CS 440: Introduction to AI

Homework 2 - Part A Solution

Due: September 23 11:59PM, 2010

Your answers must be concise and clear. Explain sufficiently that we can easily determine what you understand. We will give more points for a brief interesting discussion with no answer than for a bluffing answer.

1 Validity and Satisfiability

(16 points) Determine whether each formula is valid, satisfiable (but not valid), or unsatisfiable.

1. $P(x)$
   Satisfiable since It holds for some denotational correspondence and some world.

2. $\forall x\ P(x)$
   Satisfiable.

3. $\exists x \forall y\ P(x, y)$
   Satisfiable.

4. $\forall x\ (P(x) \vee \neg P(x))$
   Valid. This is a tautology, which is always true.

5. $\exists x \forall y\ (P(x, y) \Rightarrow \forall w \exists z\ P(w, z))$
   Satisfiable.

6. $\exists x\ (P(x) \Rightarrow \neg P(x))$
   Satisfiable.

7. $\exists x\ \neg(\neg P(x) \vee P(x))$
   Unsatisfiable. $\neg P(x) \vee P(x)$ is a tautology, and it is negated, which makes it always false.

8. $\forall x \forall y\ (\neg P(x) \vee P(y)) \Rightarrow (P(x) \Rightarrow P(y))$
   Valid. This is the same as $\Theta \Rightarrow \Theta$, which becomes $\neg\Theta \vee \Theta$, a tautology.

2 Sentence Translation

(24 points) This problem has two parts. In the first part, you are asked to translate some first order predicate calculus (FOPC) sentences into English sentences. The meanings of the predicates and functions in the FOPC sentences are given. You should give as natural the English translations as possible. Few points will be given for an English sentence like "For all x there exists a y such that either P of x and y or Q of y." In the second part, you are asked to translate English sentences into FOPC sentences. If the meaning of the English sentence is ambiguous, give all possible readings. Note that sometimes *and*, *or*, and *if* should not be directly translated into $\wedge$, $\vee$, and $\Rightarrow$ in FOPC. Please define every predicate and function you use in your FOPC sentences whenever they are not clearly self-explanatory.

If there is a sentence that cannot be translated, please state so and clearly explain why not.

If you do nat have access to the symbols used in FOPC, make the following six substitutions in part 2:

$\wedge$ /\ (slash, backslash)

$\vee$ \/ (backslash, slash)

$\forall$ *forall*

$\exists$ *exists*

$\neg$ –

$\Rightarrow$ => (equals, greater than)

Predicate symbol definitions for part 1:

Thief(x) x is a thief

Cop(x) x is a cop

Student(x) x is a student

Bike(x) x is a bike

Owns(x, y) x is the owner of y

Stolen_by(x, y) x is stolen by y

Caught_by(x, y) x is caught by y

Accomplice(x, y) x and y are accomplices

Function symbol definition for part 1:

CarOf(x) A car of x.

1. FOPC to English

   (a) $\forall x(\neg \text{Student}(x) \vee \neg \text{Cop}(x))$
   Nobody is both a student and a cop.

   (b) $\exists x \exists y(\text{Cop}(x) \wedge \text{Thief}(y) \wedge \text{Stolen\_by}(\text{CarOf}(x), y))$
   There exists a cop whose car is stolen by some thief.

   (c) $\forall x \forall y((\exists z(\text{Bike}(z) \wedge \text{Stolen\_by}(z, x) \wedge \text{Accomplice}(x, y))) \Rightarrow \text{Thief}(x \wedge y))$
   Cannot be translated. $\wedge$ can only be applied to things that are truth valuable, not to variables that denote objects in the world.

   (d) $\forall x(\text{Student}(x) \Rightarrow \exists y \exists z(\text{Bike}(y) \wedge \text{Thief}(z) \wedge \text{Owns}(x, y) \wedge \text{Stolen\_by}(y, z)))$
   Every student has a bike that is stolen by some thief.

   (e) $\neg \forall x \forall y(\text{Bike}(x) \wedge \text{Thief}(y) \wedge \text{Stolen\_by}(x, y) \Rightarrow \exists z(\text{Caught\_by}(y, \text{Cop}(z))))$
   Cannot be translated. $Cop(z)$ returns True or False, which cannot be a parameter for a predicate. Only constant symbols or variables can be a predicate's parameter.

2. English to FOPC

   (a) Evey student who takes Korean passes it.
   $\forall x \forall y$ $((\text{Student}(x) \wedge \text{Takes}(x, Korean) \wedge \text{Grade}(x, y, Korean)) \Rightarrow \text{Greater}(y, F))$
   where Grade(x,y,z) means student x gets grade y in course z, and F denotes grade F.

   (b) The Fighting Illini is the team that won the most of games in the Big Ten conference.
   $\exists x$ $(\text{Number\_of\_games\_won}(\text{FightingIllini}, x) \wedge \forall y \forall z$ $((\text{Big\_Ten\_team}(y) \wedge \text{Different}(y, \text{FightingIllini}) \wedge \text{Number\_of\_games\_won}(y, z)) \Rightarrow \text{Greater\_than}(x, z)$ $))$

   (c) There is a chef who cooks for all people in town who do not cook for themselves.
   $\exists x$ $(\text{Chef}(x) \wedge \forall y$ $((\text{Person}(y) \wedge \neg \text{Cooks}(y, y)) \Rightarrow \text{Cooks}(x, y)))$

3 Unification

(21 points) For each set of the following atomic sentences, find the *most general unifier* (MGU) and the corresponding unification instance. If certain pair is not unifiable, state so and explain why.

In the sentences below: Is_wizard, Are_brothers, Teaches, and P are predicates; *BestFriendOf*, *FatherOf*, and *SonOf* are functions; *Fred*, *George*, *Ron*, *Harry*, and *James* are constants; x, y, z, u, and v are variables which have already been standardized apart.

1. • $P(Fred, x, Ron)$

- $P(y, y, x)$

Not unifiable. Each variable should be paired at most once.

2.
   - Is_wizard(x)
   - Is_wizard($BestFriendOf(y)$)

   $\{\ x = BestFriendOf(y)\ \}$
   $Is\_wizard(BestFriendOf(y))$

3.
   - Are_brothers($Fred$, $George$)
   - Are_brothers(x, $Fred$)

   Not unifiable. Cannot unify different constants.

4.
   - $P(FatherOf(x), SonOf(x, y), x)$
   - $P(z, u, SonOf(z, u))$

   Not unifiable. Cannot have circular unifiers.

5.
   - $P(SonOf(x, y), FatherOf(Ron), u)$
   - $P(v, FatherOf(u), x)$

   $\{v = SonOf(x, y), u = Ron, x = u\}$
   $P(SonOf(Ron, y), FatherOf(Ron), Ron)$

6.
   - Teaches($FatherOf(x), x$)
   - Teaches($y, SonOf(z, u)$)
   - Teaches($w, SonOf(v, v)$)

   $\{\ y = FatherOf(x), w = y, x = SonOf(z, u), z = v, u = v\}$
   $Teaches(FatherOf(SonOf(v, v)), SonOf(v, v))$

7.
   - Teaches(x, $James$)
   - Teaches(y, $FatherOf(Harry)$)

   (We know that Harry's father is James)
   Not unifiable. Cannot unify a constant with a function.

4 Soundness and Completeness

(12 points) For each part below, give an inference rule that has the specified properties. Your inference rules must be different from all of those that appear in the lecture slides.

1. Sound but not complete
   To be sound, we need to infer something that is true.

$$\frac{\alpha \vee \beta \quad \neg\beta}{\alpha}$$

A database Δ entails a sentence ϕ ($\Delta \models \phi$) iff every possible world that satisfies δ also satisfies ϕ. In this example, you will have a truth table as follows.

α	β	$(\alpha \vee \beta) \wedge \neg\beta$
T	T	F
T	F	T
F	T	F
F	F	F

When $(\alpha \vee \beta) \wedge \neg\beta$ is True, α is also true. Therefore, this inference rule is sound. On the other hand, this inference rule is not complete since it cannot infer everything that is true.

2. Complete but not sound
   To be complete, we need to infer everything that is true. This includes all tautologies.

$$\frac{\text{(Nothing)}}{\alpha}$$

It simply says everything is true, so it's complete. It is not sound since it also infers everything that is false.

3. Neither sound nor complete

$$\frac{\begin{array}{c}\alpha \vee \beta \\ \alpha\end{array}}{\beta}$$

α	β	$(\alpha \vee \beta) \wedge \alpha$
T	T	T
T	F	T
F	T	F
F	F	F

When $(\alpha \vee \beta) \wedge \alpha$ is True, β may be False, meaning $\Delta \not\models \phi$. Thus, this inference rule is not sound. It is not complete either since it cannot infer everything that is true.