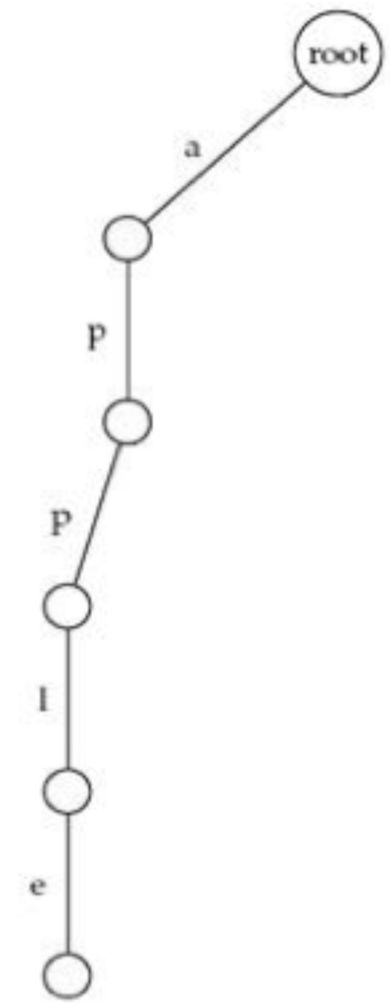
Multiple Pattern Matching

- **Solution:** k "pattern matching problems": O(kmn)

- **Another Solution:**
  - Using "Keyword trees" => O(kn+nm) where n is maximum length of $\mathbf{p}^i$
  - Preprocess all *k* patterns to construct a "keyword tree"
  - Now, any given text, all occurrences of all patterns can be found in time O(m)

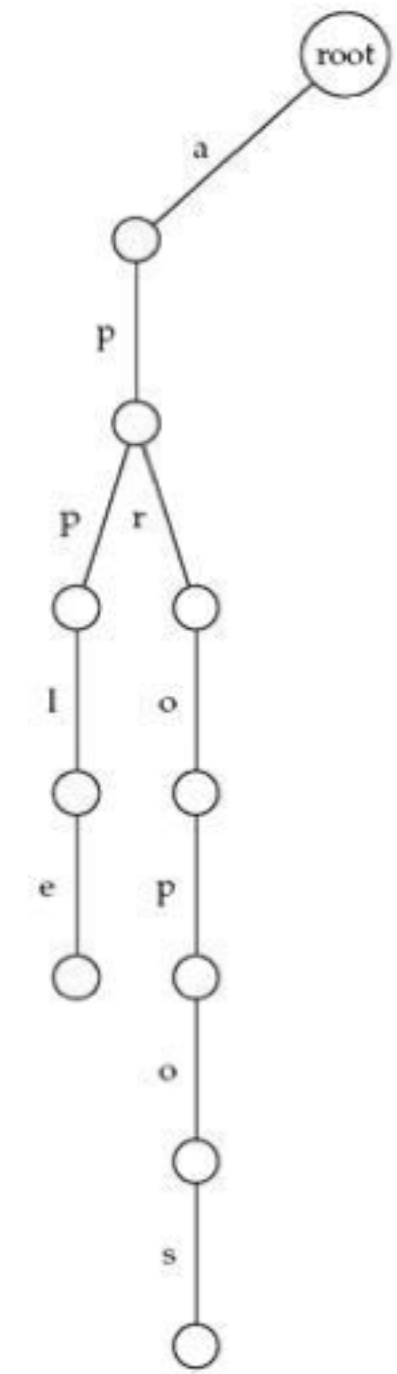
Keyword tree approach

- ***Keyword tree***:
  - Apple



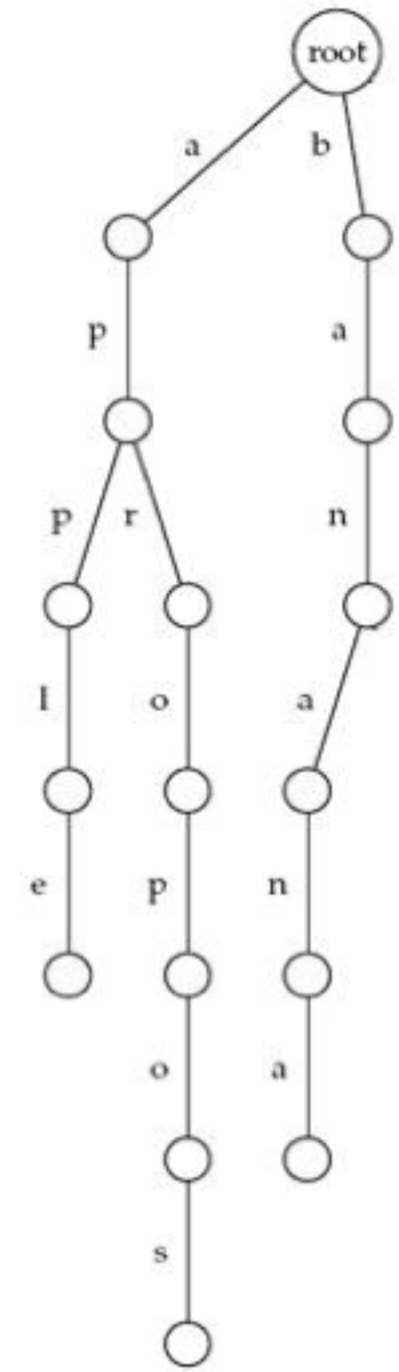
Keyword tree approach

- ***Keyword tree***:
  - Apple
  - Apropos



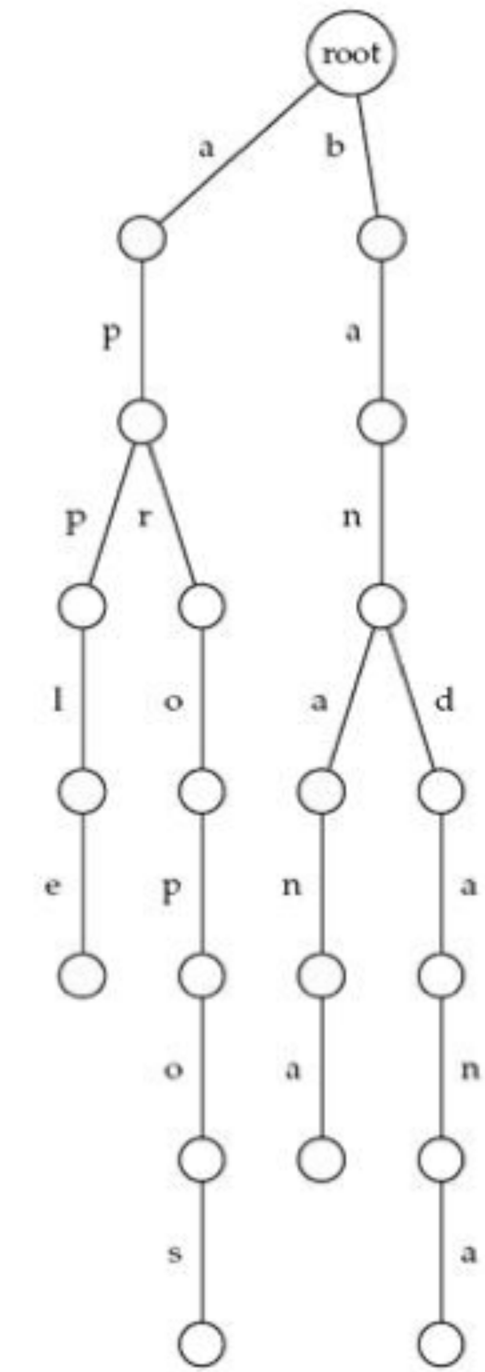
Keyword tree approach

- ***Keyword tree***:
  - Apple
  - Apropos
  - Banana



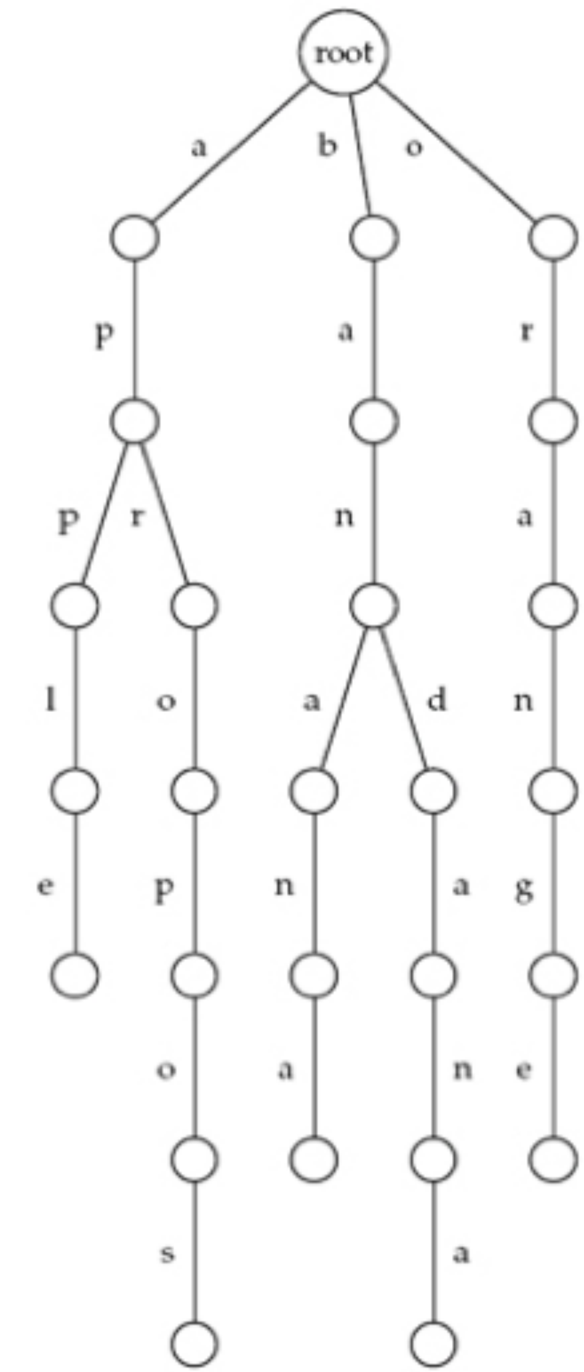
Keyword tree approach

- ***Keyword tree***:
  - Apple
  - Apropos
  - Banana
  - Bandana



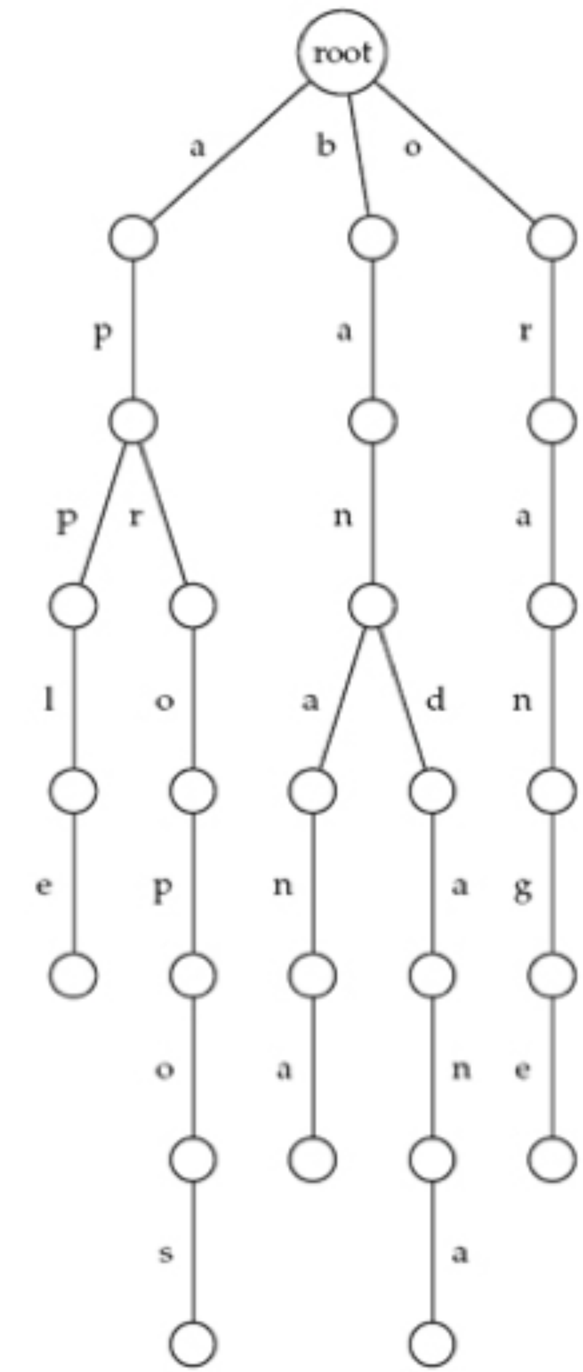
Keyword tree approach

- ***Keyword tree***:
  - Apple
  - Apropos
  - Banana
  - Bandana
  - Orange



Keyword tree approach: Properties

- Stores a set of keywords in a rooted labeled tree
- Each edge labeled with a letter from an alphabet
- Any two edges coming out of the same vertex have distinct labels
- Every keyword stored can be spelled on a path from root to some leaf



Keyword tree: Construction

Construction for $\mathcal{P} = \{P_1, \ldots, P_k\}$:

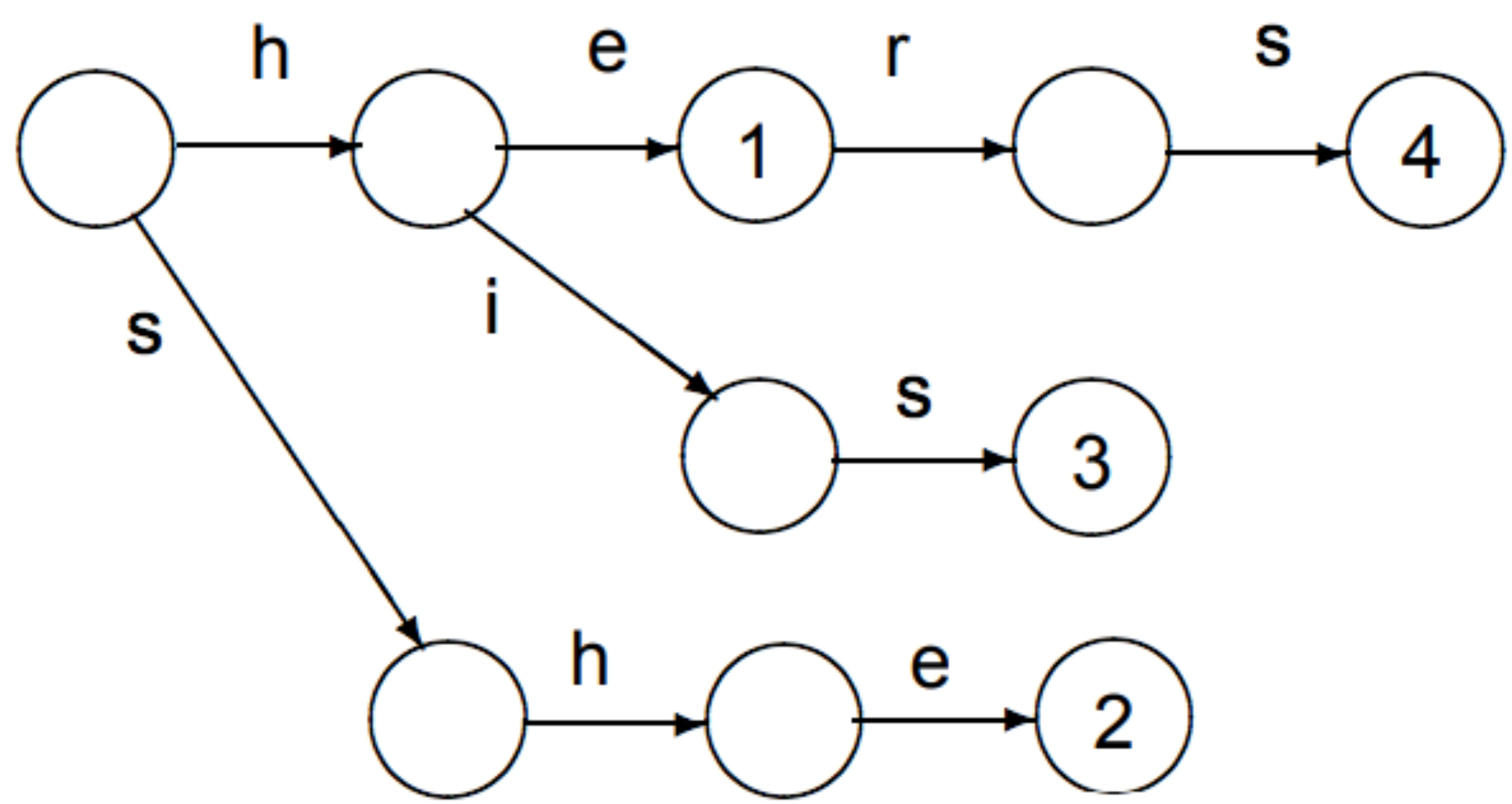
Begin with a root node only;
Insert each pattern P_i, one after the other, as follows:

Starting at the root, follow the path labeled by chars of P_i;

- If the path ends before P_i, continue it by adding new edges and nodes for the remaining characters of P_i
- Store identifier i of P_i at the terminal node of the path

This takes clearly $O(|P_1| + \cdots + |P_k|)$

A keyword tree for $\mathcal{P} = \{\text{he}, \text{she}, \text{his}, \text{hers}\}$:



Keyword tree: Lookup of a string

Lookup of a string P: Starting at root, follow the path labeled by characters of P as long as possible;

- If the path leads to a node with an identifier, P is a keyword in the dictionary
- If the path terminates before P, the string is not in the dictionary

How to check all occurrences in a text ***t***?

Keyword tree approach: Complexity

- Build keyword tree in O(kn) time; kn is total length of all patterns
- Start "threading" at each position in text; at most n steps tell us if there is a match here to any $\mathbf{p}^i$
- O(kn + nm)
  - We're down from O(kmn) to this
- The next big idea, Aho-Corasick algorithm: O(kn + m)

Aho-Corasick algorithm: Key idea

Exploit the redundancy in the patterns

HERSHE

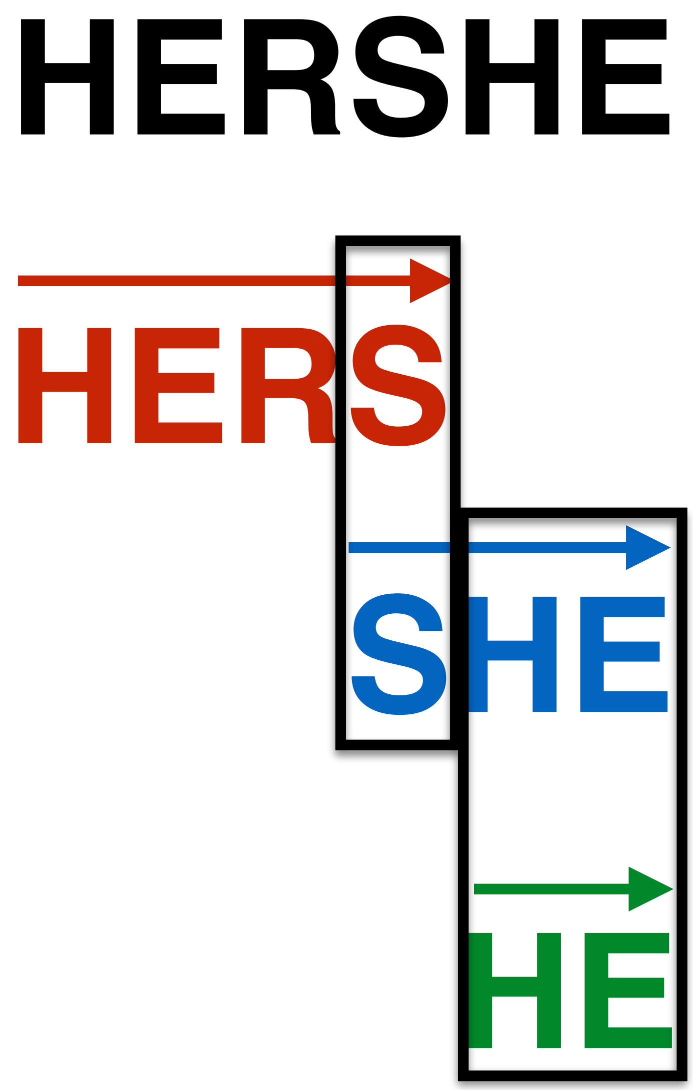
HERS

SHE

HE

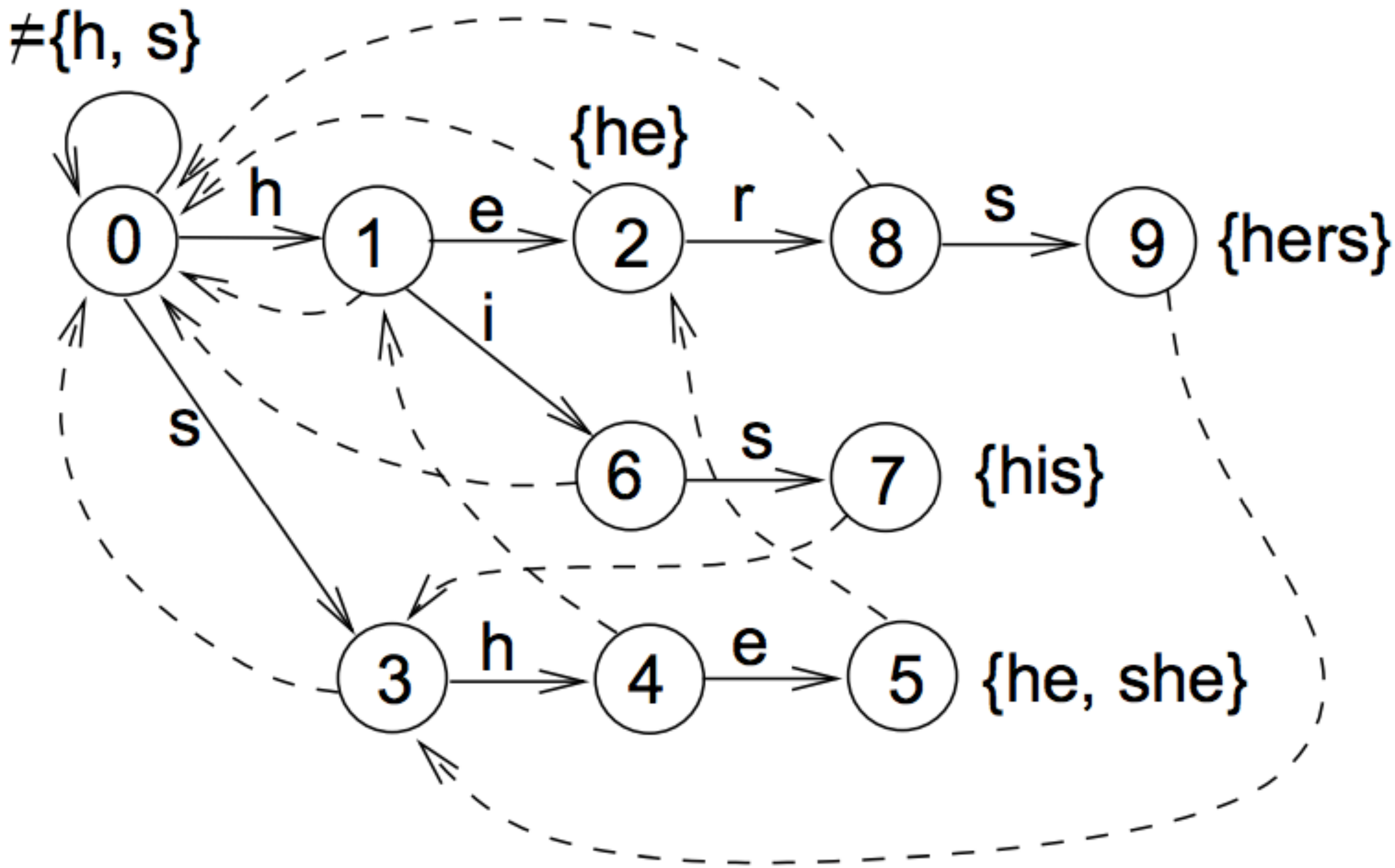
Aho-Corasick algorithm: Key idea

Exploit the redundancy in the patterns



Aho-Corasick algorithm

With failing edges and node labels



Rules

- Transition among the different nodes by following edges depending on next character seen (say “h”)
- If outgoing edge with label “h”, follow it
- If no such edge, and are at root, stay
- If no such edge, and at non-root, follow dashes edge (“fail” transition); DO NOT CONSUME THE CHARACTER (say “h”)

Consider text “**hershe**”