CS447: Natural Language Processing

http://courses.engr.illinois.edu/cs447

Lecture 13: Recurrent Neural Nets

Julia Hockenmaier

juliahmr@illinois.edu

3324 Siebel Center

Brief Recap

1D CNNs for text

Text is a (variable-length) **sequence** of words (word vectors)

[#channels = dimensionality of word vectors]

We can use a **1D CNN** to slide a window of *n* tokens across:

— Filter size n = 3, stride = 1, no padding

The quick brown fox jumps over the lazy dog

The **quick brown fox** jumps over the lazy dog

The quick **brown fox jumps** over the lazy dog

The quick brown **fox jumps over** the lazy dog

The quick brown fox **jumps over the** lazy dog

The quick brown fox jumps **over the lazy** dog

— Filter size n = 2, stride = 2, no padding:

The quick brown fox jumps over the lazy dog

The quick **brown fox** jumps over the lazy dog

The quick brown fox **jumps over** the lazy dog

The quick brown fox jumps over **the lazy** dog

Sequence Labeling

Input: a sequence of *n* tokens/words:

`Pierre Vinken , 61 years old , will join IBM 's board as a nonexecutive director Nov. 29`

Output: a sequence of *n* labels, such that each token/word is associated with a label:

POS-tagging: `Pierre_NNP Vinken_NNP ,_, 61_CD years_NNS old_JJ ,_, will_MD join_VB IBM_NNP 's_POS board_NN as_IN a_DT nonexecutive_JJ director_NN Nov._NNP 29_CD ._.`

Named Entity Recognition: `Pierre_B-PERS Vinken_I-PERS ,_O 61_O years_O old_O ,_O will_O join_O IBM_B-ORG 's_O board_O as_O a_O nonexecutive_O director_O Nov._B-DATE 29_I-DATE ._O`

Statistical POS tagging

	She	promised	to	back	the	bill
$\mathbf{w} =$	$w^{(1)}$	$w^{(2)}$	$w^{(3)}$	$w^{(4)}$	$w^{(5)}$	$w^{(6)}$
$\mathbf{t} =$	$t^{(1)}$	$t^{(2)}$	$t^{(3)}$	$t^{(4)}$	$t^{(5)}$	$t^{(6)}$
	PRP	**VBD**	**TO**	**VB**	**DT**	**NN**

What is the most likely sequence of tags $\mathbf{t} = t^{(1)} \ldots t^{(N)}$ for the given sequence of words $\mathbf{w} = w^{(1)} \ldots w^{(N)}$?

$$\mathbf{t}^* = \mathrm{argmax}_{\mathbf{t}}\, P(\mathbf{t} \mid \mathbf{w})$$

Hidden Markov Models (HMMs)

HMMs are the most commonly used generative models for POS tagging (and other tasks, e.g. in speech recognition)

HMMs make specific **independence assumptions** in $P(\mathbf{t})$ and $P(\mathbf{w}|\ \mathbf{t})$:

1) $P(\mathbf{t})$ is an *n*-gram (typically **bigram** or **trigram**) model over tags:

$$P_{\text{bigram}}(\mathbf{t}) = \prod_i P(t^{(i)} \mid t^{(i-1)})$$

$$P_{\text{trigram}}(\mathbf{t}) = \prod_i P(t^{(i)} \mid t^{(i-1)}, t^{(i-2)})$$

$P(t^{(\mathrm{i})} \mid t^{(\mathrm{i}-1)})$ and $P(t^{(\mathrm{i})} \mid t^{(\mathrm{i}-1)}, t^{(\mathrm{i}-2)})$ are called **transition probabilities**

2) In $P(\mathbf{w} \mid \mathbf{t})$, each $\mathrm{w}^{(\mathrm{i})}$ depends only on [is generated by/conditioned on] $\mathrm{t}^{(\mathrm{i})}$:

$$P(\mathbf{w} \mid \mathbf{t}) = \prod_i P(w^{(i)} \mid t^{(i)})$$

$P(w^{(\mathrm{i})} \mid t^{(\mathrm{i})})$ are called **emission probabilities**

These probabilities don't depend on the position in the sentence $^{(\mathrm{i})}$,
but are defined over word and tag types.
With subscripts $_{\mathrm{i,j,k}}$, to index word/tag types, they become $P(t_i \mid t_j)$, $P(t_i \mid t_j, t_k)$, $P(w_i \mid t_j)$

Today's lecture

Part 1: Recurrent Neural Nets for various NLP tasks

Part 2: Practicalities:

Training RNNs
Generating with RNNs
Using RNNs in complex networks

Part 3: Changing the recurrent architecture
to go beyond vanilla RNNs:
LSTMs, GRUs

Part 1: Recurrent Neural Nets for various NLP tasks

Recurrent Neural Nets (RNNs)

Feedforward nets can only handle inputs and outputs that have a **fixed size**.

Recurrent Neural Nets (RNNs) handle **variable length sequences** (as **input** and as **output**)

There are 3 main variants of RNNs, which differ in their internal structure:

Basic RNNs (Elman nets),
Long Short-Term Memory cells (LSTMs)
Gated Recurrent Units (GRUs)

RNNs in NLP

RNNs are used for…

… language modeling and generation, including…

… auto-completion and…

… machine translation

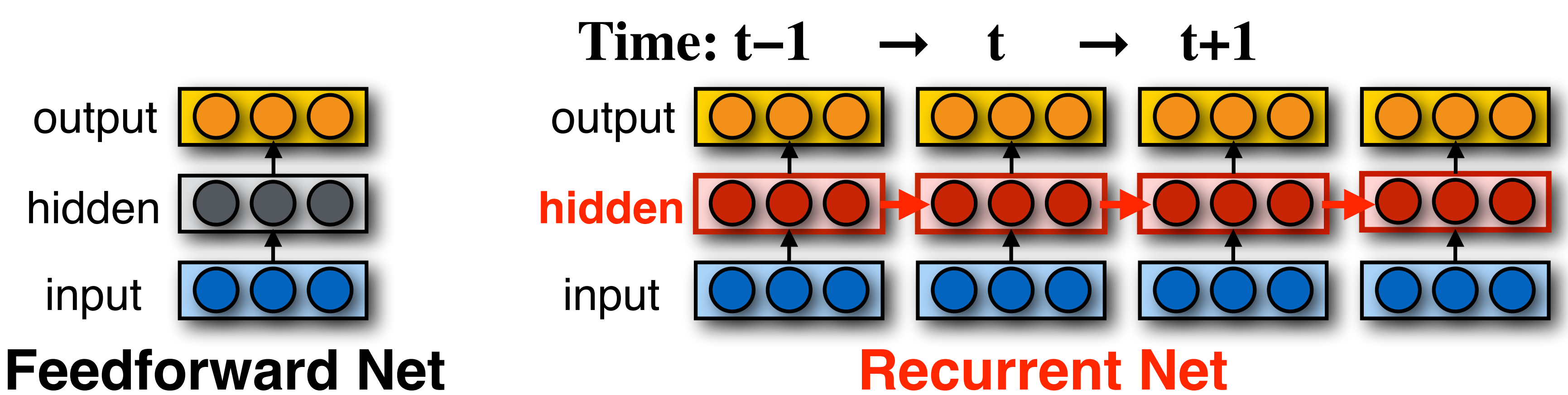
… sequence classification (e.g. sentiment analysis)

… sequence labeling (e.g. POS tagging)

Recurrent neural networks (RNNs)

Basic RNN: Process a **sequence of *T* inputs**
and/or generate a **sequence of *T* outputs**
by running a [variant of a feedforward] net T times.

Recurrence:
The **hidden state** computed at the **previous step ($\mathbf{h}^{(t-1)}$)**
is fed into **the hidden state** at the **current step ($\mathbf{h}^{(t)}$)**
With H hidden units, this requires additional H^2 parameters

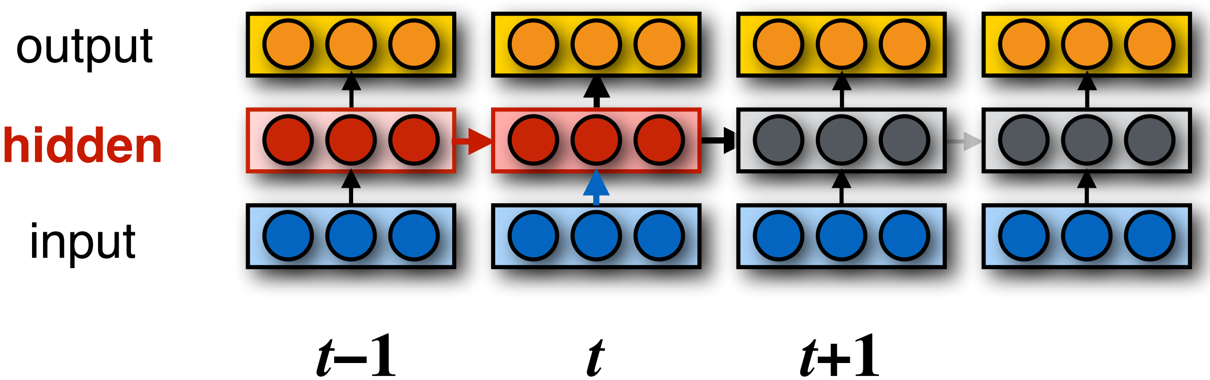


Basic RNNs

Basic RNNs are feedforward nets where
the **hidden layer** gets its input at **time *t*…**

… from the activations of **the input layer**
computed **at the *same* time *t*,** and

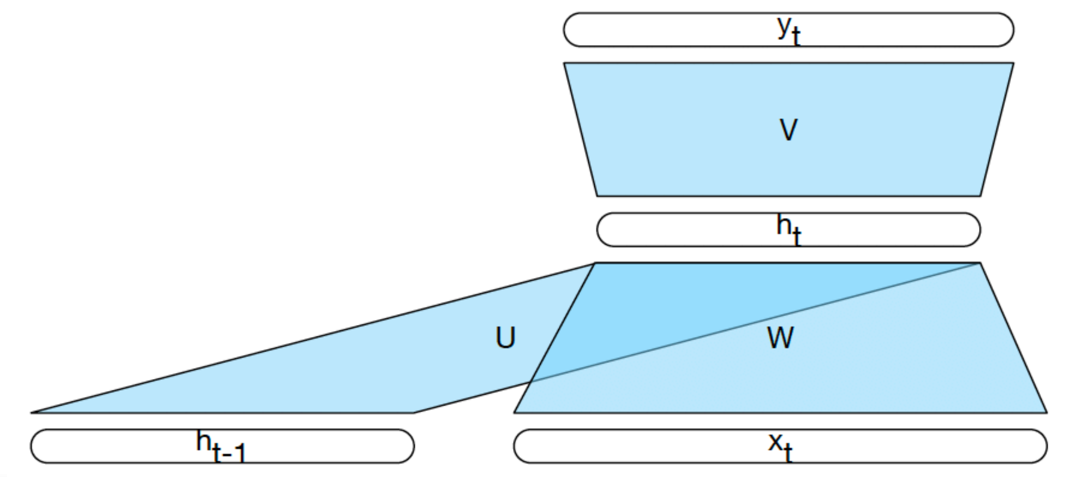
… from the activations of **the same hidden layer**
at **the *previous* time *t*–1**:



The input may change at each time step ***t*,**
but the feedforward net is the same at each time ***t***

Basic RNNs

Each time step t corresponds to a **feedforward net** whose **hidden layer h**$^{(t)}$ gets input from the **layer below** (**x**$^{(t)}$) and from the output of the **hidden layer at the previous time step h**$^{(t-1)}$

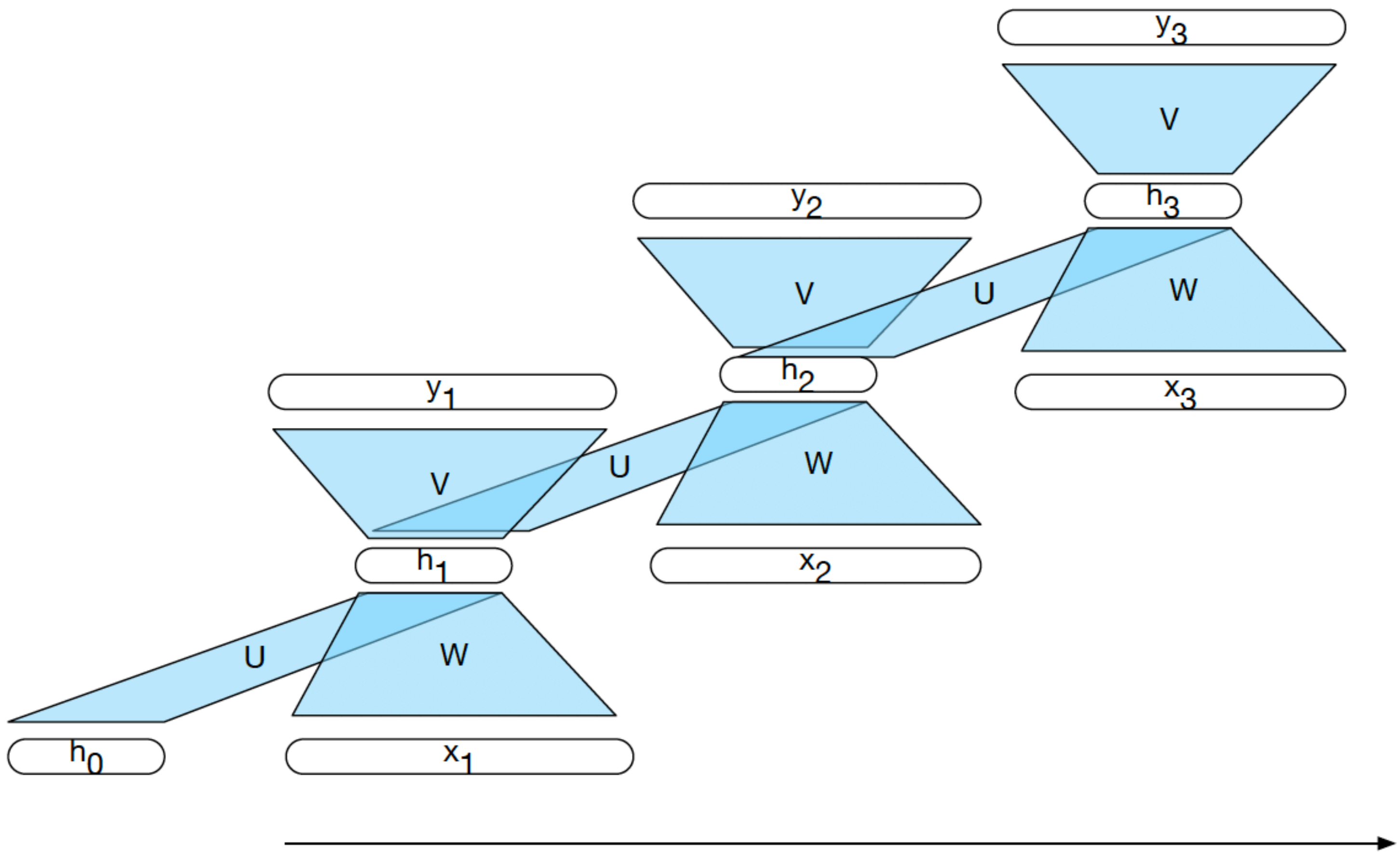


Computing the **vector of hidden states** at time *t*

$$\mathbf{h}^{(t)} = g(\mathbf{U}\mathbf{h}^{(t-1)} + \mathbf{W}\mathbf{x}^{(t)})$$

The ***i*-th element** of **h**$_t$: $h_i^{(t)} = g\left(\sum_j U_{ji} h_j^{(t-1)} + \sum_k W_{ki} x_k^{(t)}\right)$

A basic RNN unrolled in time



RNNs for language modeling

If our vocabulary consists of V words, the output layer (at each time step) has V units, one for each word.

The softmax gives a distribution over the V words for the next word.

To compute the **probability of string** $w^{(0)}w^{(1)}\ldots w^{(n)}w^{(n+1)}$ (where $w^{(0)} = \langle s \rangle$, and $w^{(n+1)} = \langle \backslash s \rangle$), feed in $w^{(i-1)}$ as input at time step i and compute

$$\prod_{i=1}^{n+1} P\left(w^{(i)} \mid w^{(0)} \ldots w^{(i-1)}\right)$$

RNNs for language generation

To **generate** $w^{(0)}w^{(1)}\ldots w^{(n)}w^{(n+1)}$
(where $w^{(0)} = $ <s>, and $w^{(n+1)} = $ <\s>)…

…Give $w^{(0)}$ as first input, and

… Choose the next word $w^{(i)}$ according to the probability

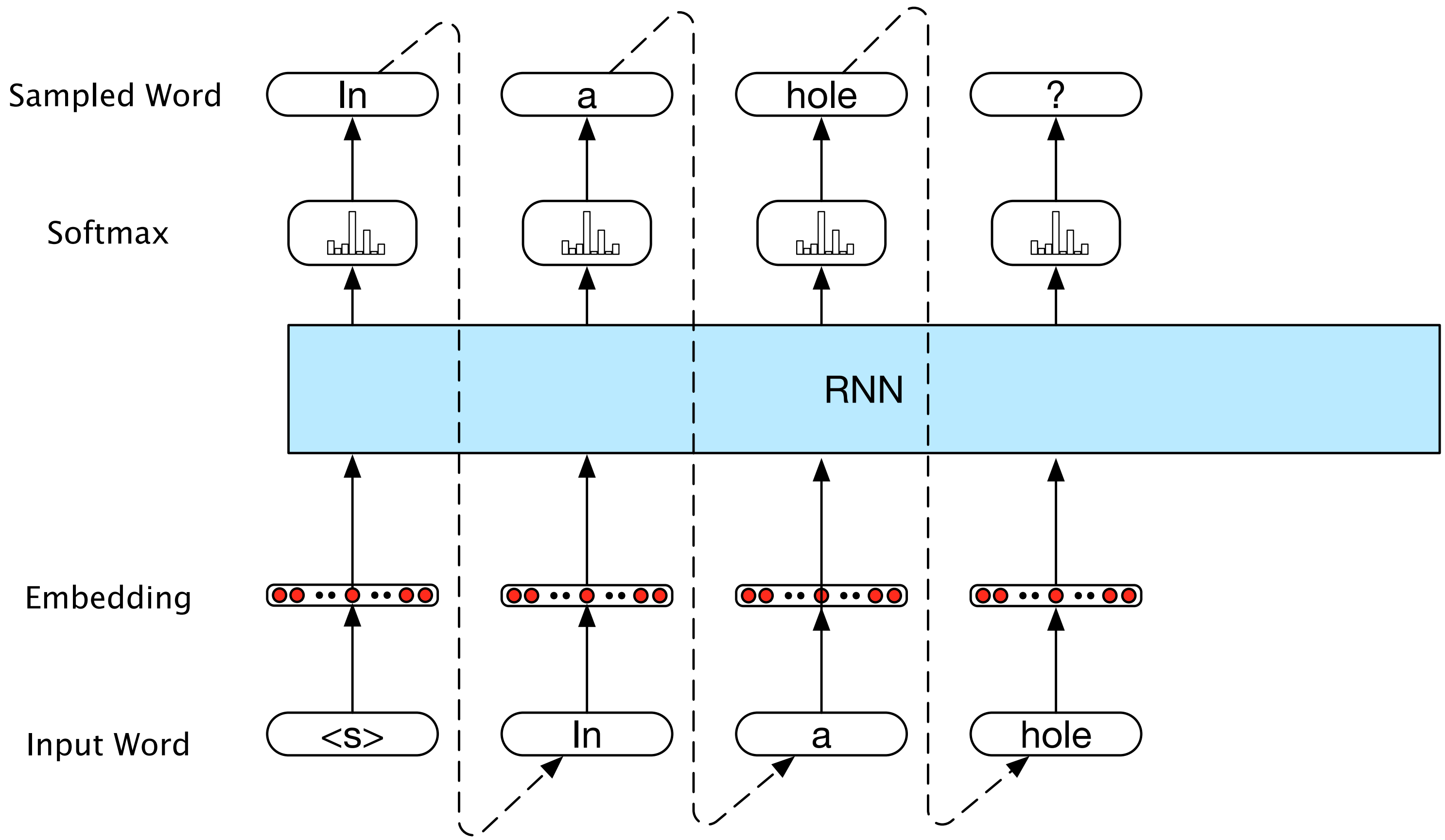
$$P(w^{(i)} \mid w^{(0)} \ldots w^{(i-1)})$$

…Feed the predicted word $w^{(i)}$ in as input
at the next time step.

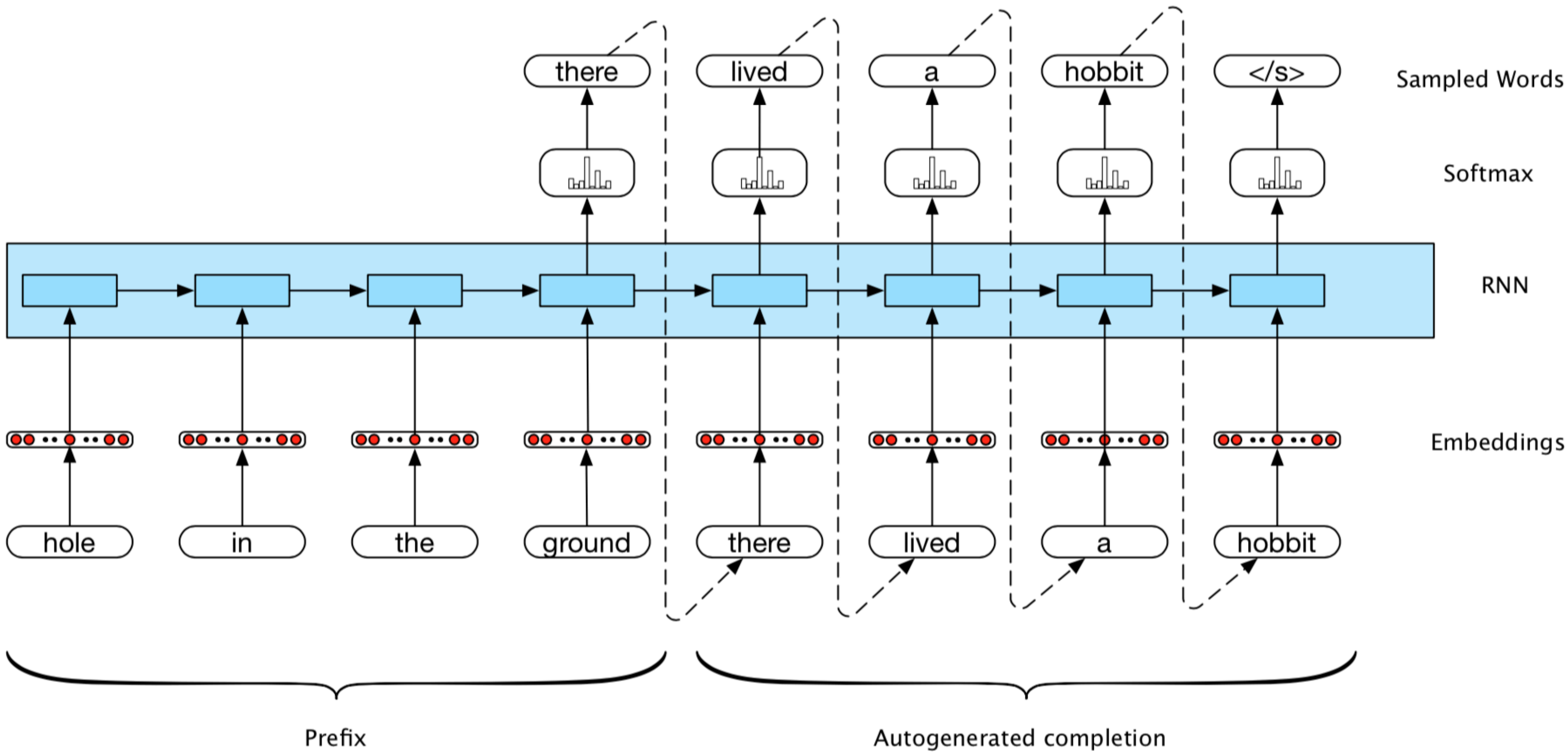
… Repeat until you generate <\s>

RNNs for language generation

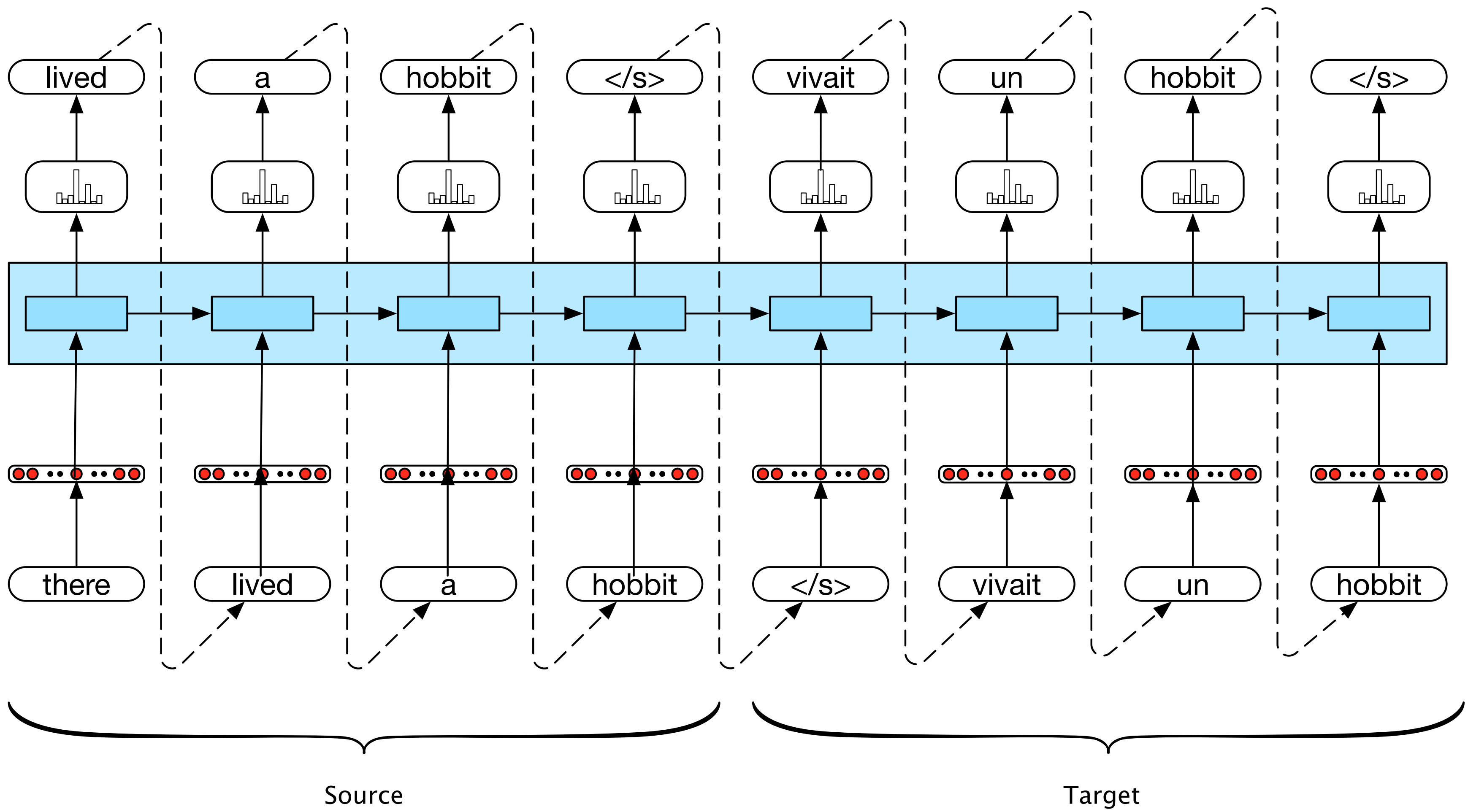
AKA “autoregressive generation”



RNN for Autocompletion



An RNN for Machine Translation



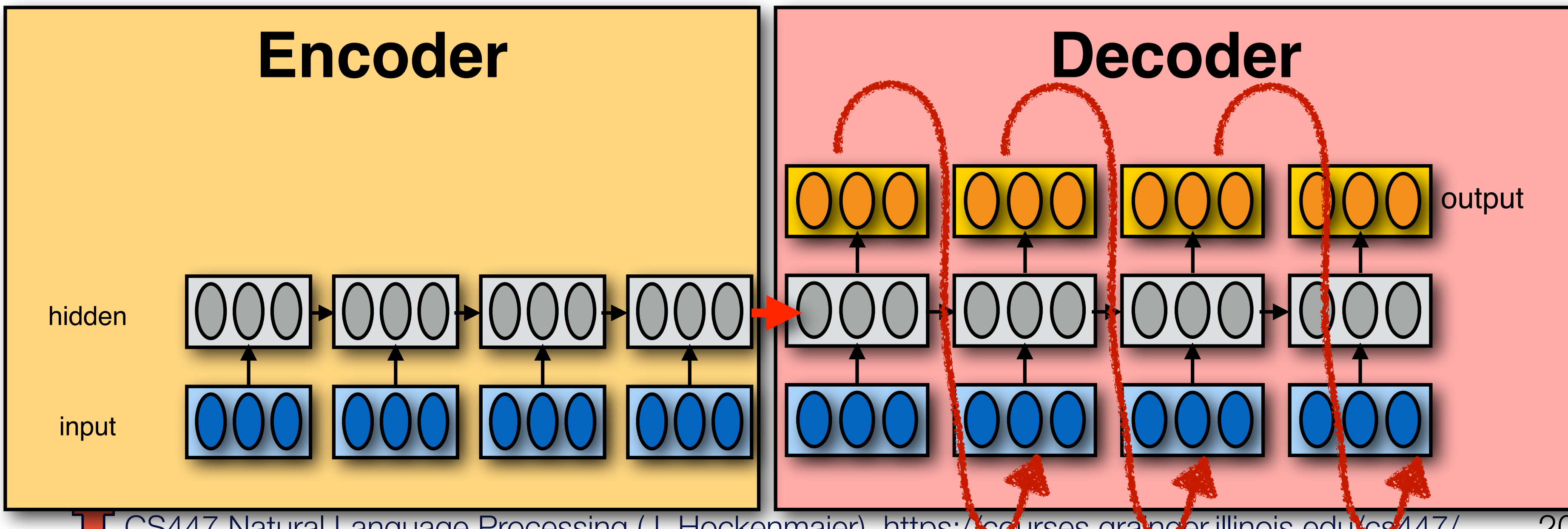
Encoder-Decoder (seq2seq) model

Task: Read an input sequence
and return an output sequence

- Machine translation: translate source into target language
- Dialog system/chatbot: generate a response

Reading the input sequence: **RNN Encoder**

Generating the output sequence: **RNN Decoder**



Encoder-Decoder (seq2seq) model

Encoder RNN:

reads in the input sequence

passes its last hidden state to the initial hidden state of the decoder

Decoder RNN:

generates the output sequence

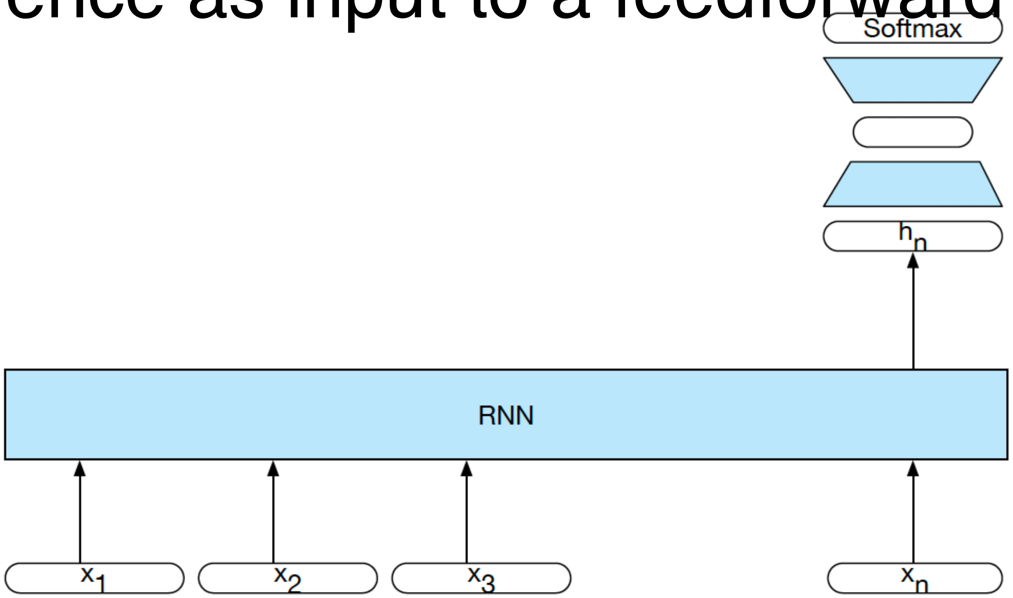
typically uses different parameters from the encoder

may also use different input embeddings

RNNs for sequence classification

If we just want to assign **one label** to the entire sequence, we don't need to produce output at each time step, so we can use a simpler architecture.

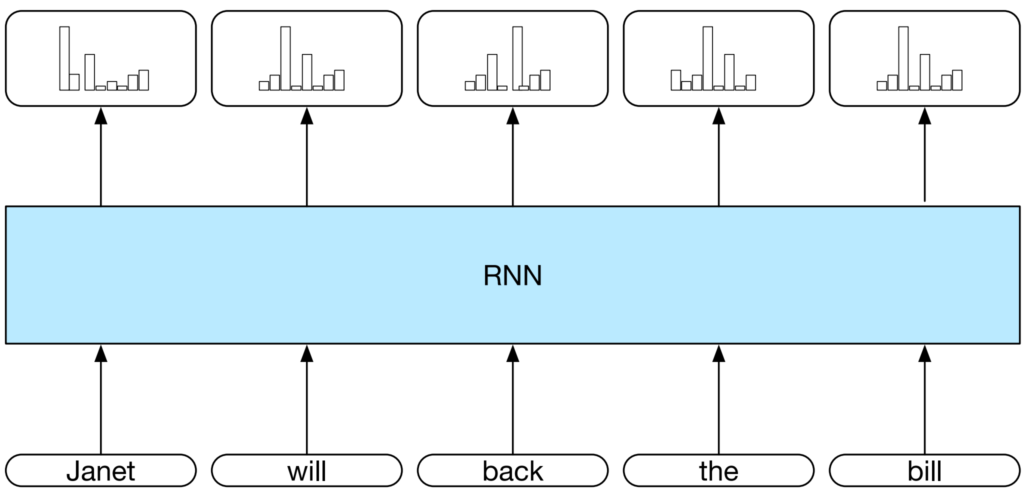
We can use the hidden state of the last word in the sequence as input to a feedforward net:



Basic RNNs for sequence labeling

Sequence labeling (e.g. POS tagging):
Assign **one label to each element** in the sequence.

RNN Architecture:
Each time step has a distribution over output classes



Extension: add a CRF layer to capture dependencies among labels of adjacent tokens.

RNNs for sequence labeling

In sequence labeling, we want to assign a label or tag $t^{(i)}$ to each word $w^{(i)}$

Now the output layer gives a (softmax) distribution over the T possible tags,
and the hidden layer contains information about the previous words and the previous tags.

To compute the probability of a tag sequence $t^{(1)} \dots t^{(n)}$ for a given string $w^{(1)} \dots w^{(n)}$, feed in $w^{(i)}$ (and possibly $t^{(i-1)}$) as input at time step i and compute $P(t^{(i)} \mid w^{(1)} \dots w^{(i-1)}, t^{(1)} \dots t^{(i-1)})$

Part 2: Recurrent Neural Net Practicalities

RNN Practicalities

This part will discuss how to train and use RNNs.

We will also discuss how to go beyond basic RNNs.

The last part used a simple RNN with one layer to illustrate how RNNs can be used for different NLP tasks.

In practice, more complex architectures are common.

Three complementary ways to extend basic RNNs:

— Using RNNs in **more complex networks** (bidirectional RNNs, stacked RNNs) [This Part]

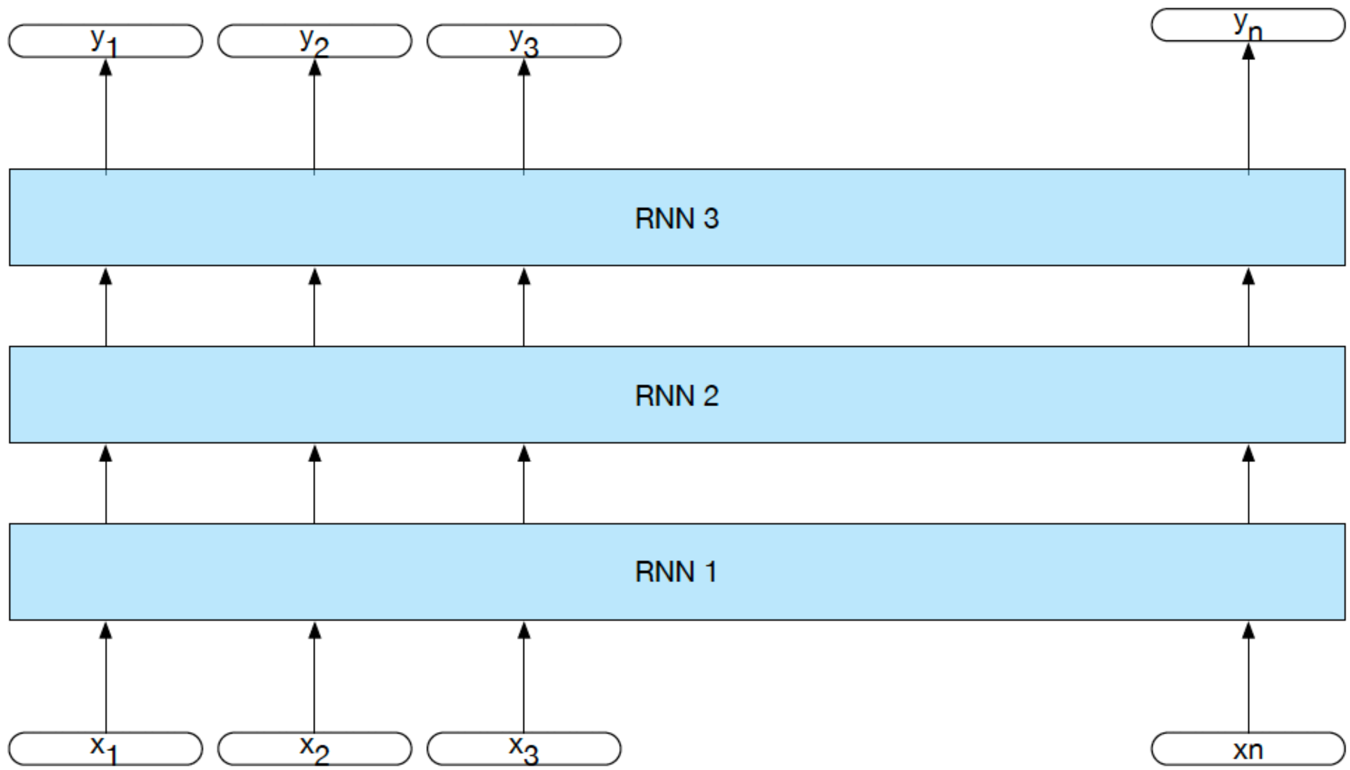
— Modifying the **recurrent** architecture (LSTMs, GRUs) [Part 3]

— Adding **attention** mechanisms [Next Lecture]

Using RNNs in more complex architectures

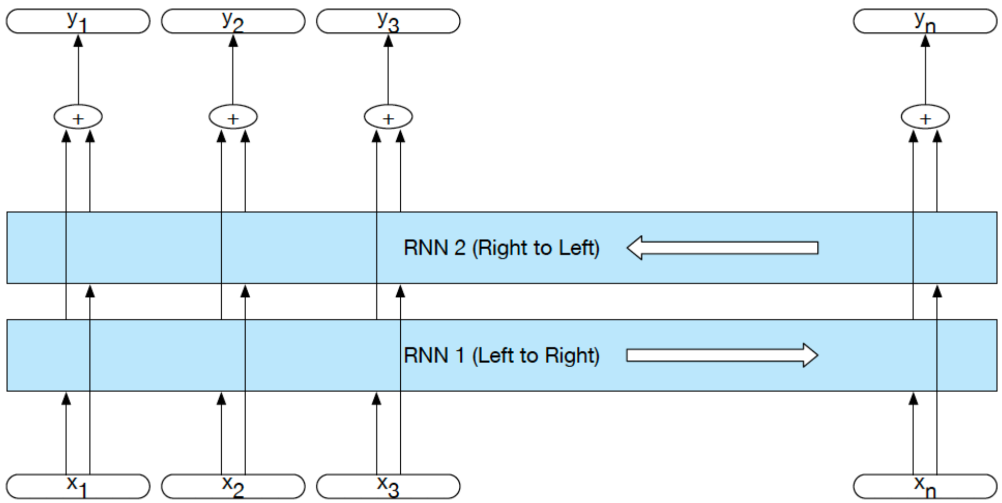
Stacked RNNs

We can create an RNN that has **“vertical” depth** (at each time step) by stacking multiple RNNs:



Bidirectional RNNs

Unless we need to generate a sequence, we can run
***two* RNNs over the input sequence,**
one in the **forward** direction, and one in the **backward** direction.
Their hidden states will capture **different context** information

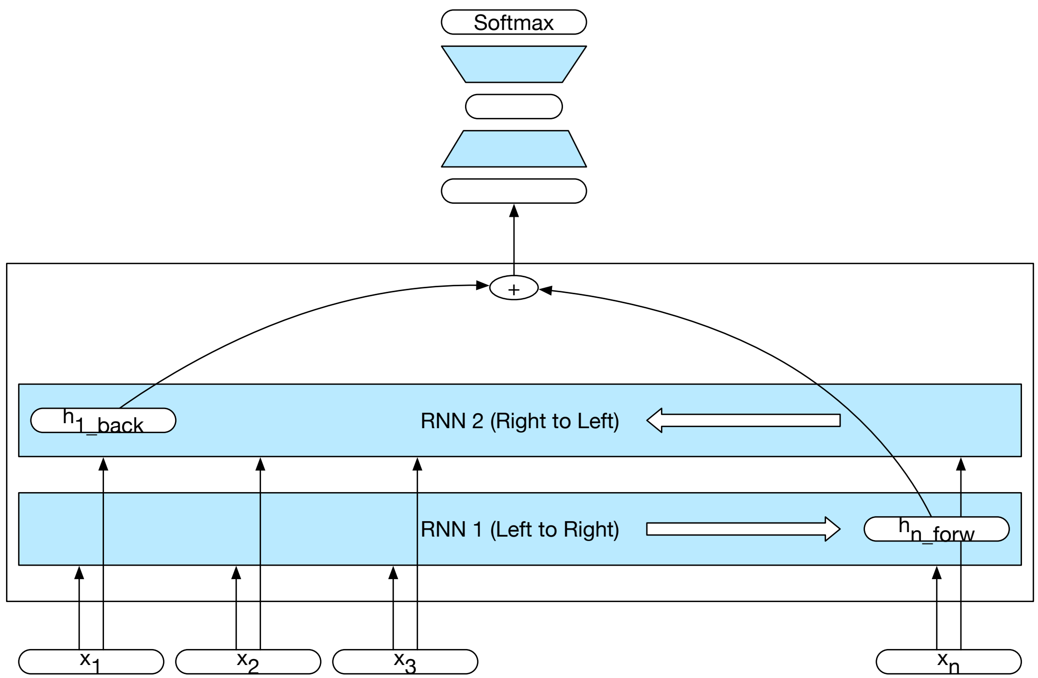


To obtain a **single hidden state** at time t: $\mathbf{h}_{\mathbf{bi}}^{(\mathbf{t})} = \mathbf{h}_{\mathbf{fw}}^{(\mathbf{t})} \oplus \mathbf{h}_{\mathbf{bw}}^{(\mathbf{t})}$

where $\oplus$ is typically concatenation

Bidirectional RNNs for sequence classification

Combine…
…the forward RNN's hidden state for the last word, and
…the backward RNN's hidden state for the first word
into a single vector



Training and Generating Sequences with RNNs

How to generate with an RNN

Greedy decoding:

Always pick the word with the highest probability

(if you start from <s>, this only generates a single sentence)

Sampling:

Sample a word according to the given distribution

Beam search decoding:

Keep a number of hypotheses after each time step

— **Fixed-width beam**: keep the top *k* hypotheses

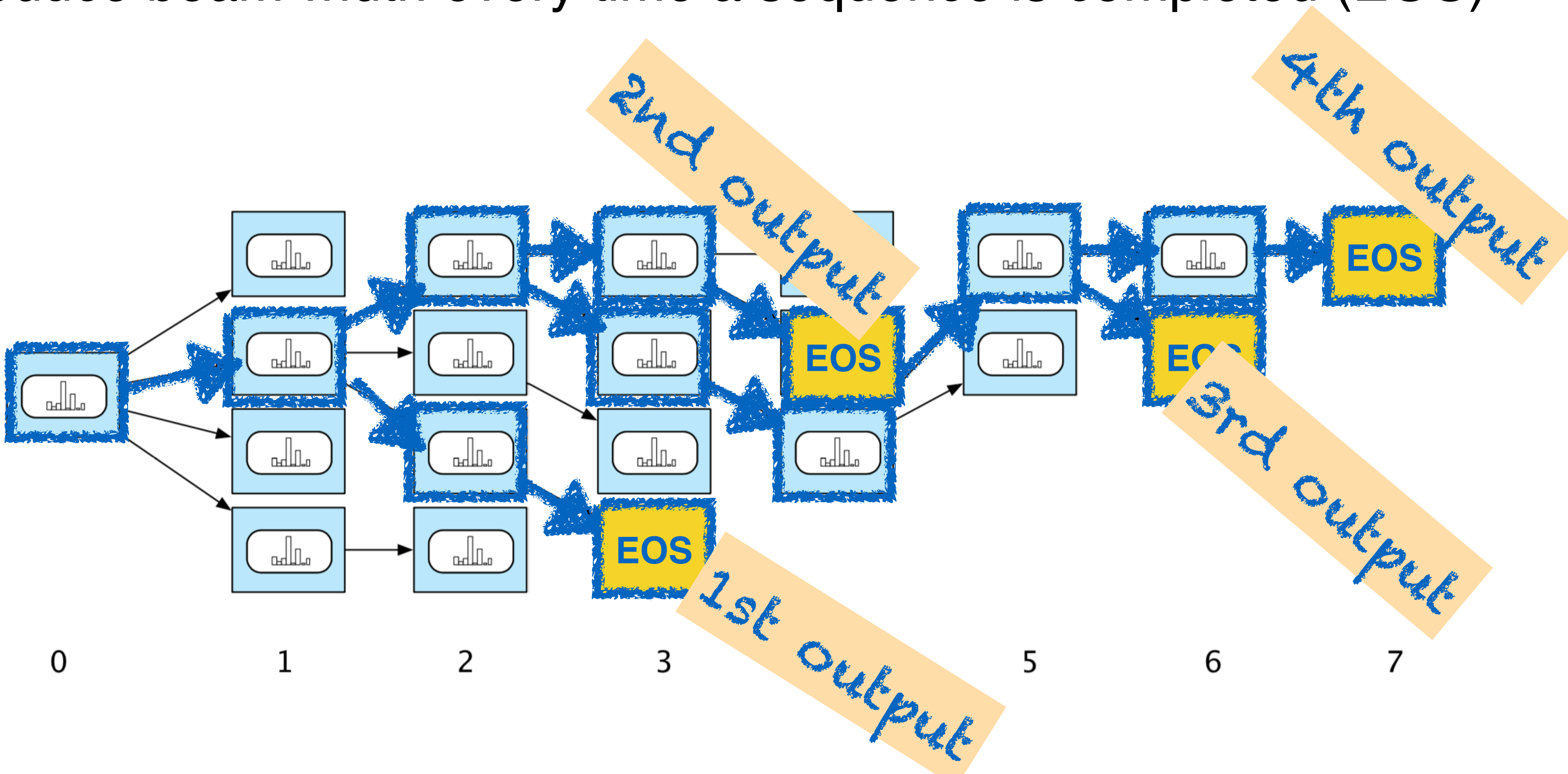
— **Variable-width beam**: keep all hypotheses whose score is with a certain factor of the best score

Beam Decoding (fixed width *k*=4)

Keep the *k* best options around at each time step.

Operate breadth-first: keep the *k* best next hypotheses among the best continuations for each of the current *k hypotheses*.

Reduce beam width every time a sequence is completed (EOS)



Training RNNs for generation

Maximum likelihood estimation (MLE):

Given training samples $w^{(1)}w^{(2)}\ldots w^{(T)}$, find the parameters θ^* that assign **the largest probability to these training samples:**

$$\theta^* = \text{argmax}_\theta P_\theta(w^{(1)}w^{(2)}\ldots w^{(T)}) = \text{argmax}_\theta \prod_{t=1..T} P_\theta(w^{(t)} \mid w^{(1)}\ldots w^{(t-1)})$$

Since $P_\theta(w^{(1)}w^{(2)}\ldots w^{(T)})$ is factored into $P_\theta(w^{(t)} \mid w^{(1)}\ldots w^{(t-1)})$, we can train models to assign a higher probability to the word $w^{(t)}$ that occurs in the training data after $w^{(1)}\ldots w^{(t-1)}$ than any other word $w_i \in V$:

$$\forall_{i=1\ldots|V|} P_\theta\left(w^{(t)} \mid w^{(1)}\ldots w^{(t-1)}\right) \geq P_\theta\left(w_i \mid w^{(1)}\ldots w^{(t-1)}\right)$$

This is also called **teacher forcing**.

Teacher forcing

Each training sequence $w^{(1)}w^{(2)}\ldots w^{(T)}$ turns into T training items:

Give $w^{(1)}w^{(2)}\ldots w^{(t-1)}$ as input to the RNN, and train it to maximize the probability of $w^{(t)}$

(as you would in standard classification,
or when training an n-gram language model).

Problems with teacher forcing

Exposure bias:

When we *train* an RNN for sequence generation, the prefix $y^{(1)} \ldots y^{(t-1)}$ that we condition on comes from the original data

When we *use* an RNN for sequence generation, the prefix $y^{(1)} \ldots y^{(t-1)}$ that we condition on is also generated by the RNN,

— The model is used on data that may look quite different from the data it was trained on.

— The model is not trained to predict the best next token within a generated sequence, or to predict the best sequence

— Errors at earlier time-steps propagate through the sequence.

Remedies

Minimum risk training:

(Shen et al. 2016, https://www.aclweb.org/anthology/P16-1159.pdf)

— define a loss function (e.g. negative BLEU) to compare generated sequences against gold sequences

—Minimize risk (expected loss on training data) such that candidates outputs with a smaller loss (higher BLEU score) have higher probability.

Reinforcement learning-based approaches:

(Ranzato et al. 2016 https://arxiv.org/pdf/1511.06732.pdf)

— use BLEU as a reward (i.e. like MRT)

— perhaps pre-train model first with standard teacher forcing.

GAN-based approaches ("professor forcing")

(Goyal et al. 2016, http://papers.nips.cc/paper/6099-professor-forcing-a-new-algorithm-for-training-recurrent-networks.pdf)

— combine standard RNN with an adversarial model that aims to distinguish original from generated sequences

Part 3: RNN Variants

RNN variants: LSTMs, GRUs

Long Short-Term Memory networks (**LSTMs**)
are RNNs with a more complex recurrent architecture

Gated Recurrent Units (GRUs)
are a simplification of LSTMs

Both contain “**Gates**” to control how much of the input or previous hidden state to forget or remember

From RNNs to LSTMs

In **Vanilla (Elman) RNNs**, the current hidden state $\mathbf{h}^{(t)}$ is a nonlinear function of the previous hidden state $\mathbf{h}^{(t-1)}$ and the current input $\mathbf{x}^{(t)}$:

$$\mathbf{h}^{(t)} = g\left(\mathbf{U}\mathbf{h}^{(t-1)} + \mathbf{W}\mathbf{x}^{(t)} + b_h\right)$$

With g**=tanh** (the original definition):
⇒ Models suffer from the *vanishing gradient* problem: they can't be trained effectively on long sequences.

With g**=ReLU**
⇒ Models suffer from the *exploding gradient* problem: they can't be trained effectively on long sequences.

From RNNs to LSTMs

LSTMs (Long Short-Term Memory networks)
were introduced to overcome the vanishing gradient problem.
Hochreiter and Schmidhuber, Neural Computation 9(8), 1997
https://www.bioinf.jku.at/publications/older/2604.pdf

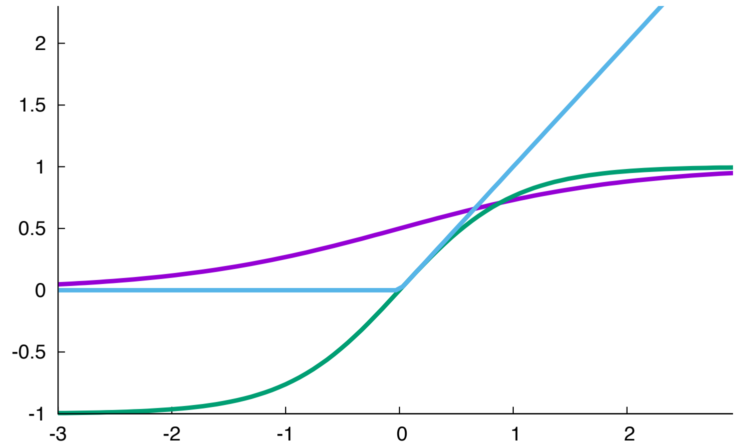
Like RNNs, LSTMs contain a **hidden state** that gets passed through the network and updated at each time step

LSTMs contain an additional **cell state** that also gets passed through the network and updated at each time step

LSTMs contain three different **gates** (**input/forget/output**) that read in the previous hidden state and current input to decide how much of the past hidden and cell states to keep.

These gates mitigate the vanishing/exploding gradient problem

Recap: Activation functions



Hyperbolic Tangent: $\tanh(x) = \dfrac{\exp(2x) - 1}{\exp(2x) + 1} \in [-1, +1]$

Rectified Linear Unit: $\mathrm{ReLU(x)} = \max(0, \mathrm{x}) \in [0, +\infty]$

Sigmoid (logistic function): $\sigma(x) = \dfrac{1}{1 + \exp(-x)} \in [0,1]$

RNN variants: LSTMs, GRUs

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Gated Recurrent Units (GRUs) are a simplification of LSTMs

Both contain "**Gates**" to control how much of the input or past hidden state to forget or remember

A **gate** performs **element-wise multiplication** of

a) a *d*-dimensional **sigmoid** layer ***g***
(all elements between 0 and 1), and

b) a *d*-dimensional **input** vector ***u***

Result: *d*-dimensional **output** vector ***v*** which is like the input ***u***,
but **elements** where $g_i \approx 0$ are (partially) **"forgotten"**

Gating: element-wise product

$$\mathbf{v} = \mathbf{g} \otimes \mathbf{u} = [g_1 u_1,\ g_2 u_2,\ \ldots,\ g_d u_d]$$

A **gate** performs **element-wise multiplication** of

a) a *d*-dimensional **sigmoid** layer ***g***
(all elements between 0 and 1), and

b) a *d*-dimensional **input** vector ***u***

Result: *d*-dimensional **output** vector ***v*** which is like the input ***u***,
but **elements** where $g_i \approx 0$ are (partially) **“forgotten”**

Gating mechanisms

Gates are trainable layers with a **sigmoid** activation function
often determined by the current input $\mathbf{x}^{(t)}$ and the (last) hidden state $\mathbf{h}^{(t-1)}$ eg.:

$$\mathbf{g}_k^{(t)} = \sigma(\mathbf{W}_k\mathbf{x}^{(t)} + \mathbf{U}_k\mathbf{h}^{(t-1)} + b_k)$$

$\mathbf{g}$ is a vector of (Bernoulli) probabilities ($\forall i : 0 \leq g_i \leq 1$)

Unlike traditional (0,1) gates, neural gates are differentiable (we can train them)

$\mathbf{g}$ is combined with another vector $\mathbf{u}$ (of the same dimensionality)
by **element-wise multiplication** (Hadamard product): $\mathbf{v} = \mathbf{g} \otimes \mathbf{u}$

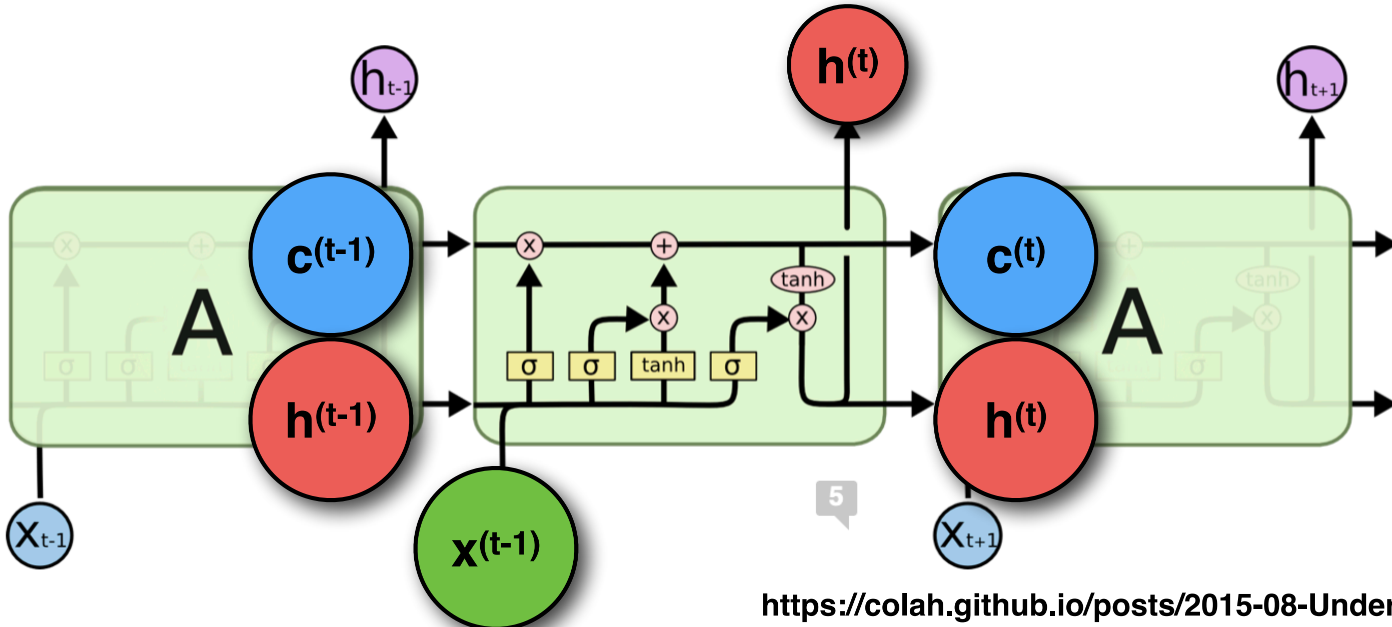
If $g_i \approx 0$, $v_i \approx 0$, and if $g_i \approx 1$, $v_i \approx u_i$

Each g_i has its own set of trainable parameters to determine how much of u_i to keep

Gates can also be used to form
linear combinations of two input vectors $\mathbf{t}, \mathbf{u}$:

- **Addition of two independent gates**: $\mathbf{v} = \mathbf{g}_1 \otimes \mathbf{t} + \mathbf{g}_2 \otimes \mathbf{u}$
- **Linear interpolation (coupled** gates): $\mathbf{v} = \mathbf{g} \otimes \mathbf{t} + (\mathbf{1} - \mathbf{g}) \otimes \mathbf{u}$

Long Short-Term Memory Networks (LSTMs)



https://colah.github.io/posts/2015-08-Understanding-LSTMs/

At time t, the LSTM cell reads in

— a c-dimensional ***previous*** **cell state vector** $\mathbf{c}^{(t-1)}$

— an h-dimensional ***previous*** **hidden state vector** $\mathbf{h}^{(t-1)}$

— a d-dimensional ***current*** **input vector** $\mathbf{x}^{(t)}$

At time t, the LSTM cell returns

— a c-dimensional ***new*** **cell state vector** $\mathbf{c}^{(t)}$

— an h-dimensional ***new*** **hidden state vector** $\mathbf{h}^{(t)}$
(which may also be passed to an **output layer**)

LSTM operations

Based on the previous cell state $\mathbf{c}^{(t-1)}$, previous hidden state $\mathbf{h}^{(t-1)}$ and the current input $\mathbf{x}^{(t)}$, the LSTM computes:

… A new **intermediate cell state** $\tilde{\mathbf{c}}^{(t)}$ that depends on $\mathbf{h}^{(t-1)}$ and $\mathbf{x}^{(t)}$**:**

$$\tilde{\mathbf{c}}^{(t)} = \tanh\left(\mathbf{W}_c\mathbf{x}^{(t)} + \mathbf{U}_c\mathbf{h}^{(t-1)} + b_c\right)$$

… **Three gates** $\mathbf{f}^{(t)}, \mathbf{i}^{(t)}, \mathbf{o}^{(t)}$, which each depend on $\mathbf{h}^{(t-1)}$ and $\mathbf{x}^{(t)}$:

- The **forget gate** $\mathbf{f}^{(t)} = \sigma\left(\mathbf{W}_f\mathbf{x}^{(t)} + \mathbf{U}_f\mathbf{h}^{(t-1)} + b_f\right)$ decides how much of the **last** $\mathbf{c}^{(t-1)}$ to remember in the new cell state: $\mathbf{f}^{(t)} \otimes \mathbf{c}^{(t-1)}$
- The **input gate** $\mathbf{i}^{(t)} = \sigma\left(\mathbf{W}_i\mathbf{x}^{(t)} + \mathbf{U}_i\mathbf{h}^{(t-1)} + b_i\right)$ decides how much of the **intermediate** $\tilde{\mathbf{c}}^{(t)}$ to use in the new cell state: $\mathbf{i}^{(t)} \otimes \tilde{\mathbf{c}}^{(t)}$
- The **output gate** $\mathbf{o}^{(t)} = \sigma\left(\mathbf{W}_o\mathbf{x}^{(t)} + \mathbf{U}_o\mathbf{h}^{(t-1)} + b_o\right)$ decides how much of the **new** $\mathbf{c}^{(t)}$ to use in the next hidden state: $\mathbf{h}^{(t)} = \mathbf{o}^{(t)} \otimes \mathbf{c}^{(t)}$

The new **cell state** $\mathbf{c}^{(t)} = \tanh\left(\mathbf{f}^{(t)} \otimes \mathbf{c}^{(t-1)} + \mathbf{i}^{(t)} \otimes \tilde{\mathbf{c}}^{(t)}\right)$ is a **linear combination of cell states** $\mathbf{c}^{(t-1)}$ and $\tilde{\mathbf{c}}^{(t)}$ that depends on forget gate $\mathbf{f}^{(t)}$ and input gate $\mathbf{i}^{(t)}$

The new **hidden state** $\mathbf{h}^{(t)} = \mathbf{o}^{(t)} \otimes \mathbf{c}^{(t)}$ depends on $\mathbf{c}^{(t)}$ and the output gate $\mathbf{o}^{(t)}$

Gated Recurrent Units (GRUs)

Based on $\mathbf{h}^{(t-1)}$ and $\mathbf{x}^{(t)}$, a GRU computes:

— a **reset gate** $\mathbf{r}^{(t)}$ to determine how much of $\mathbf{h}^{(t-1)}$ to keep in $\tilde{\mathbf{h}}^{(t)}$

$$\mathbf{r}^{(t)} = \sigma\left(\mathbf{W}_r\mathbf{x}^{(t)} + \mathbf{U}_r\mathbf{h}^{(t-1)} + b_r\right)$$

— an intermediate **hidden state** $\tilde{\mathbf{h}}^{(t)}$ that depends on $\mathbf{x}^{(t)}$ and $\mathbf{r}^{(t)} \otimes \mathbf{h}^{(t-1)}$

$$\tilde{\mathbf{h}}^{(t)} = \phi\left(\mathbf{W}_h\mathbf{x}^{(t)} + \mathbf{U}_h(\mathbf{r}^{(t)} \otimes \mathbf{h}^{(t-1)}) + b_r\right) \quad [\phi = \tanh \text{ or } \mathrm{ReLU}]$$

— an **update gate** $\mathbf{z}^{(t)}$ to determine how much of $\mathbf{h}^{(t-1)}$ to keep in $\mathbf{h}^{(t)}$

$$\mathbf{z}^{(t)} = \sigma\left(\mathbf{W}_z\mathbf{x}^{(t)} + \mathbf{U}_z\mathbf{h}^{(t-1)} + b_r\right)$$

— a **new hidden state** $\mathbf{h}^{(t)}$ as a linear interpolation of $\mathbf{h}^{(t-1)}$ and $\tilde{\mathbf{h}}^{(t)}$ with weights determined by the (coupled) update gate $\mathbf{z}^{(t)}$

$$\mathbf{h}^{(t)} = \mathbf{z}^{(t)} \otimes \mathbf{h}^{(t-1)} + \left(\mathbf{1} - \mathbf{z}^{(t)}\right) \otimes \tilde{\mathbf{h}}^{(t)}$$

Cho et al. (2014) Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation
https://arxiv.org/pdf/1406.1078.pdf

LSTMs vs GRUs

LSTMs are more expressive than GRUs
and basic RNNs (they're better at learning long-range dependencies)

But GRUs are easier to train than LSTMs
(useful when training data is limited)