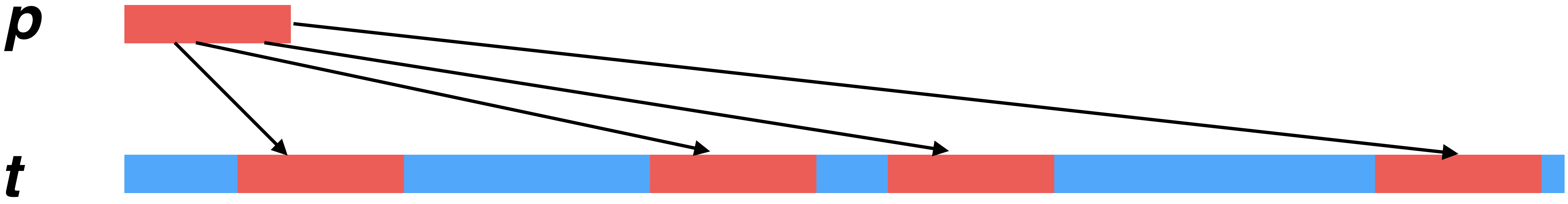
Exact Pattern Matching



Goal: *Find all occurrences of a pattern in a text*

Input: Pattern $\boldsymbol{p} = p_1 \ldots p_n$ and text $\boldsymbol{t} = t_1 \ldots t_m$

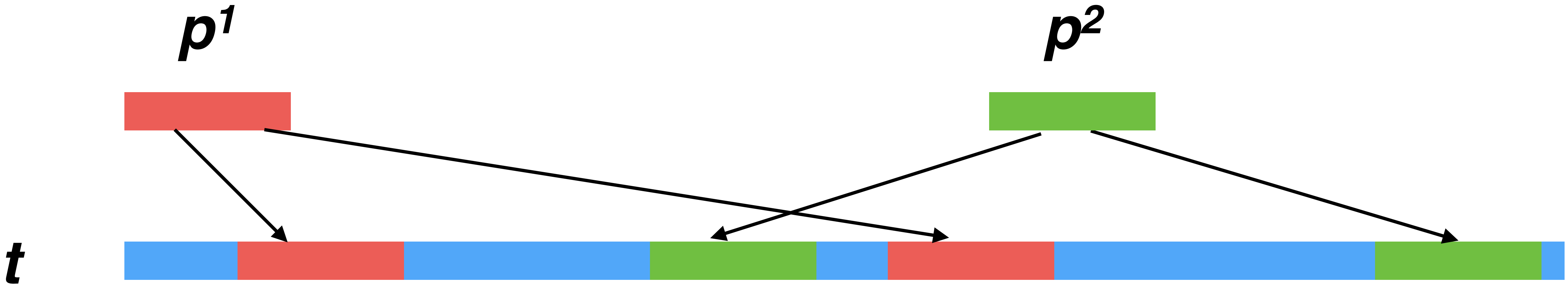
Output: All positions $1 \leq i \leq (m - n + 1)$ such that the n-letter substring of $\boldsymbol{t}$ starting at i matches $\boldsymbol{p}$

Motivation: Searching database for a known pattern

Pattern Matching: Running Time

- Naïve runtime: O(nm)
  - How?
- On average, it should be close to O(m)
  - Why?
- Can solve problem in O(m) time ?
  - Yes, we’ll see how (in a later lecture)

Multiple Pattern Matching



Goal: Given a set of patterns and a text, find all occurrences of any of patterns in text

Input: k patterns $\mathbf{p}^1,\ldots,\mathbf{p}^k$, and text $\mathbf{t} = t_1\ldots t_m$

Output: Positions $1 \leq i \leq m$ where substring of $\mathbf{t}$ starting at i matches $\mathbf{p}_j$ for $1 \leq j \leq k$

Motivation: Searching database for known multiple patterns

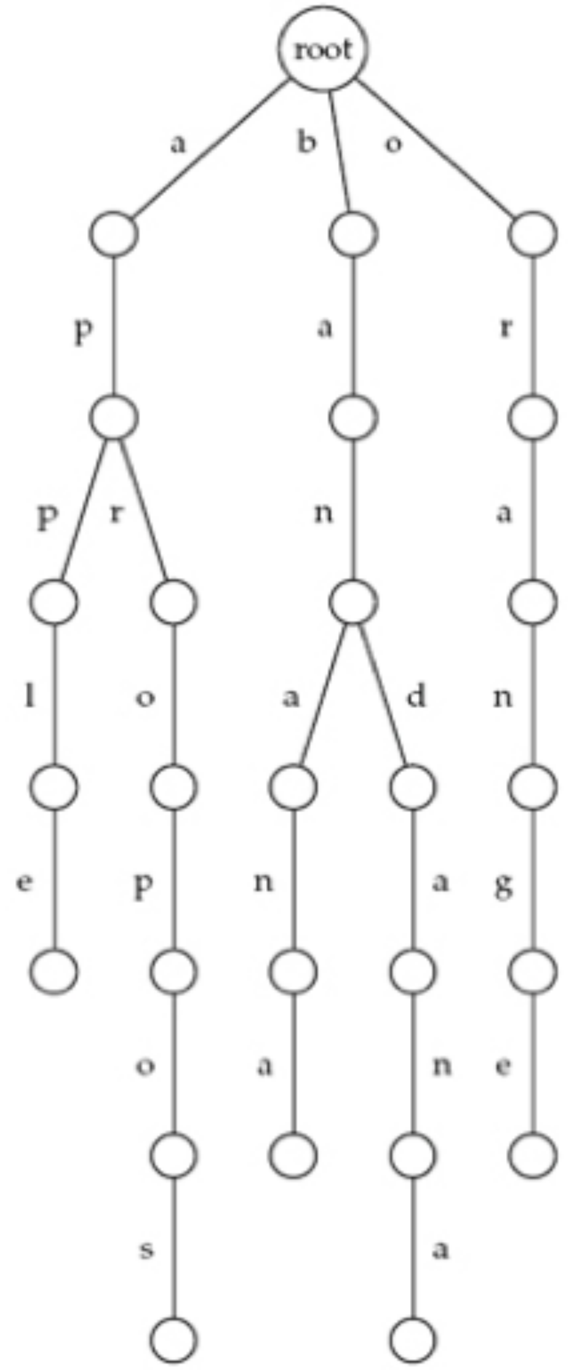
Multiple Pattern Matching

- **Solution:** k “pattern matching problems”: O(kmn)

- **Another Solution:**
  - Using “Keyword trees” => O(kn+nm) where n is maximum length of $\mathbf{p}^i$
  - Preprocess all *k* patterns to construct a “keyword tree”
  - Now, any given text, all occurrences of all patterns can be found in time O(m)

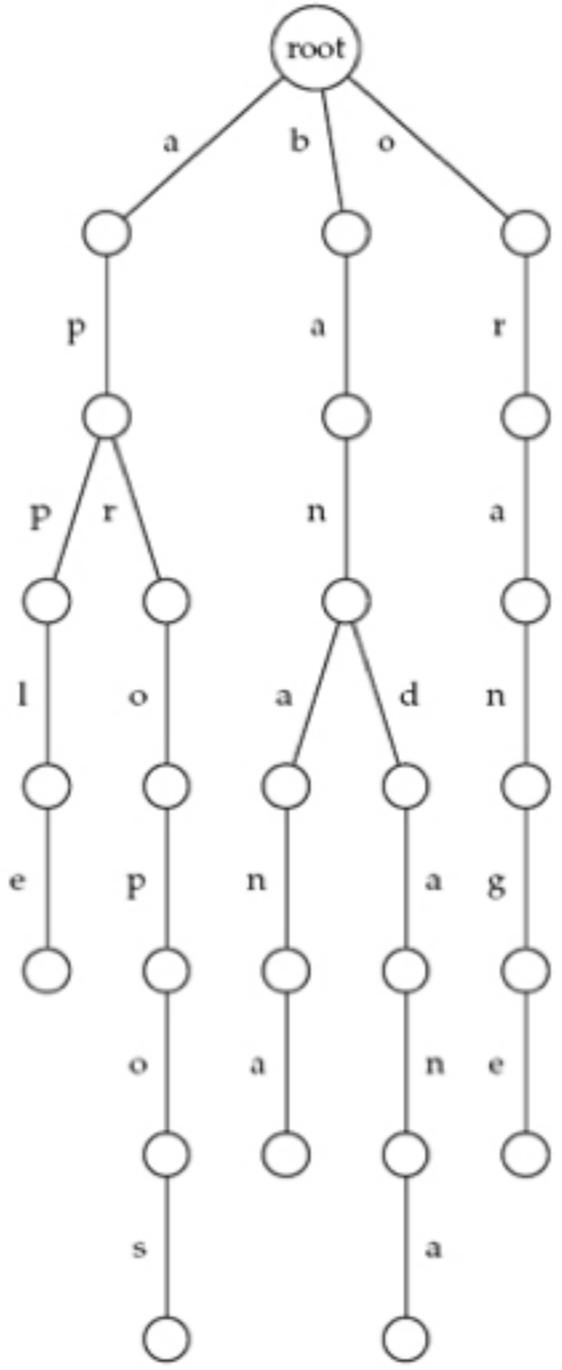
Keyword tree approach

- ***Keyword tree***:
  - Apple
  - Apropos
  - Banana
  - Bandana
  - Orange



Keyword tree approach: Properties

- Stores a set of keywords in a rooted labeled tree
- Each edge labeled with a letter from an alphabet
- Any two edges coming out of the same vertex have distinct labels
- Every keyword stored can be spelled on a path from root to some leaf



Keyword tree: Construction

Construction for $\mathcal{P} = \{P_1, \ldots, P_k\}$:

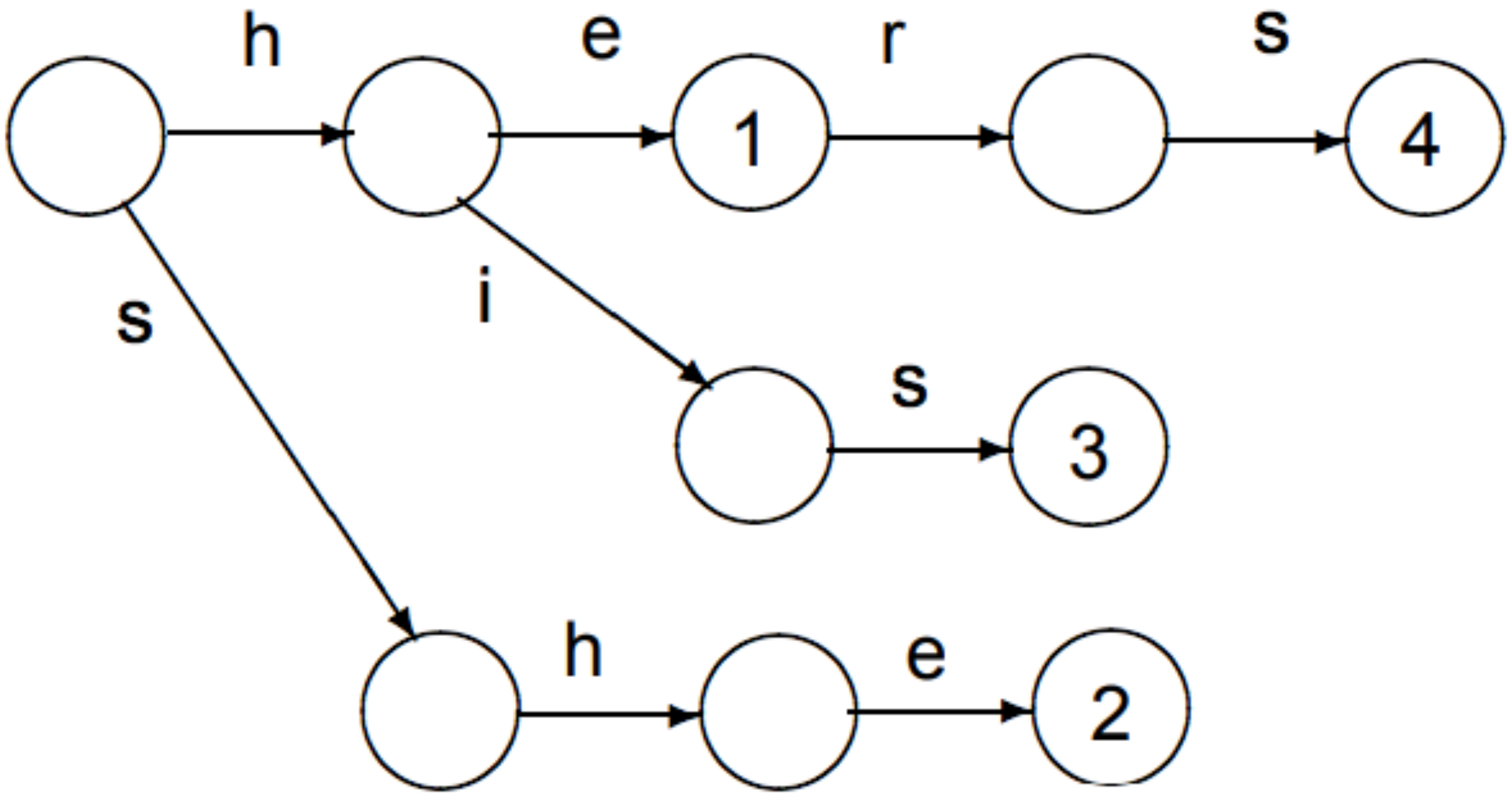
Begin with a root node only;
Insert each pattern P_i, one after the other, as follows:

Starting at the root, follow the path labeled by chars of P_i;

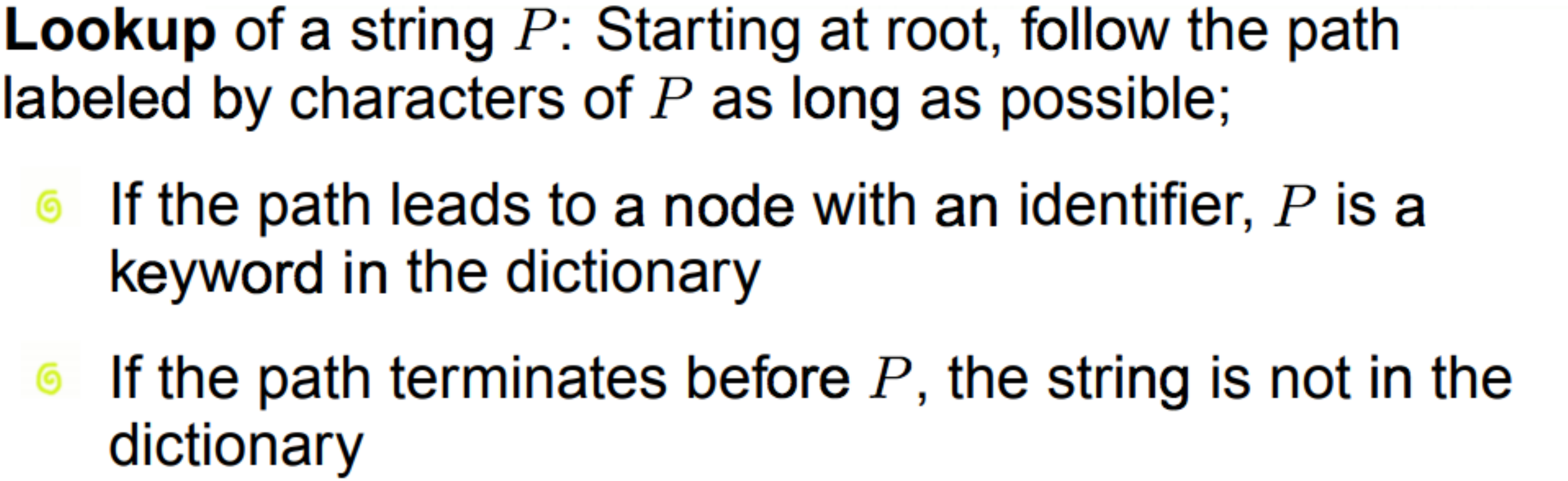
- If the path ends before P_i, continue it by adding new edges and nodes for the remaining characters of P_i
- Store identifier i of P_i at the terminal node of the path

This takes clearly $O(|P_1| + \cdots + |P_k|)$

A keyword tree for $\mathcal{P} = \{\text{he}, \text{she}, \text{his}, \text{hers}\}$:



Keyword tree: Lookup of a string



Lookup of a string P: Starting at root, follow the path labeled by characters of P as long as possible;

- If the path leads to a node with an identifier, P is a keyword in the dictionary
- If the path terminates before P, the string is not in the dictionary

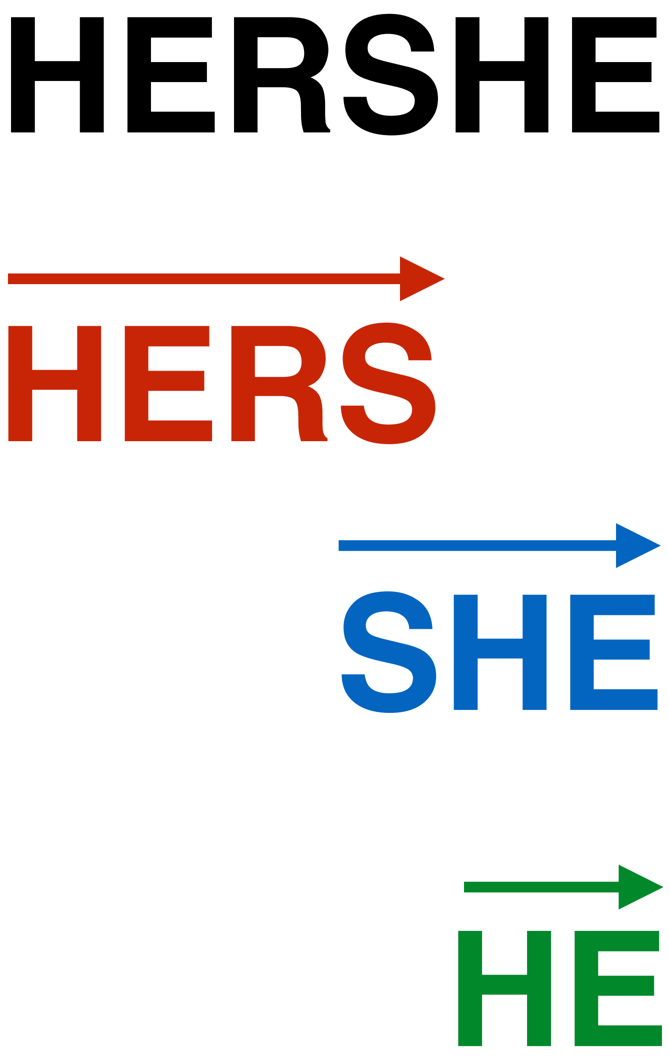
How to check all occurrences in a text ***t***?

Keyword tree approach: Complexity

- Build keyword tree in O(kn) time; kn is total length of all patterns
- Start “threading” at each position in text; at most n steps tell us if there is a match here to any $\mathbf{p}^i$
- O(kn + nm)
  - We’re down from O(kmn) to this
- The next big idea, Aho-Corasick algorithm: O(kn + m)

Aho-Corasick algorithm: Key idea

Exploit the redundancy in the patterns



HERSHE

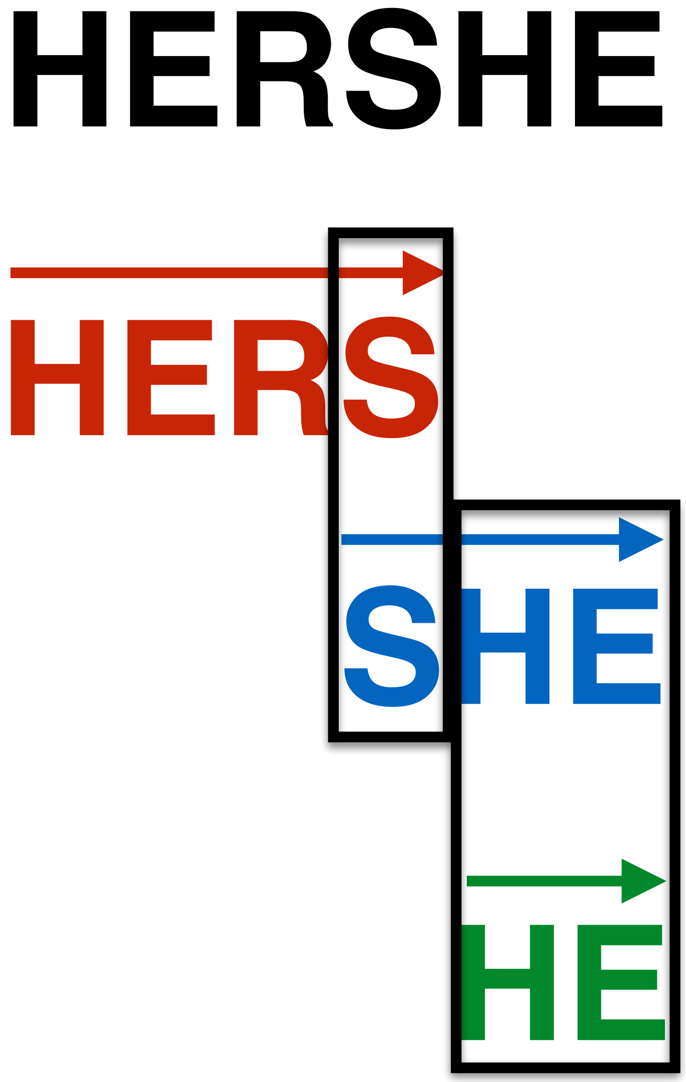
HERS

SHE

HE

Aho-Corasick algorithm: Key idea

Exploit the redundancy in the patterns



HERSHE

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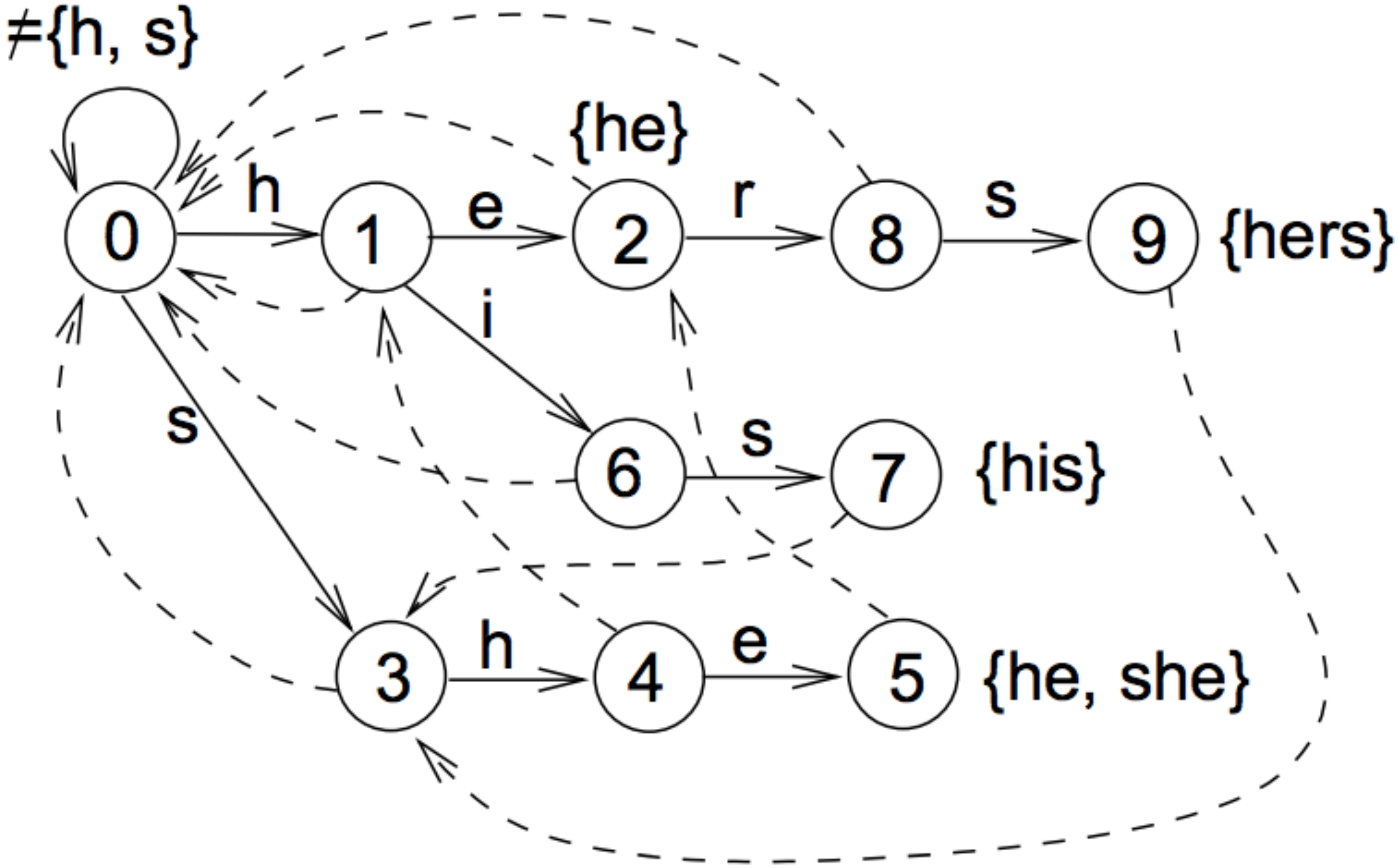
SHE

HE

Aho-Corasick algorithm

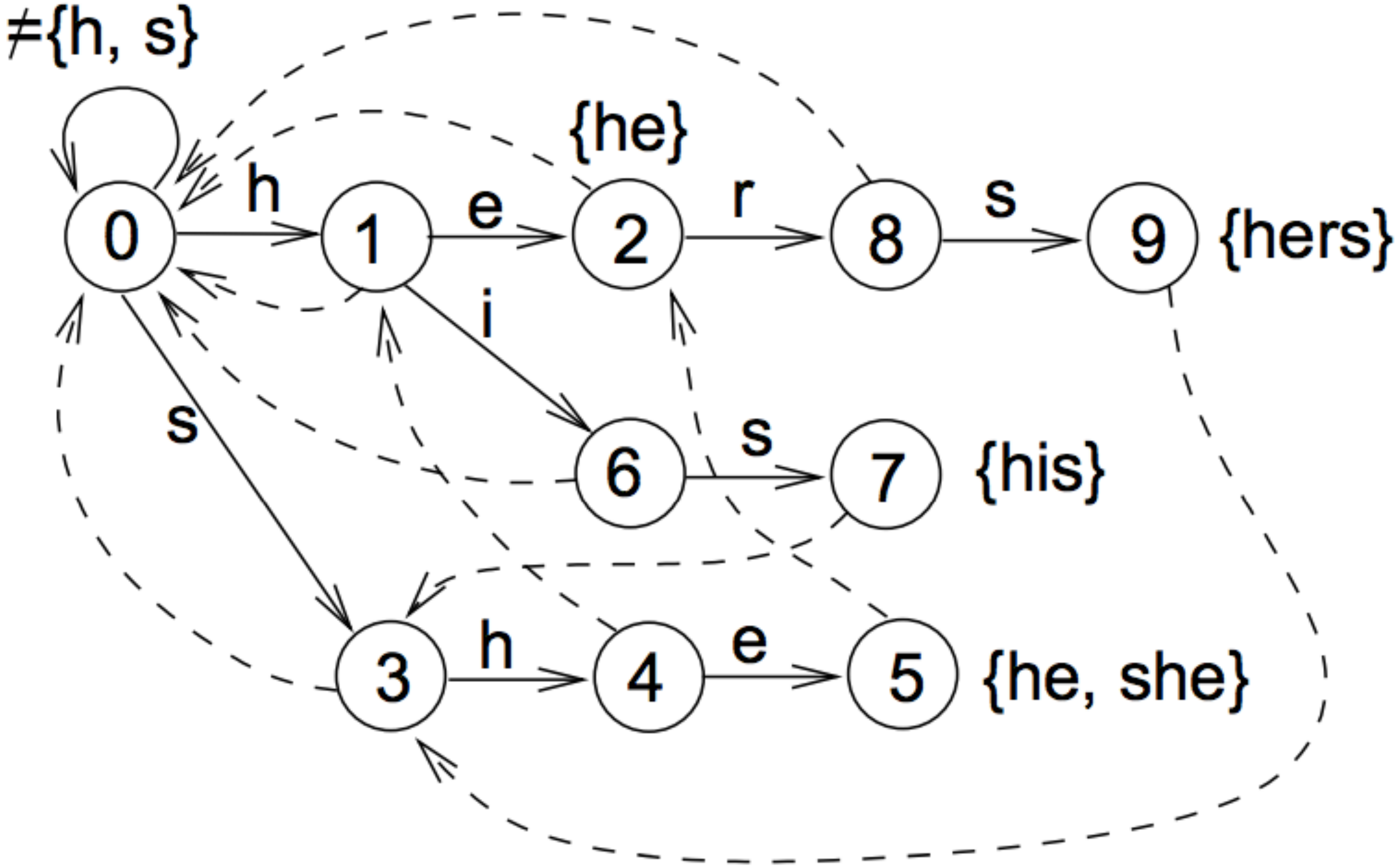
With failing edges and node labels



Rules

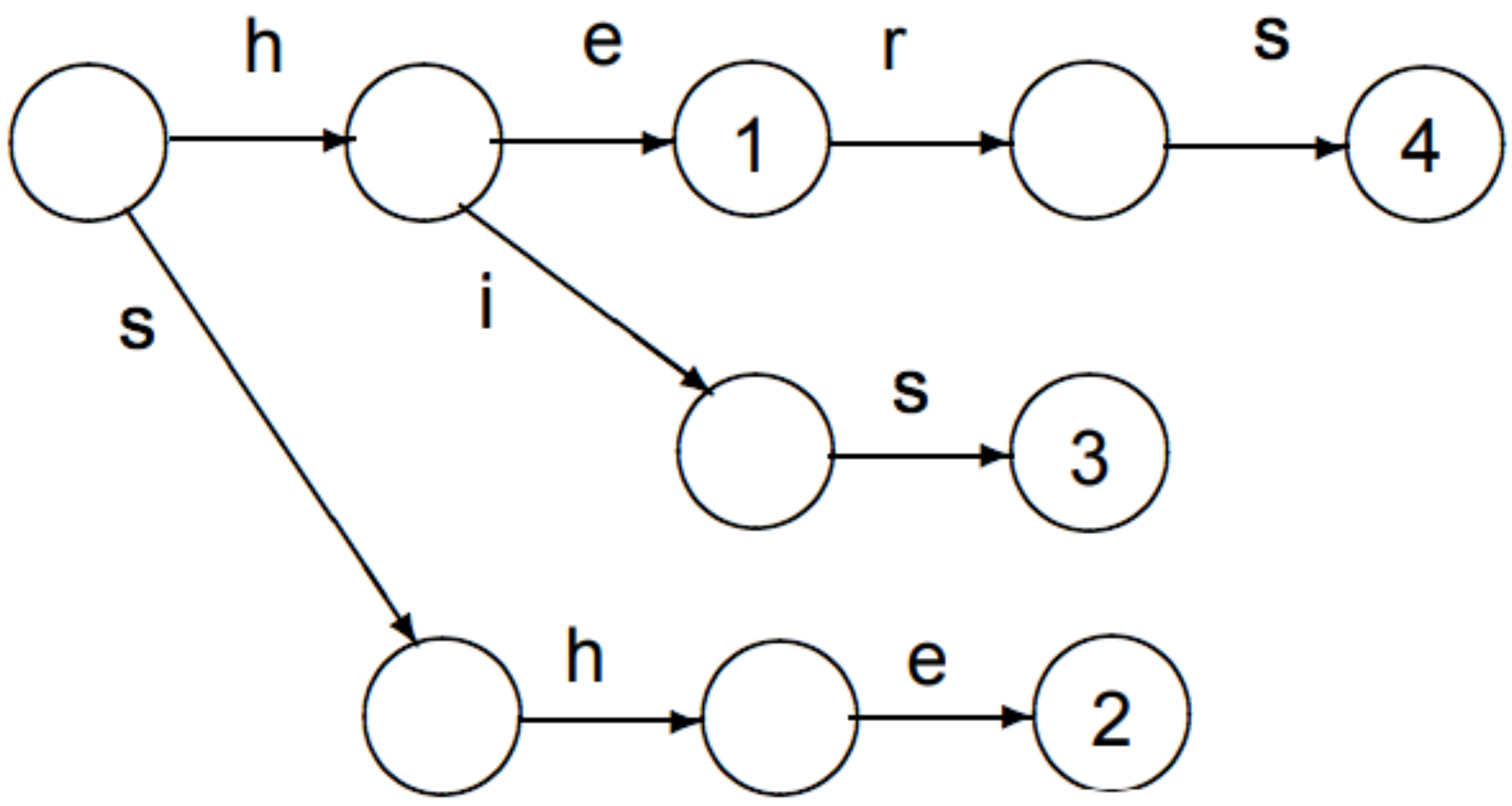
- Transition among the different nodes by following edges depending on next character seen (say “h”)
- If outgoing edge with label “h”, follow it
- If no such edge, and are at root, stay
- If no such edge, and at non-root, follow dashes edge (“fail” transition); DO NOT CONSUME THE CHARACTER (say “h”)

Consider text “**hershe**”



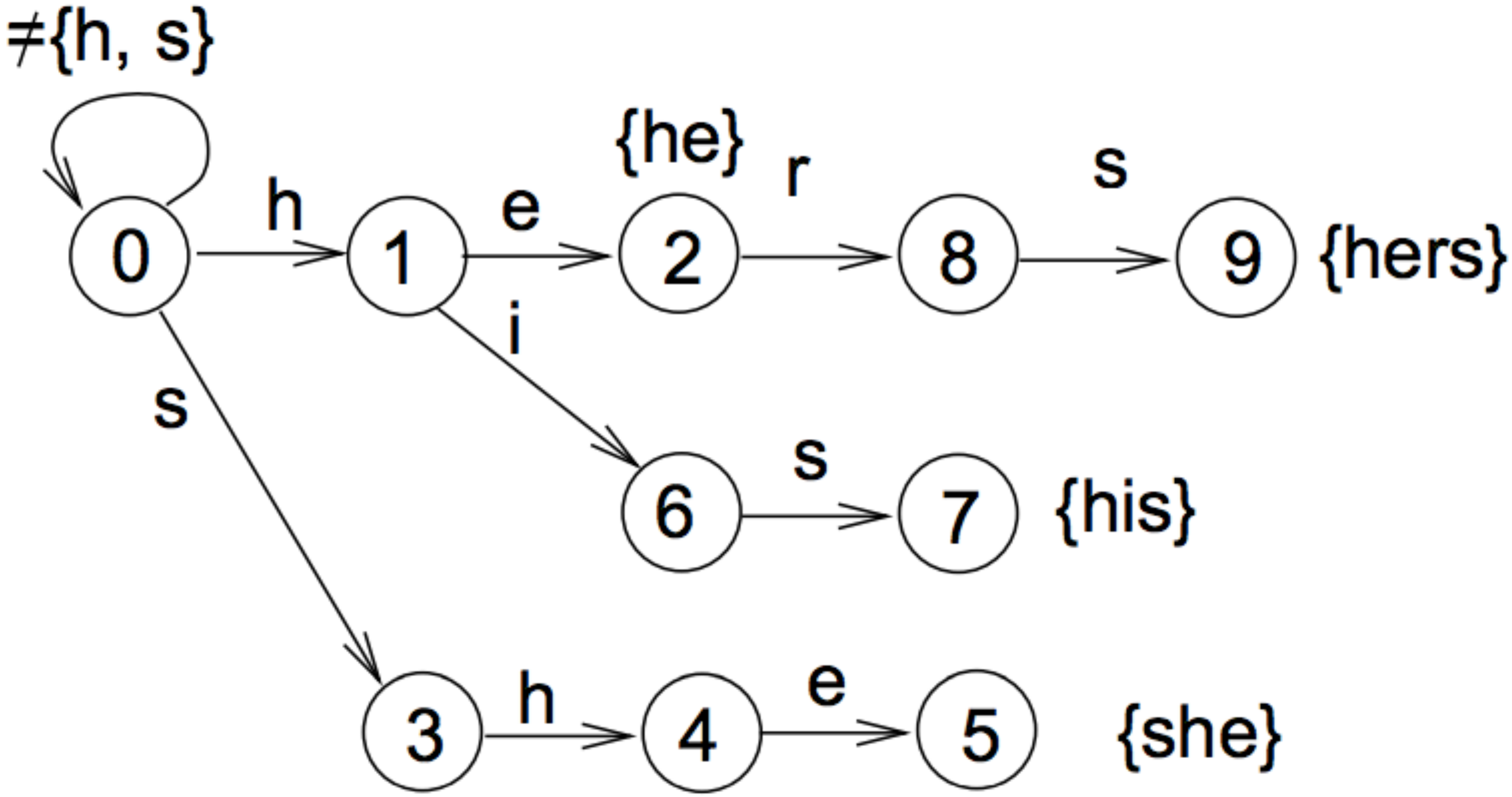
Aho-Corasick algorithm

A keyword tree for $\mathcal{P} = \{\text{he}, \text{she}, \text{his}, \text{hers}\}$:



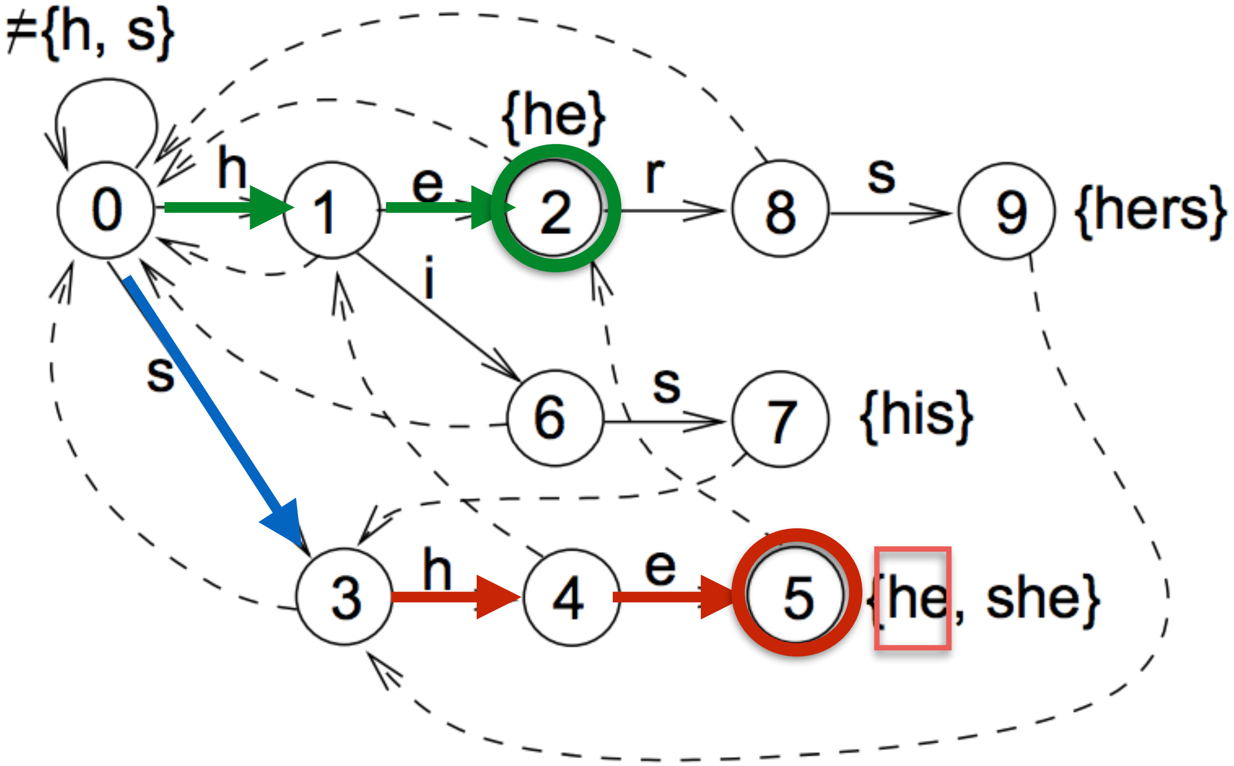
Aho-Corasick algorithm

Add pattern labels



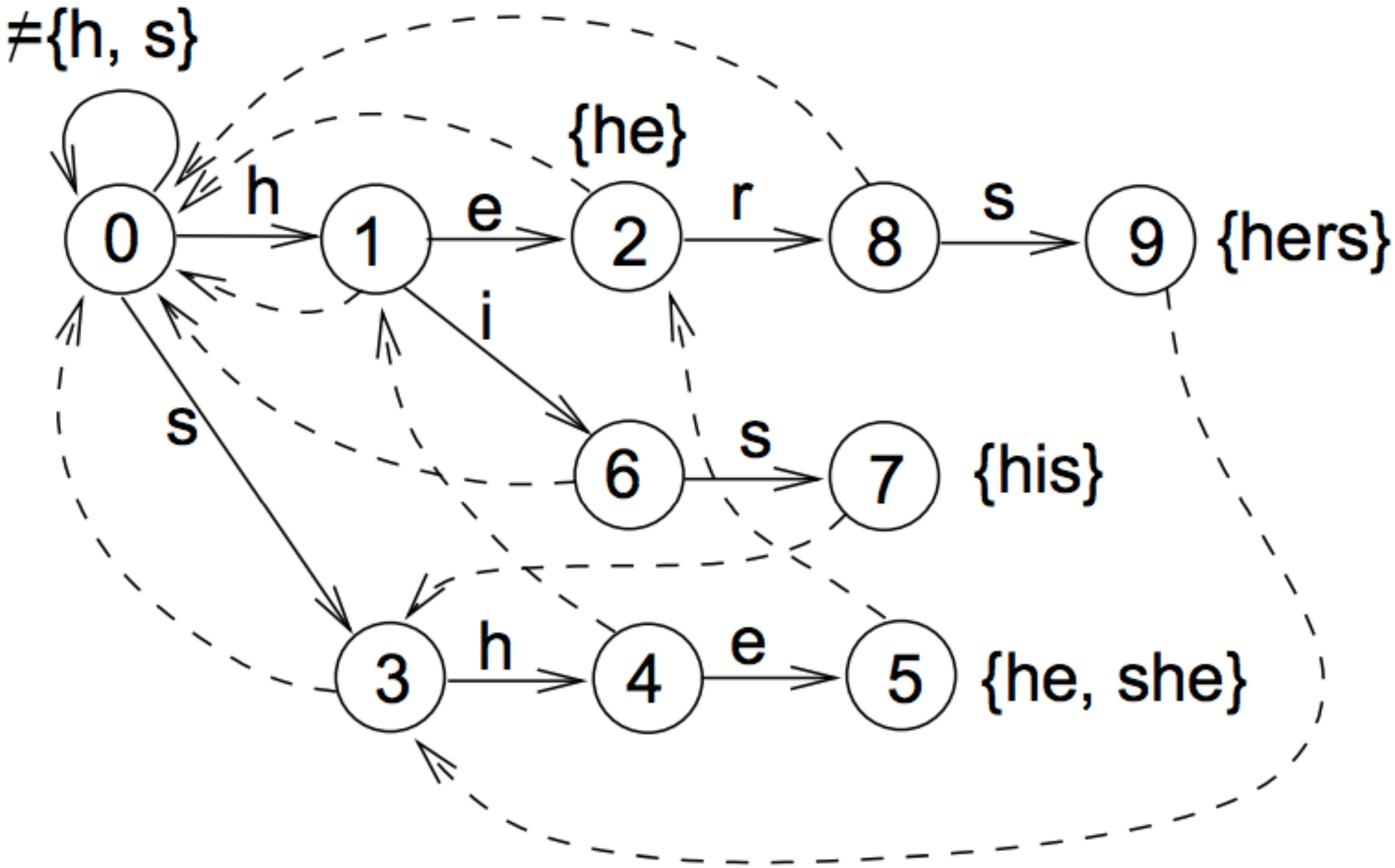
Adding failing edges

- If currently at node q representing word L(q), find the longest proper suffix of L(q) that is a prefix of some pattern, and go to the node representing that prefix. Insert the labels of the pointed node (if there is any) to node q's set of labels.
- Example: node q = 5, L(q) = she; longest proper suffix that is a prefix of some pattern: "he". Dashed edge to node q'=2



Aho-Corasick Algorithm

Add Failing Edges and Labels



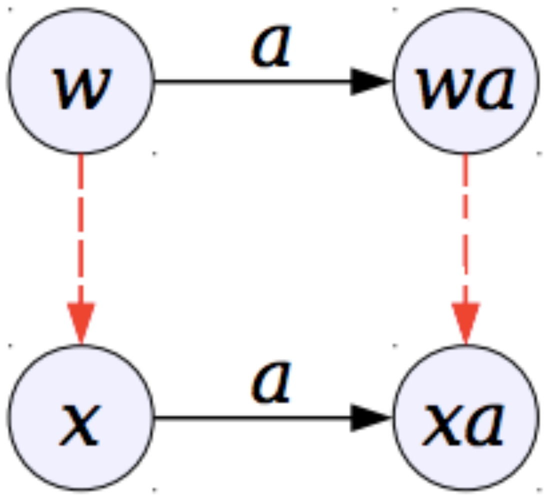
Aho-Corasick Algorithm: Construction

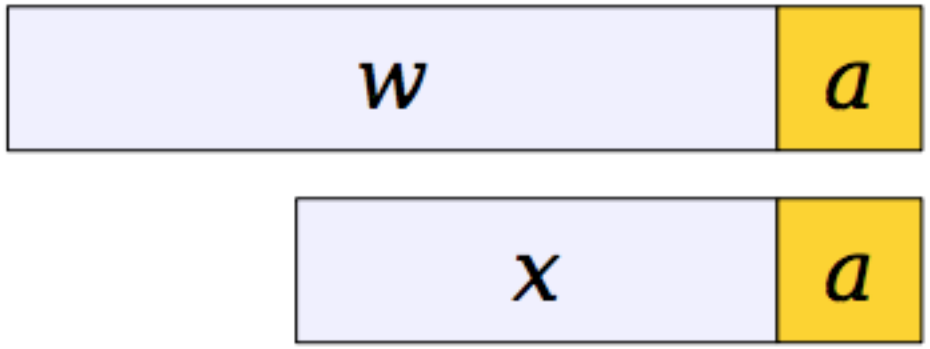
What about a naive algorithm?

A better algorithm: intuition

Suppose we already know the failing edge from a node **w** to **x**. If we follow a solid edge with label **a**, there are two possibilities:

- ***Case 1:*** *xa* exists.

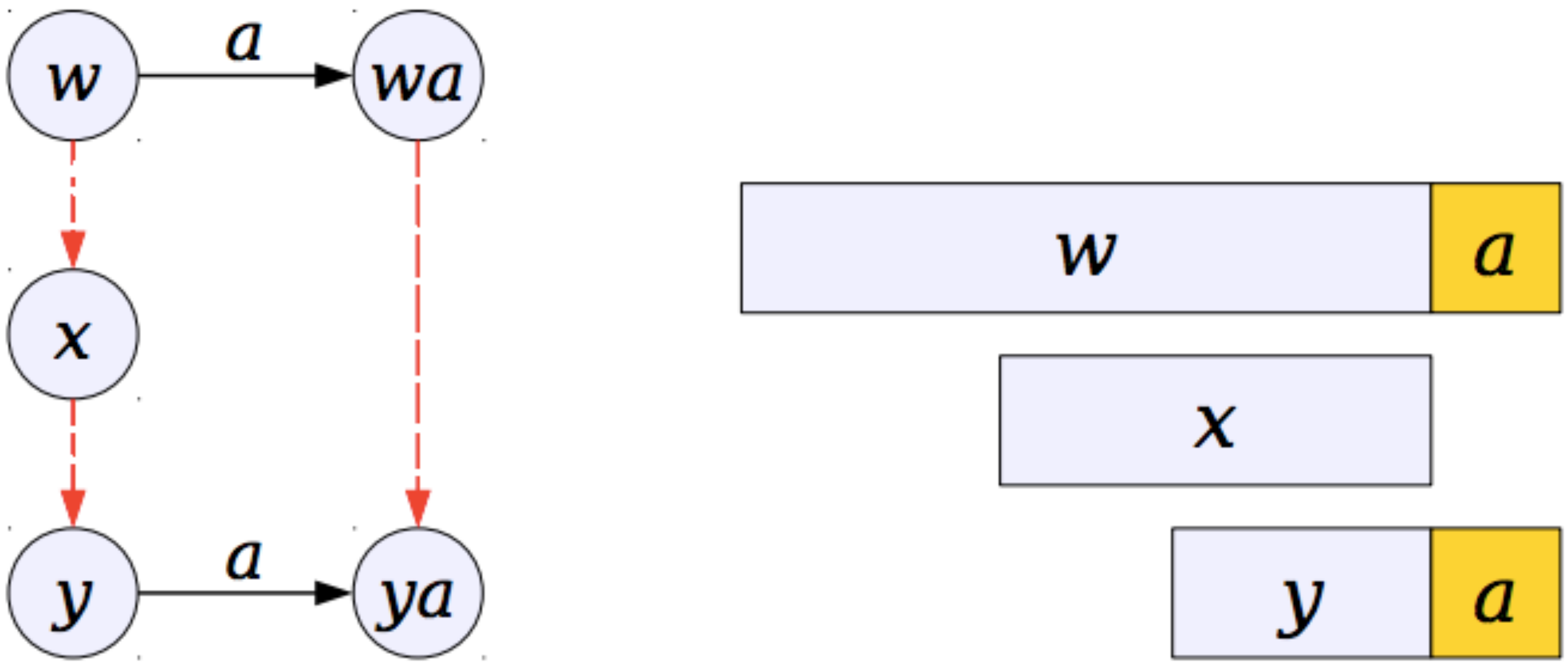




A better algorithm: intuition

Suppose we already know the failing edge from a node **w** to **x**. If we follow a solid edge with label **a**, there are two possibilities:

- ***Case 2:*** *xa* does not exist.



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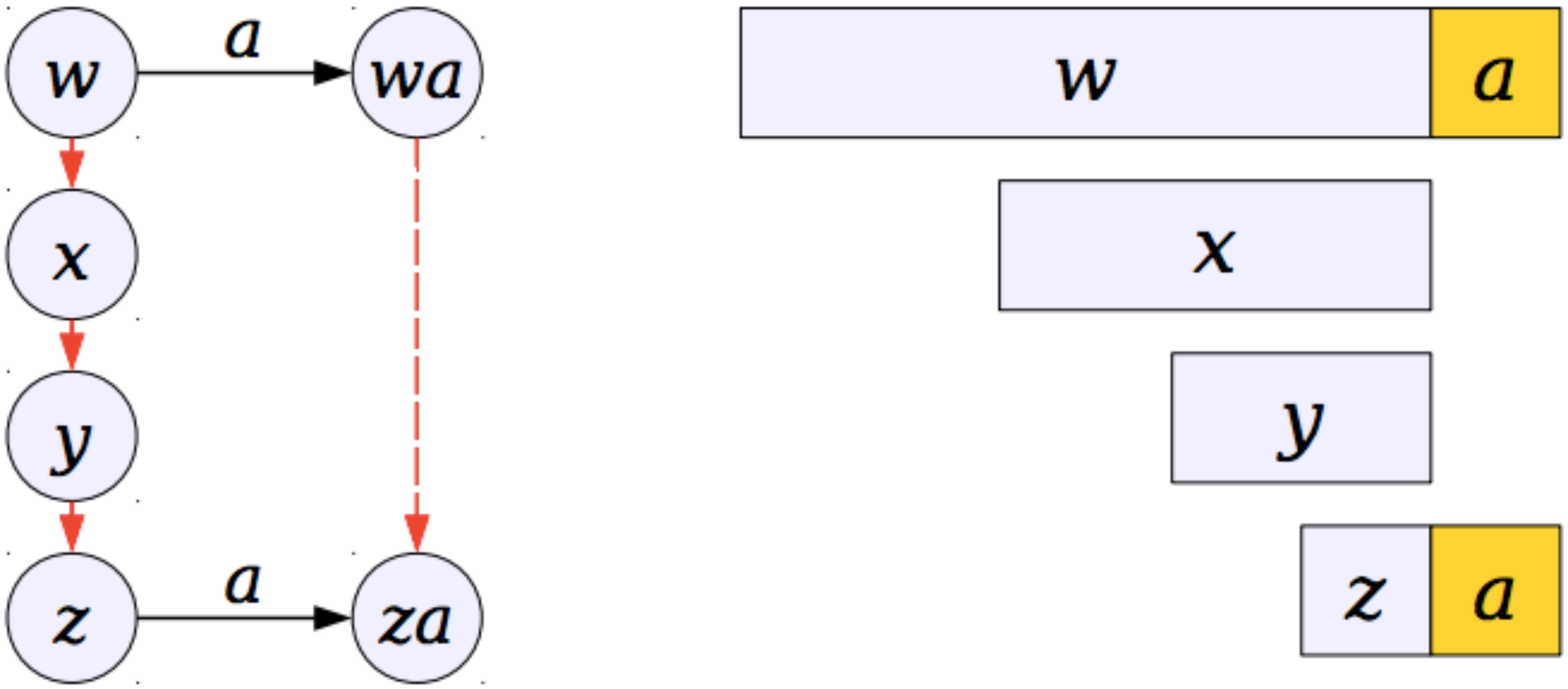

Constructing failing edge for a node

- To construct the failing edge for a node **wa**:
  - Follow **w**'s failing edge to node **x**.
  - If node **xa** exists, **wa** has a failing edge to **xa**.
  - Otherwise, follow **x**'s failing edge and repeat.
  - If you need to follow all the way back to the root, then **wa**’s failing edge points to the root.

- *Observation* 1: Failing edges point from longer strings to shorter strings.
- *Observation* 2: If we precompute failing edges for nodes in ascending order of string length, all of the information needed for the above approach will be available at the time we need it.

Complexity

- Focus on the time to fill in the failing edges for a single pattern of length **n**.
  - The failing edges moves one-step backward because it always points to a shorter string.
  - The solid edges moves one-step forward.
  - We cannot take more steps backward than forward. Therefore, across the entire construction, we can take at most **n** steps backward for this pattern.

- Total time required to construct failing edges for a pattern of length n: O(n).
- Total time required to construct failing edges for all k patterns: O(kn).