CS440/ECE448: Intro to Artificial Intelligence

Lecture 16
Exact inference in Bayes Nets

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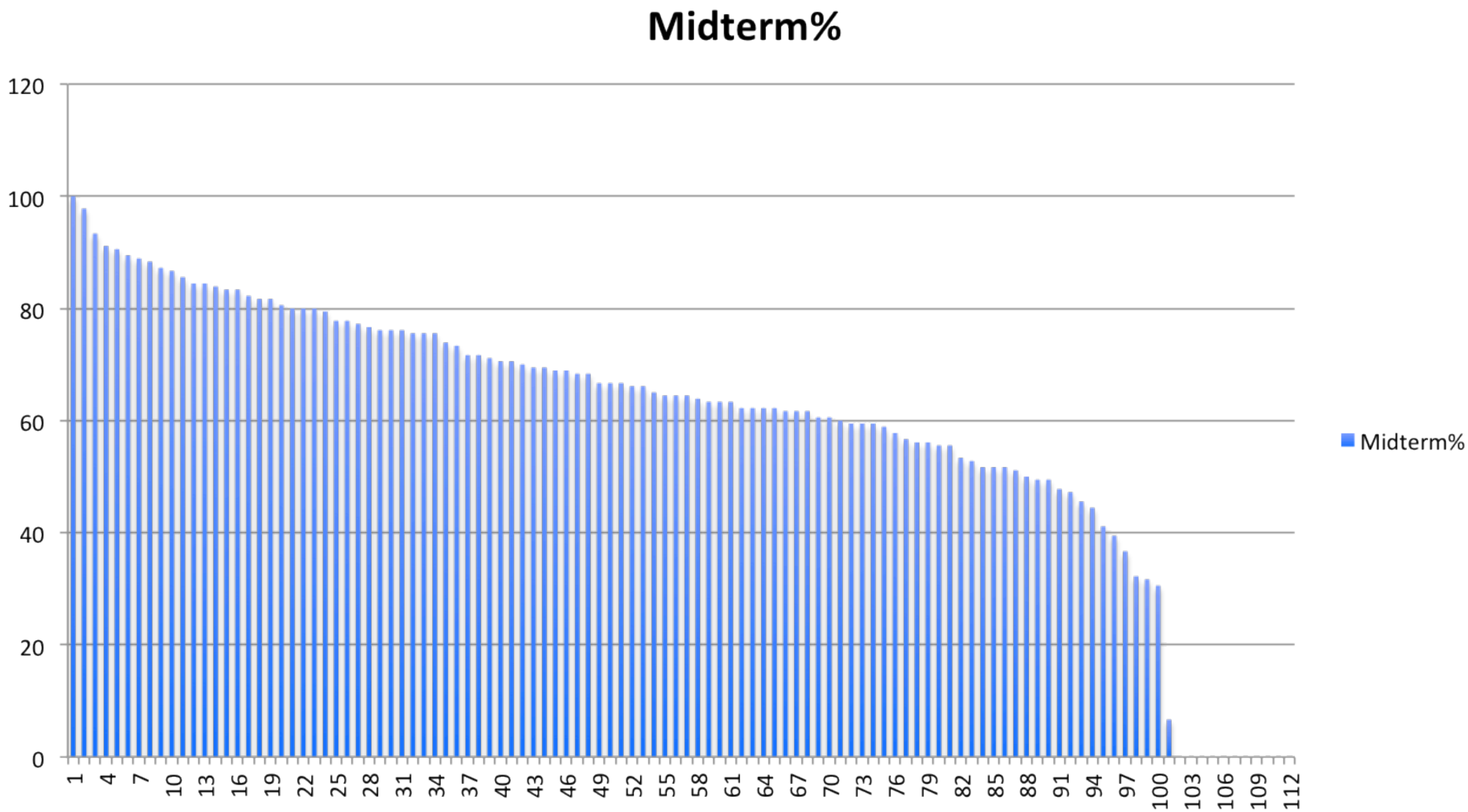
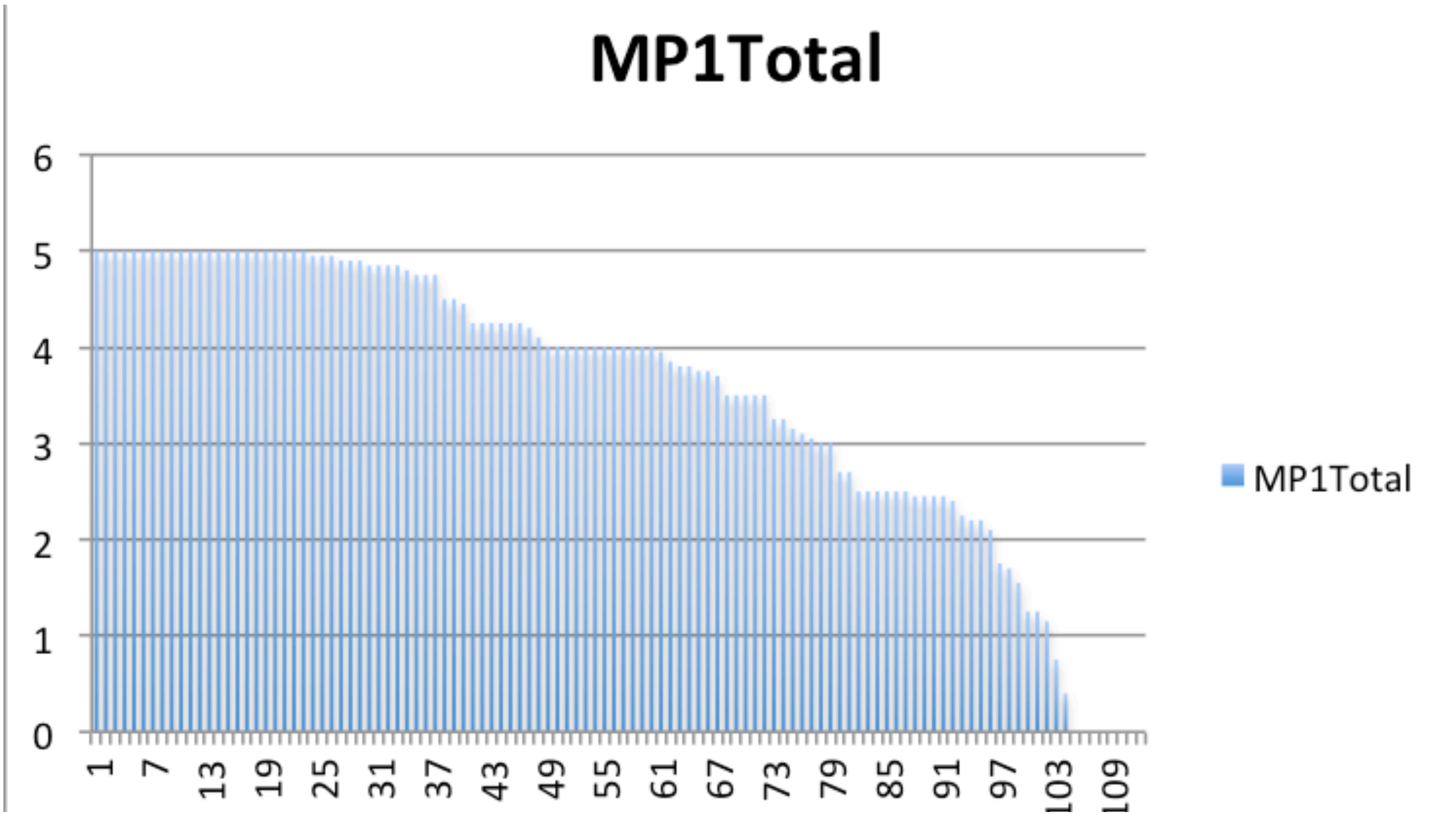
http://cs.illinois.edu/fa11/cs440

Grades….

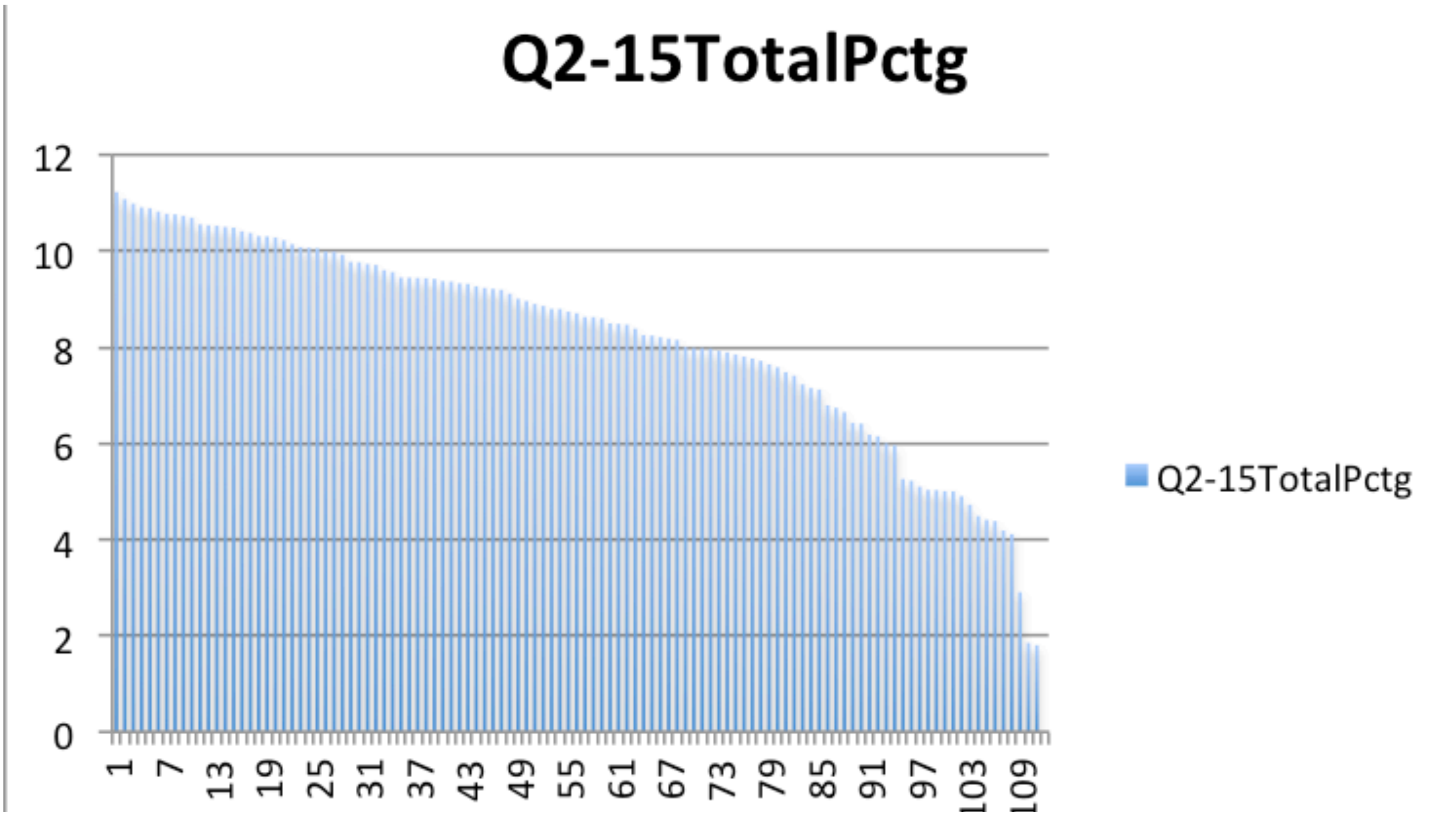
Your midterm percentages



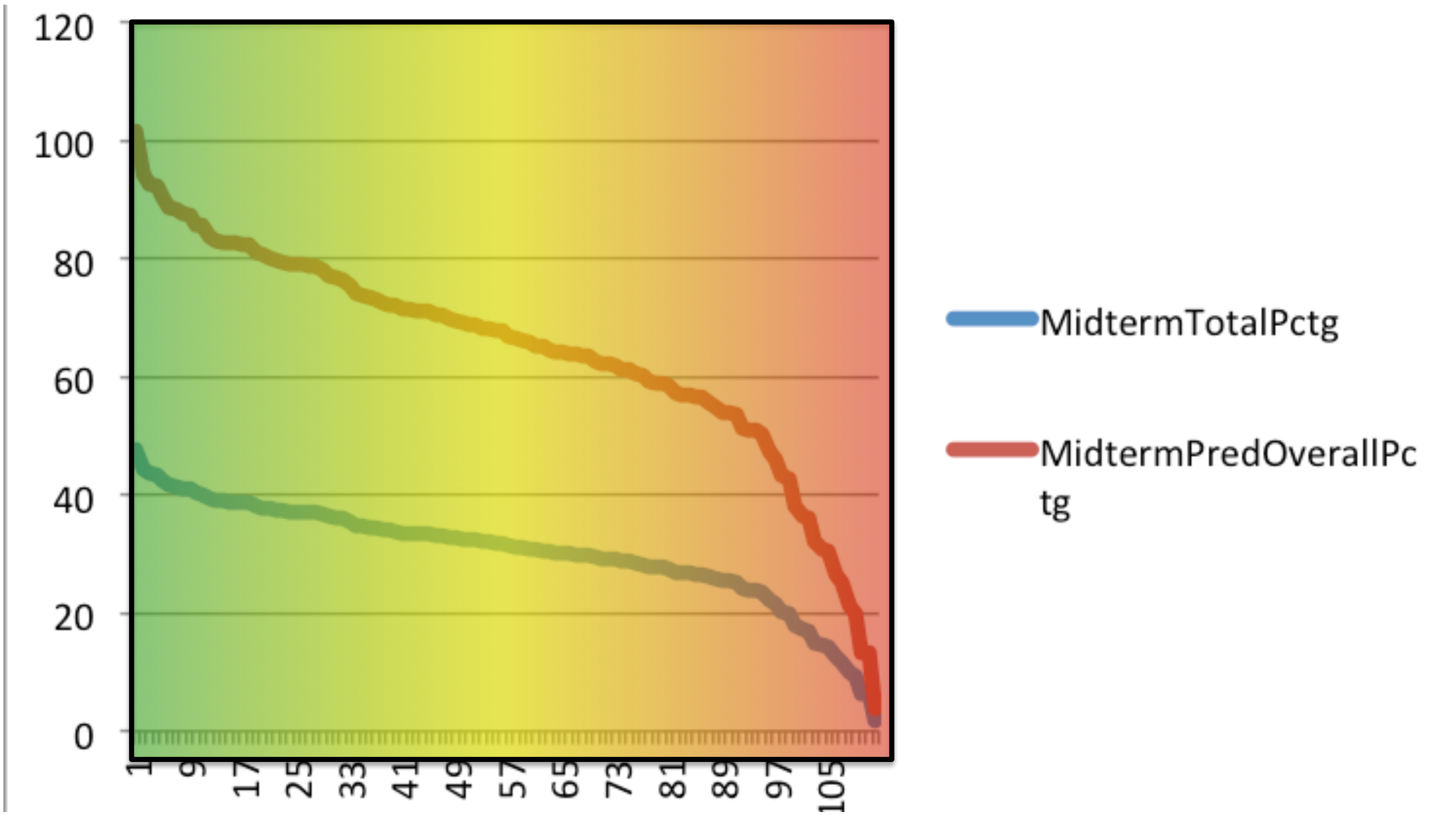
Your MP percentages



Your Quiz totals

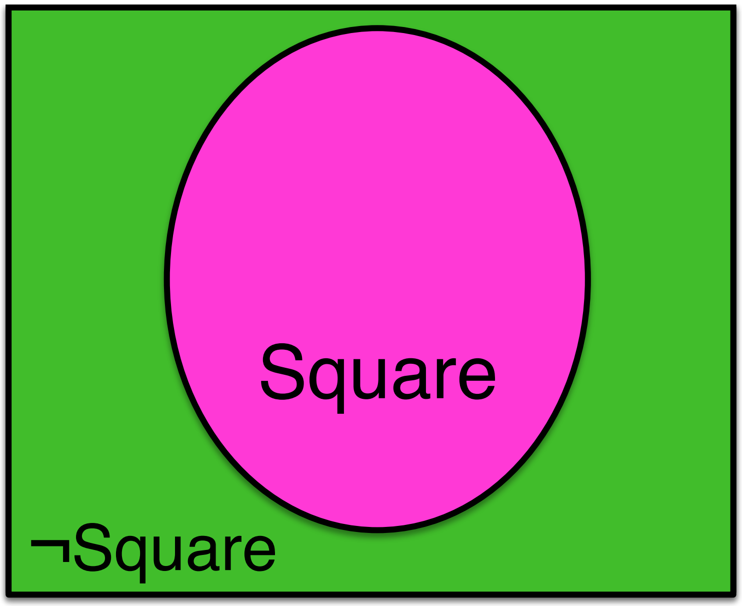


Your current and predicted final grades

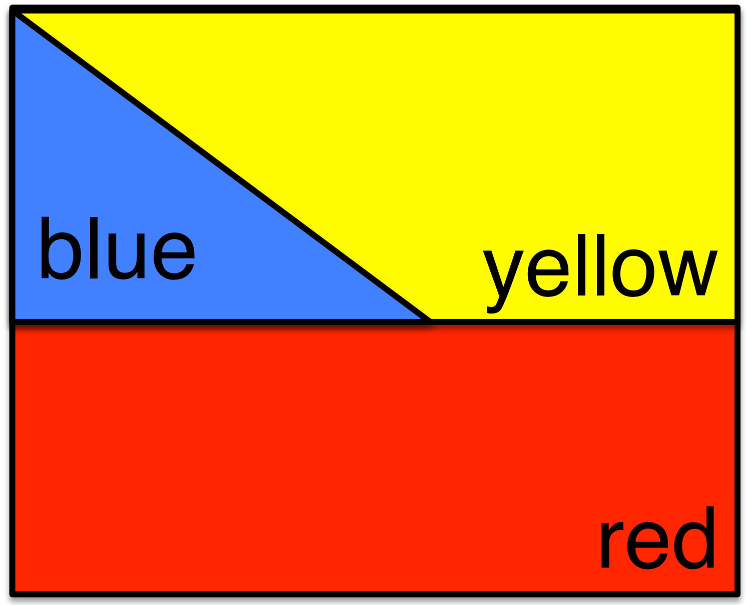


Probability review

Atomic events

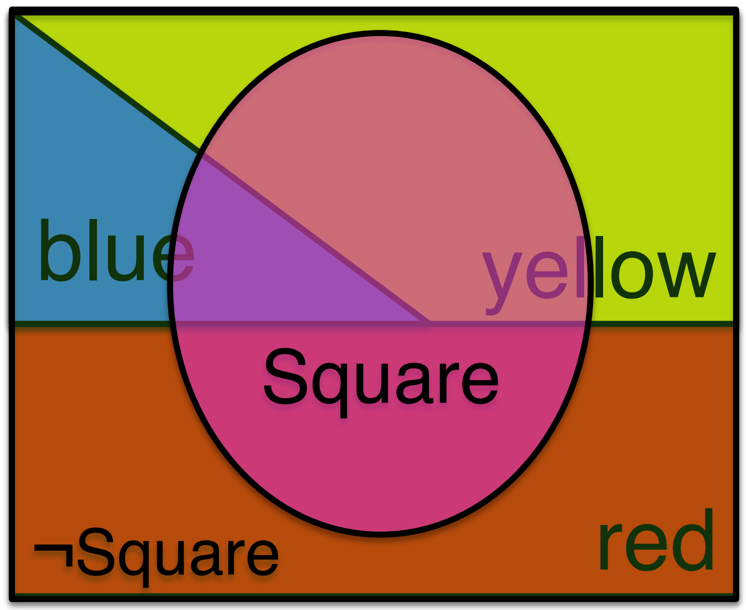


Boolean random variable *Square*



Categorical random variable *Color*

Complex events

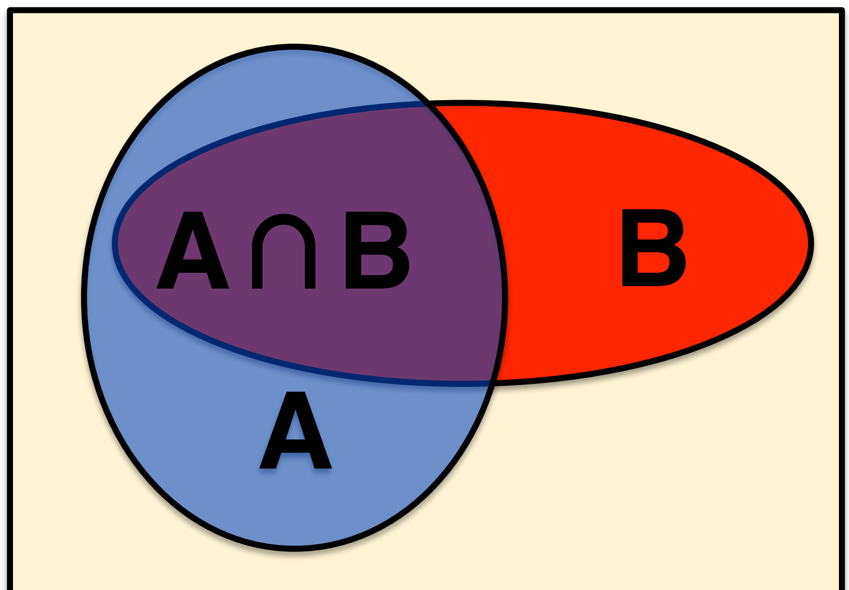


Joint probability P(A,B)

$P(A \cap B) = P(A, B)$

If A and B are boolean variables:

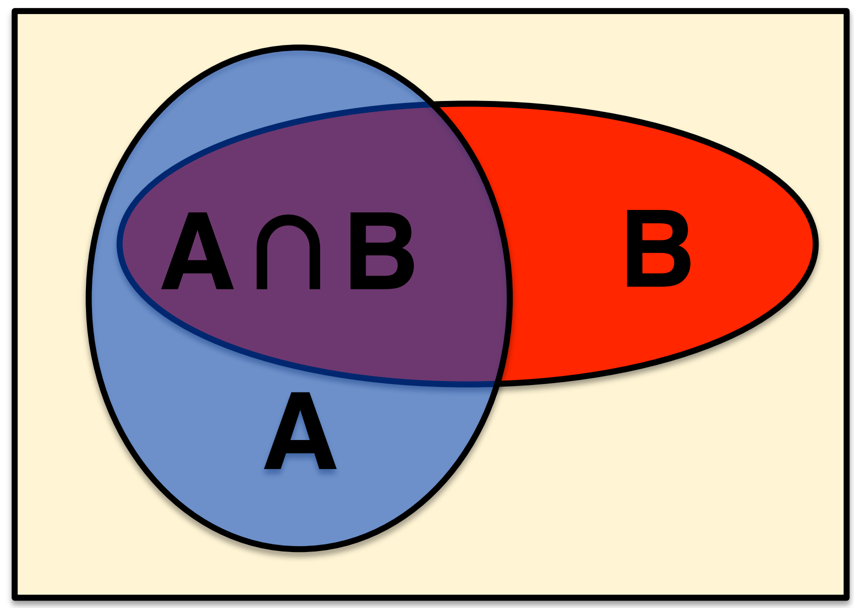
$P(A,B) = P(A \land B)$



Conditional probability $P(A|B)$

Definition:

$$P(A \mid B) = \frac{P(A,B)}{P(B)}$$



Product rule

$$P(A,B) = P(A \mid B)P(B)$$

The full joint distribution

		Weather			
		Sunny	Cloudy	Rainy	Snowy
Fun?	Yes	0.25	0.15	0.05	0.13
	No	0.05	0.1	0.25	0.02

From the full joint distribution, we can obtain:

– Conditional distributions *P(Fun? | Weather)*

– Marginal distributions *P(Weather)*

Independence

Random variables X and Y are **independent** $(X \perp Y)$ if $P(X,Y) = P(X) \times P(Y)$

NB.: Since X and Y are R.V.s (not individual events), $P(X,Y) = P(X)\times P(Y)$ is an abbreviation for:
$\forall x \forall y\ P(X=x,Y=y) = P(X=x)\times P(Y=y)$

X and Y are ***conditionally*** **independent** given Z $(X \perp Y \mid Z)$ if $P(X,Y \mid Z) = P(X \mid Z) \times P(Y \mid Z)$

Conditional Independence

X and Y are ***conditionally* independent** given Z
$(X \perp Y \mid Z)$ if $P(X,Y \mid Z) = P(X \mid Z) \times P(Y \mid Z)$

The value of X depends on the value of Z,
and the value of Y depends on the value of Z,
so X and Y are not independent.

Bayesian networks

Insight: (Conditional) independence assumptions are essential for probabilistic modeling

Bayes Net: a directed graph which represents the joint distribution of a number of random variables in a directed graph

- Nodes = random variables
- Directed edges = dependencies

The *Student* scenario
(Koller & Friedman'09)

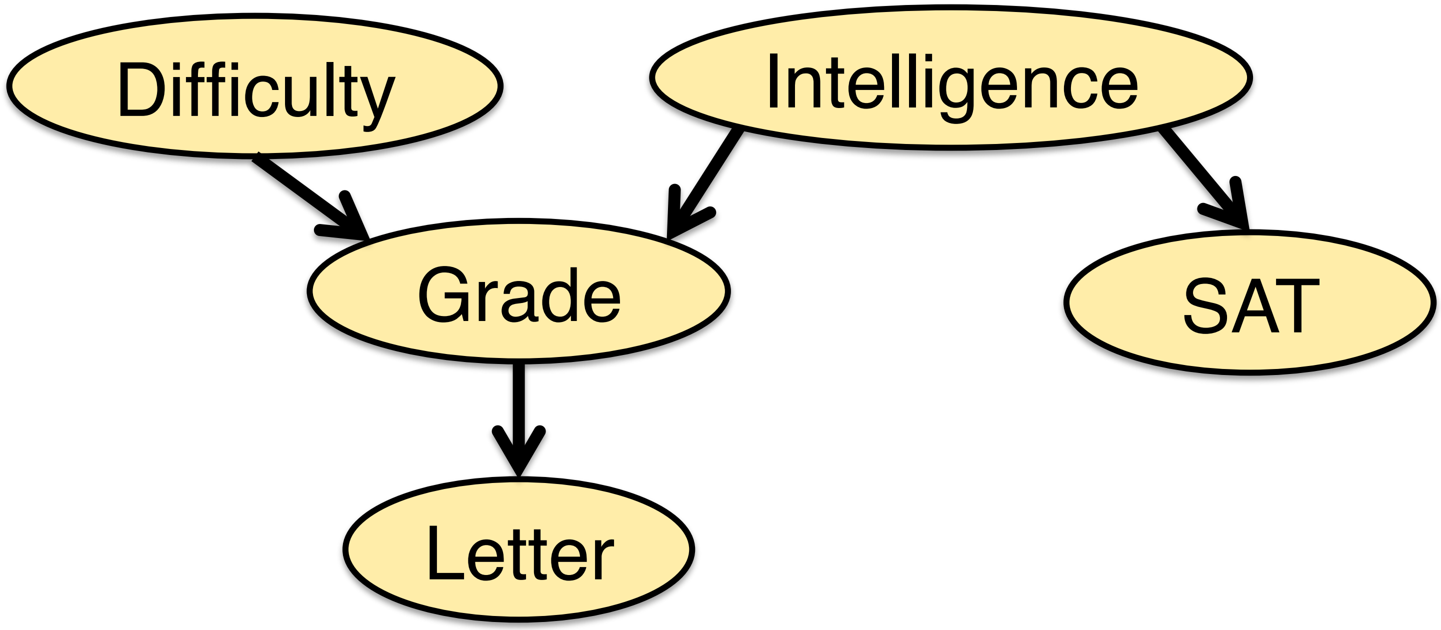
A company wants to hire intelligent CS grads.

Each student has an **SAT score** and a **recommendation letter** from a professor that they took a class from.

The SAT score depends on the student's intelligence

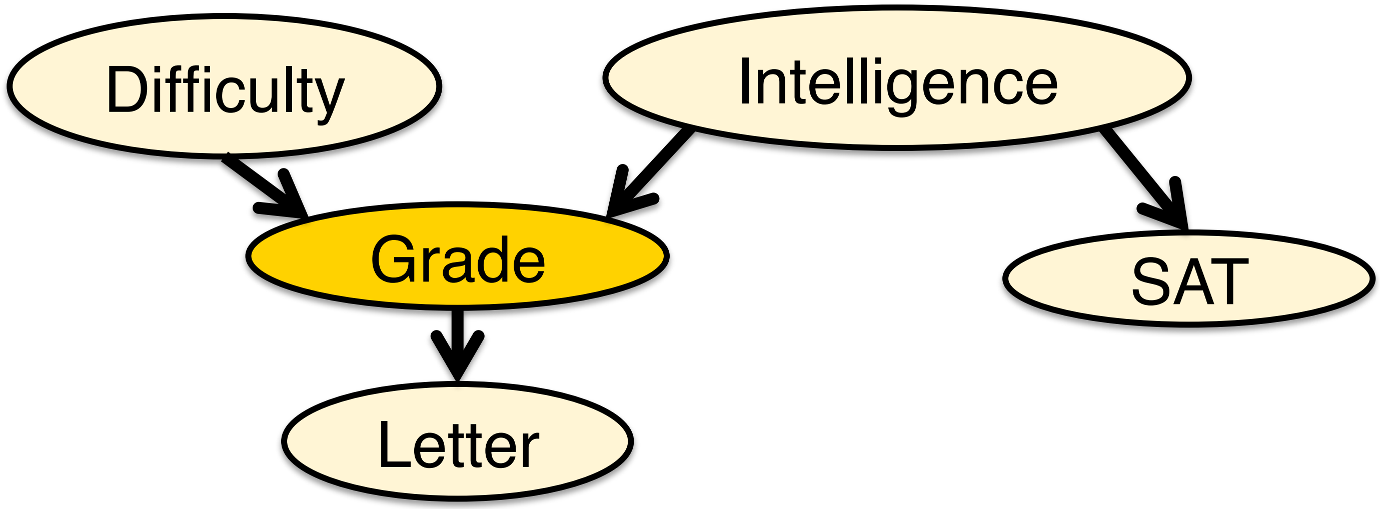
The professor's recommendation depends purely on the student's **grade**.

The student's grade in the class depends on their **intelligence** as well as the **difficulty** of the class.

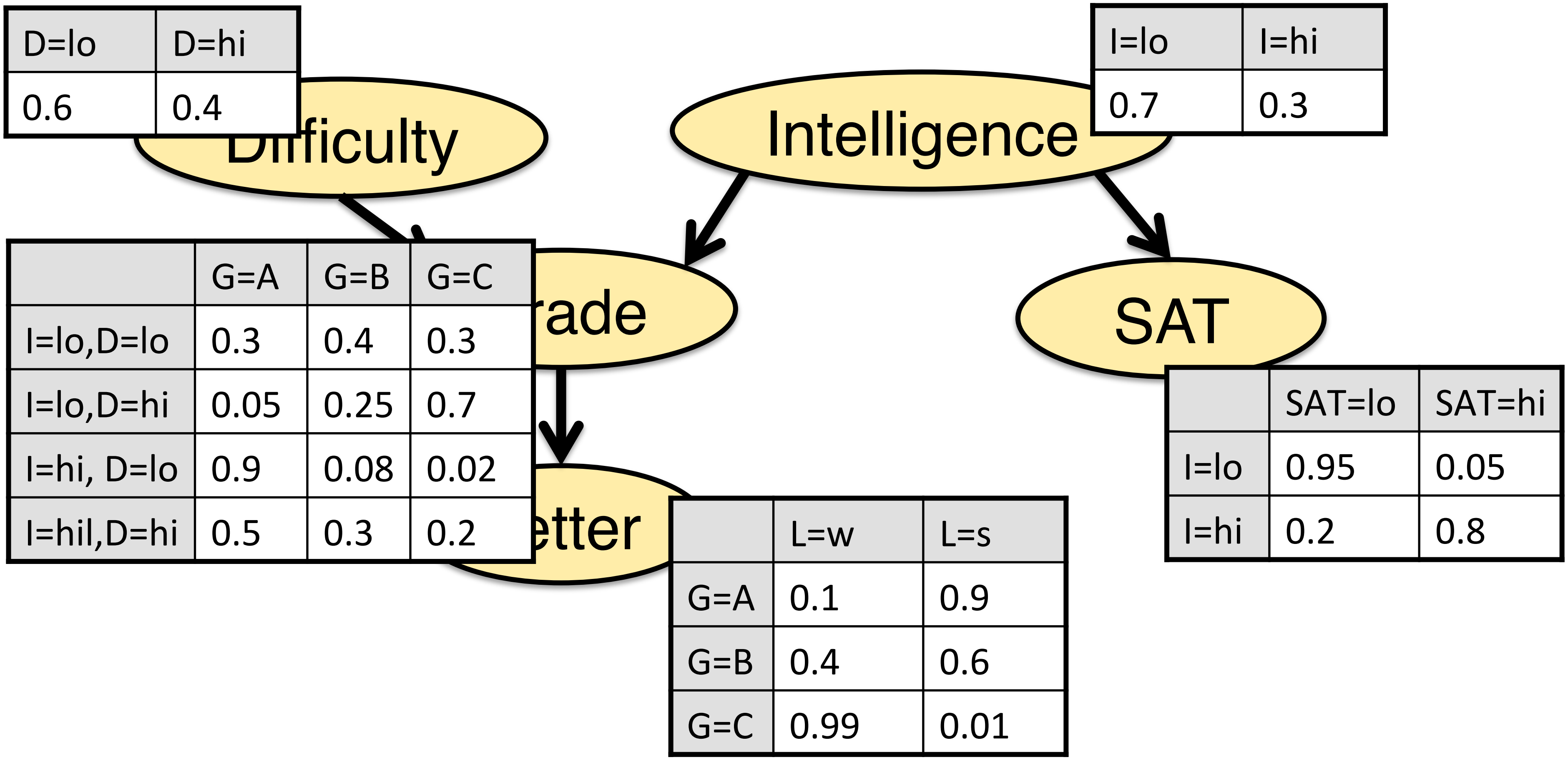


Each student has an **SAT score**
and a **recommendation letter.**
The SAT score depends on their **intelligence.**
The recommendation depends on their **grade**.
The grade depends on the student's **intelligence**
as well as the **difficulty** of the class.

Some terminology



Difficulty and *Intelligence* are **parents** of *Grade.*
Letter is a (direct) **descendant** of *Grade.*
SAT is a **non-descendant** of *Grade.*



D=lo	D=hi
0.6	0.4

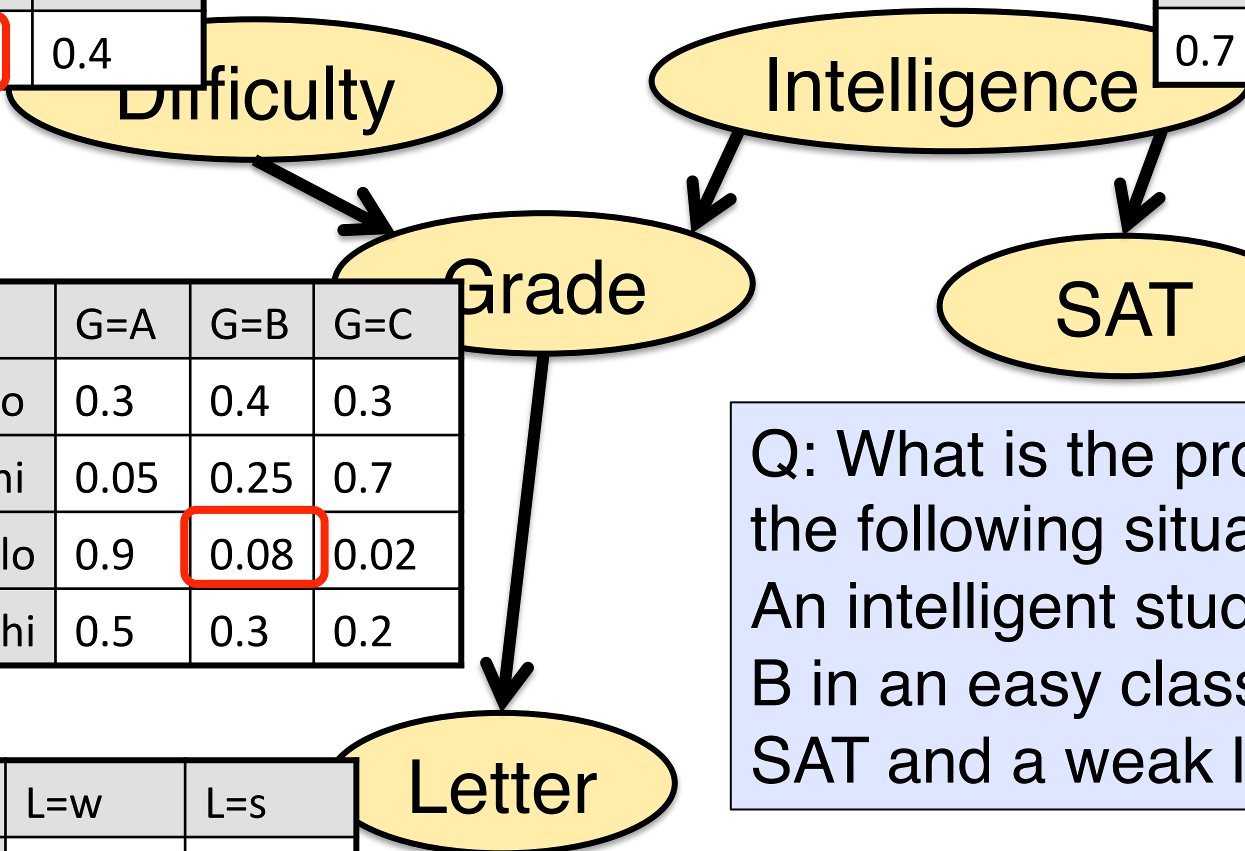
I=lo	I=hi
0.7	0.3

	G=A	G=B	G=C
I=lo,D=lo	0.3	0.4	0.3
I=lo,D=hi	0.05	0.25	0.7
I=hi, D=lo	0.9	0.08	0.02
I=hil,D=hi	0.5	0.3	0.2

	SAT=lo	SAT=hi
I=lo	0.95	0.05
I=hi	0.2	0.8

	L=w	L=s
G=A	0.1	0.9
G=B	0.4	0.6
G=C	0.99	0.01

Difficulty is a binary R.V. *(easy/hard)*
Intelligence is a binary R.V. *(low/high)*
SAT is a binary R.V. *(low/high)*
There are three **grades**. *(A,B,C)*
Letter is a binary R.V. *(weak rec./strong recc)*



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Q: What is the probability of the following situation: An intelligent student gets a B in an easy class, a high SAT and a weak letter?

Answer:

$P(I=hi)P(D=lo)P(G=B|I=hi,D=lo)P(S=hi)P(L=w|G=B)$

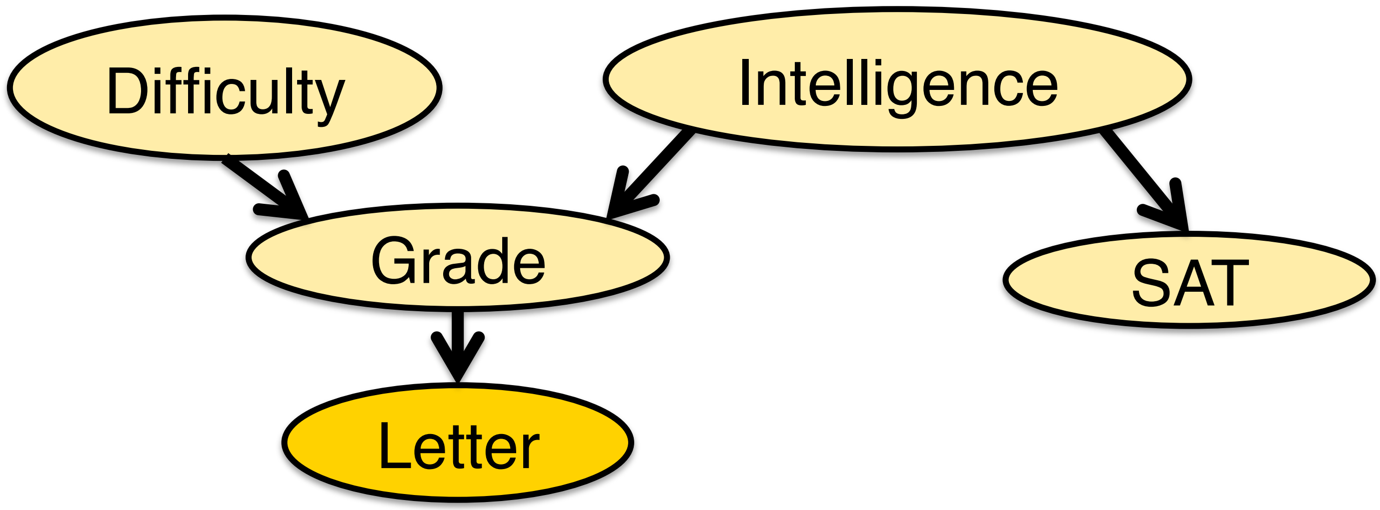
$= 0.3 \times 0.6 \times 0.08 \times 0.8 \times 0.4 = 0.004608$

The chain rule for BNs

In order to compute the joint probability of the random vars $X_1 \ldots X_n$ in a Bayes Net, we multiply the conditional probabilities of each R.V. X_i given its parents $Pa(X_i)$:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid Pa(X_i))$$

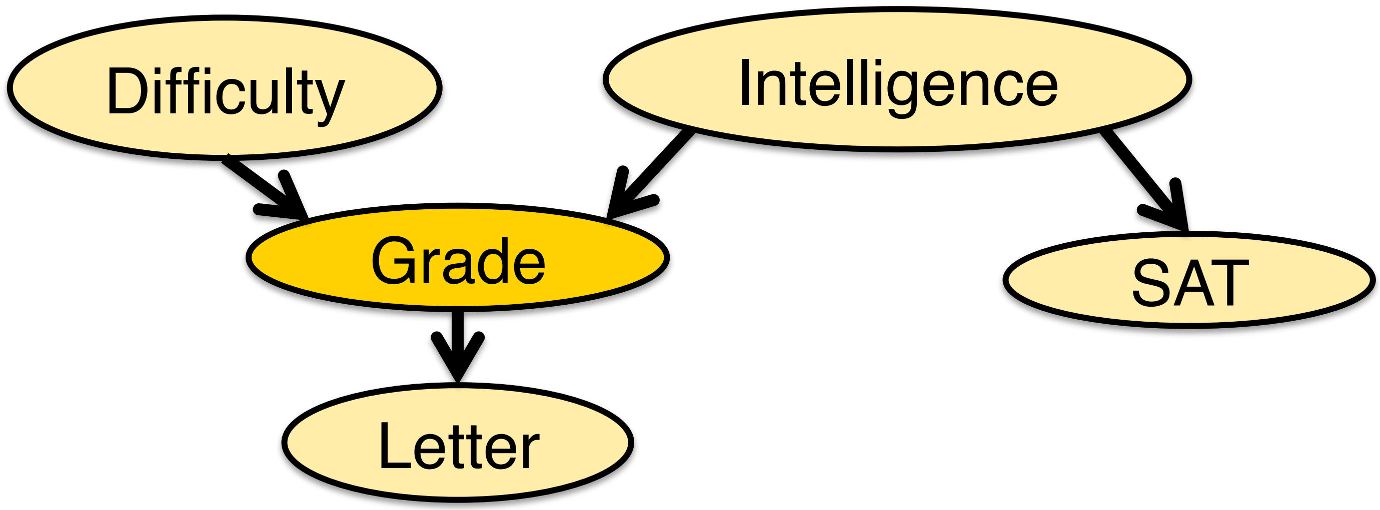
Conditional independences



Each node depends directly only on its parents.

Letter is conditionally independent of all other nodes given its parent:
$(Letter \perp Intelligence, Difficulty, SAT \mid Grade)$

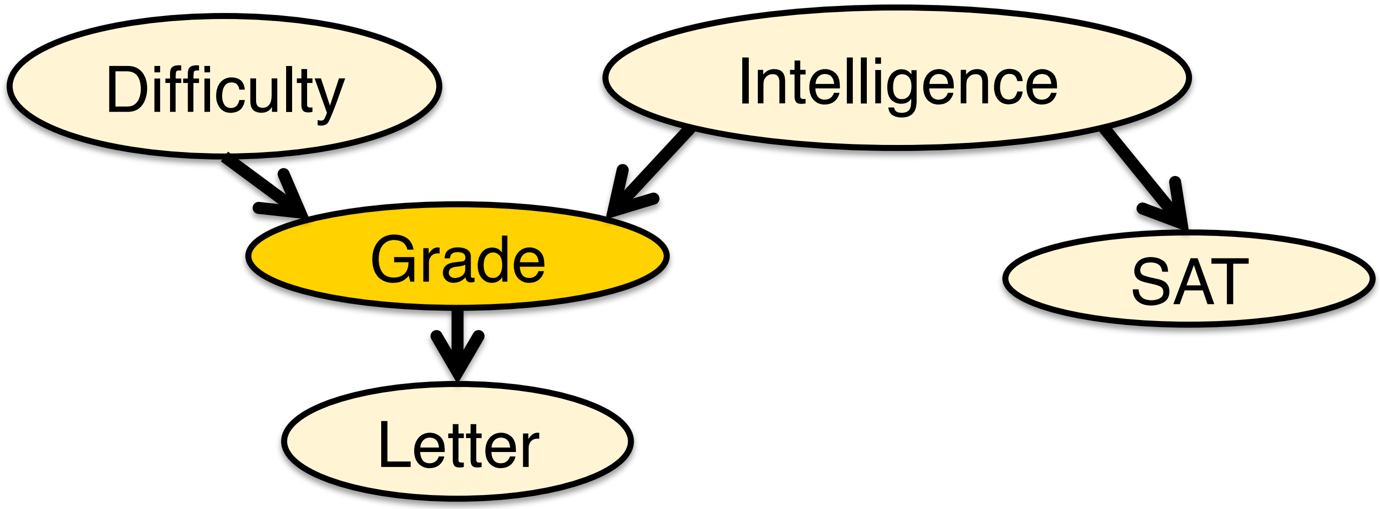
Conditional independences



What about *Grade?*

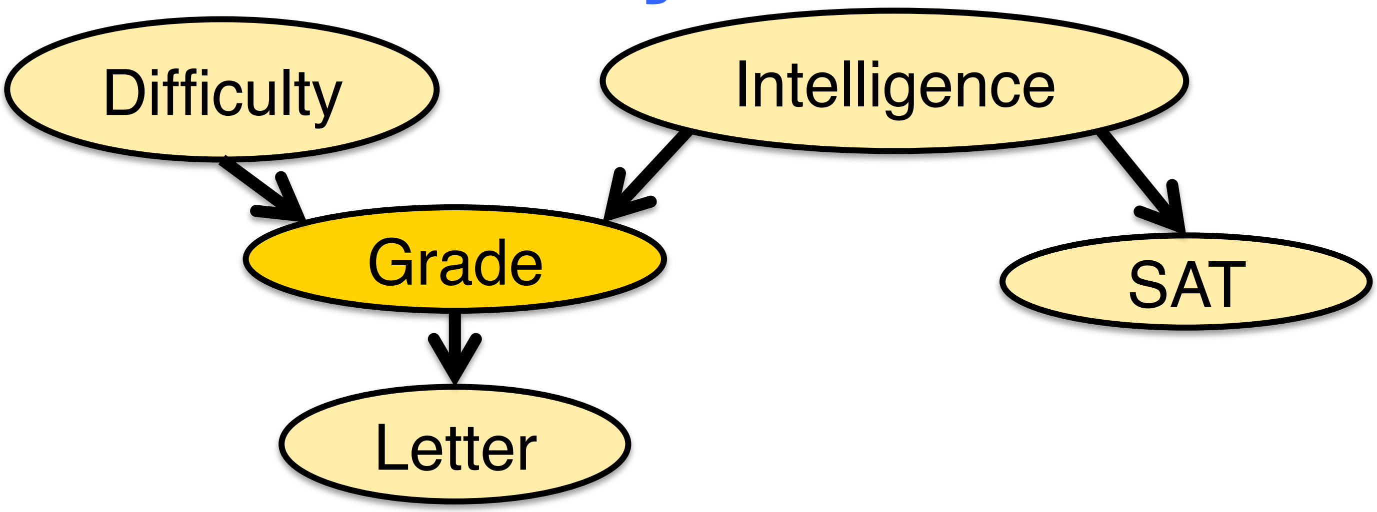
Grade is conditionally independent of SAT given Intelligence, Letter (and Difficulty)
$(Grade \perp SAT \mid Letter, Intelligence, Difficulty)$

More terminology



Difficulty and *Intelligence* are **parents** of *Grade.*
Letter is a (direct) **descendant** of *Grade.*
The parents and direct descendant of a node form its **Markov blanket.**

Conditional independences in Bayes Nets



Each node is conditionally independent of its non-descendants given its Markov blanket.

Inference in Bayes Nets

More generally, we want to know the distribution of a set of **query variables** given some observed **event.**

What is the probability of getting a strong letter if you are an intelligent student?

An event is an assignment of values to a set of **evidence variables.** *(here: intelligence)*

Computing inferences in Bayes Nets

From the joint to the conditional:
$P(X \mid Y) = P(X,Y) / P(Y)$

How do we compute $P(Y)$?
Answer: Marginalization

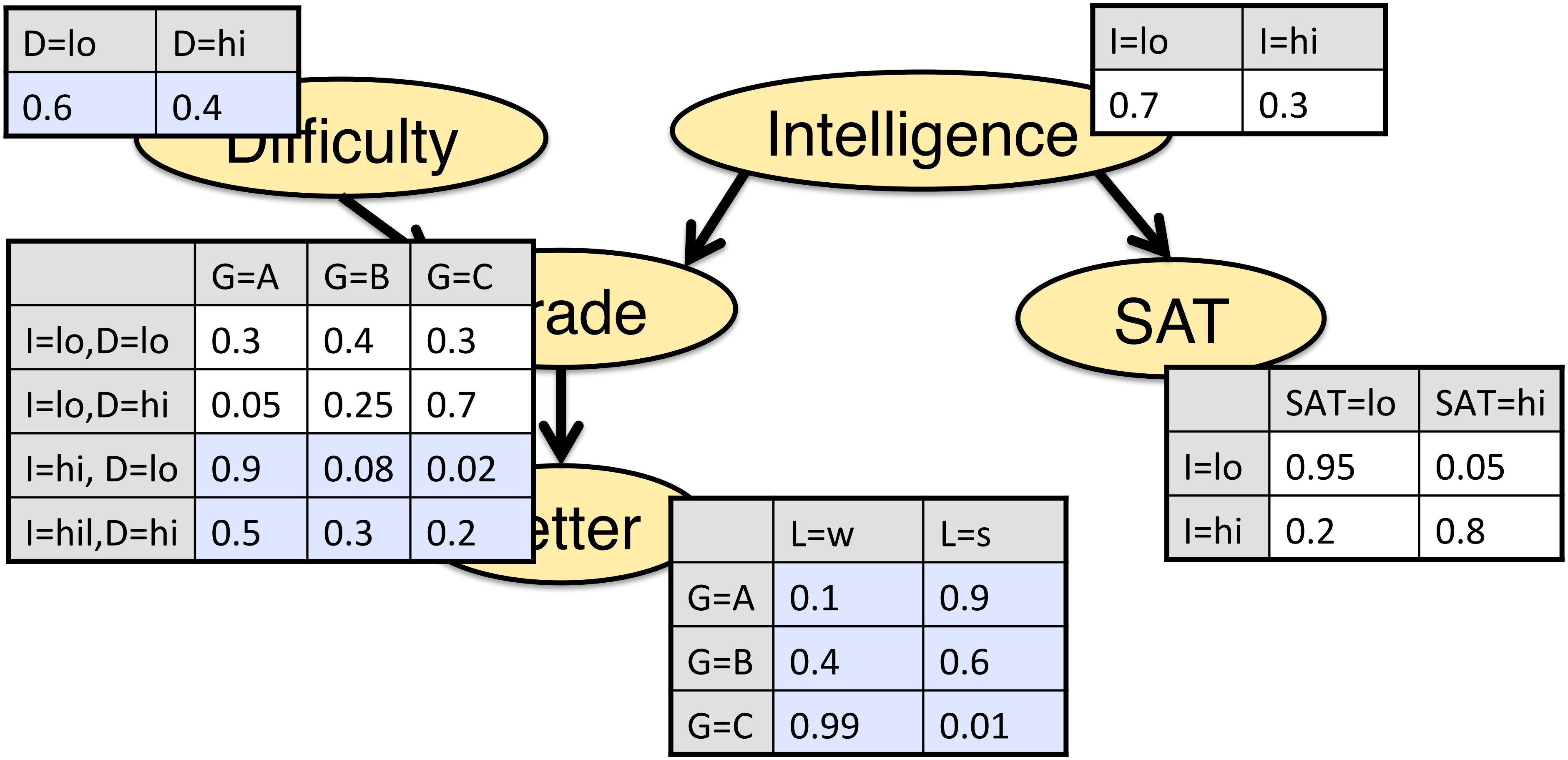
Do we care about $P(Y)$?
Answer: not necessarily if we just want to compare $P(X \mid Y=y)$ for the same set of y's.

Computing inferences in Bayes Nets

What is the probability of getting a strong letter if you are an intelligent student?

What about the other, hidden, variables ?
Answer: we have to marginalize them out.

$$P(X,\mathbf{E}) = \sum_H P(X,H,E)$$



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What is the probability of getting a strong letter if you are an intelligent student?