- Homework 2A is available
- Read Chapter 10

Clause Form

(also clausal form)

- Set notation of CNF (conjunctive normal form - also POS)
- R & N stop with CNF – we do not
- Write axioms as a conjunction of sentences
- Each sentence is a disjunction of literals
  (recall literal: atomic WFF or negated atomic WFF)
- Braces { } denote sets; comma separates literals

Convert FOPC to Clause Form

1. **Eliminate equivalence $\Leftrightarrow$ and implication $\Rightarrow$ symbols**
2. **Move $\neg$ inwards forming literals**
3. **Standardize variables apart - unique variable names eliminating scoping conflicts**
4. **Skolemize**
5. **Drop universal quantifiers**
6. **Distribute AND $\wedge$ over OR $\vee$**
7. **Flatten nested ANDs $\wedge$ and ORs $\vee$ yielding CNF (POS)**
8. **Write in set notation standardizing variables apart in different clauses**

Example: $\forall x\, \exists y\, [(\exists z\, (R(x, z) \vee P(y, z))) \Rightarrow \forall z\, Q(y,z)]$

1. $\forall x\, \exists y\, [(\neg(\exists z\, (R(x, z) \vee P(y, z)))) \vee \forall z\, Q(y,z)]$

2. $\forall x\, \exists y\, [(\forall z\, (\neg R(x, z) \wedge \neg P(y, z))) \vee \forall z\, Q(y,z)]$

3. $\forall x1\, \exists y1\, [(\forall z1\, (\neg R(x1, z1) \wedge \neg P(y1, z1))) \vee \forall z2\, Q(y1, z2)]$

4. $\forall x1\, [(\forall z1\, (\neg R(x1, z1) \wedge \neg P(Sk1(x1), z1))) \vee$
   $\forall z2\, Q(Sk1(x1), z2)]$

5. $[(\neg R(x1, z1) \wedge \neg P(Sk1(x1), z1)) \vee Q(Sk1(x1), z2)]$

6. $[(\neg R(x1, z1) \vee Q(Sk1(x1), z2)) \wedge$
   $(\neg P(Sk1(x1), z1) \vee Q(Sk1(x1), z2))]$

7. $[(\neg R(x1, z1) \vee Q(Sk1(x1), z2)) \wedge$
   $(\neg P(Sk1(x1), z1) \vee Q(Sk1(x1), z2))]$

8. $\{\neg R(x2, z3), Q(Sk1(x2), z4)\}$
   $\{\neg P(Sk1(x3), z5), Q(Sk1(x3), z6)\}$

7. [(¬R(x1, z1) ∨ Q(Sk1(x1), z2)) ∧
(¬P(Sk1(x1), z1) ∨ Q(Sk1(x1), z2))]

8. {¬R(x2, z3), Q(Sk1(x2), z4)}
{¬P(Sk1(x3), z5), Q(Sk1(x3), z6)}

$\forall$x L(x) ∧ M(x) is the same as $\forall$w L(w) ∧ $\forall$v M(v)

THIS DOES NOT WORK WITH ∨
Don't rename between ORs

$\forall$x L(x) ∨ M(x) is NOT the same as $\forall$w L(w) ∨ $\forall$v M(v)

7. $[(\neg R(x1, z1) \vee Q(Sk1(x1), z2)) \wedge$
$(\neg P(Sk1(x1), z1) \vee Q(Sk1(x1), z2))]$

$$\forall x1\ \forall z1\ \forall z2\ [(\neg R(x1, z1) \vee Q(Sk1(x1), z2)) \wedge (\neg P(Sk1(x1), z1) \vee Q(Sk1(x1), z2))]$$

$$\forall x2\ \forall z3\ \forall z4\ (\neg R(x2, z3) \vee Q(Sk1(x2), z4)) \wedge \forall x3\ \forall z5\ \forall z6\ (\neg P(Sk1(x3), z5) \vee Q(Sk1(x3), z6))]$$

Refutation Proofs

- Assume negated goal
  - augment the axiom set with the negated goal
- Derive a contradiction
- Why it works:
  - If Δ is satisfiable
    then $\Delta \Dashv \theta \equiv \Delta \wedge \neg\theta$ is unsatisfiable
- Infer a contradiction ≡ “False” ≡ { }
- Unifier gives sufficient constraints to get { }

Refutation Inference Procedures and Completeness

Recall difficulty achieving both soundness & completeness

Start with an empty axiom set

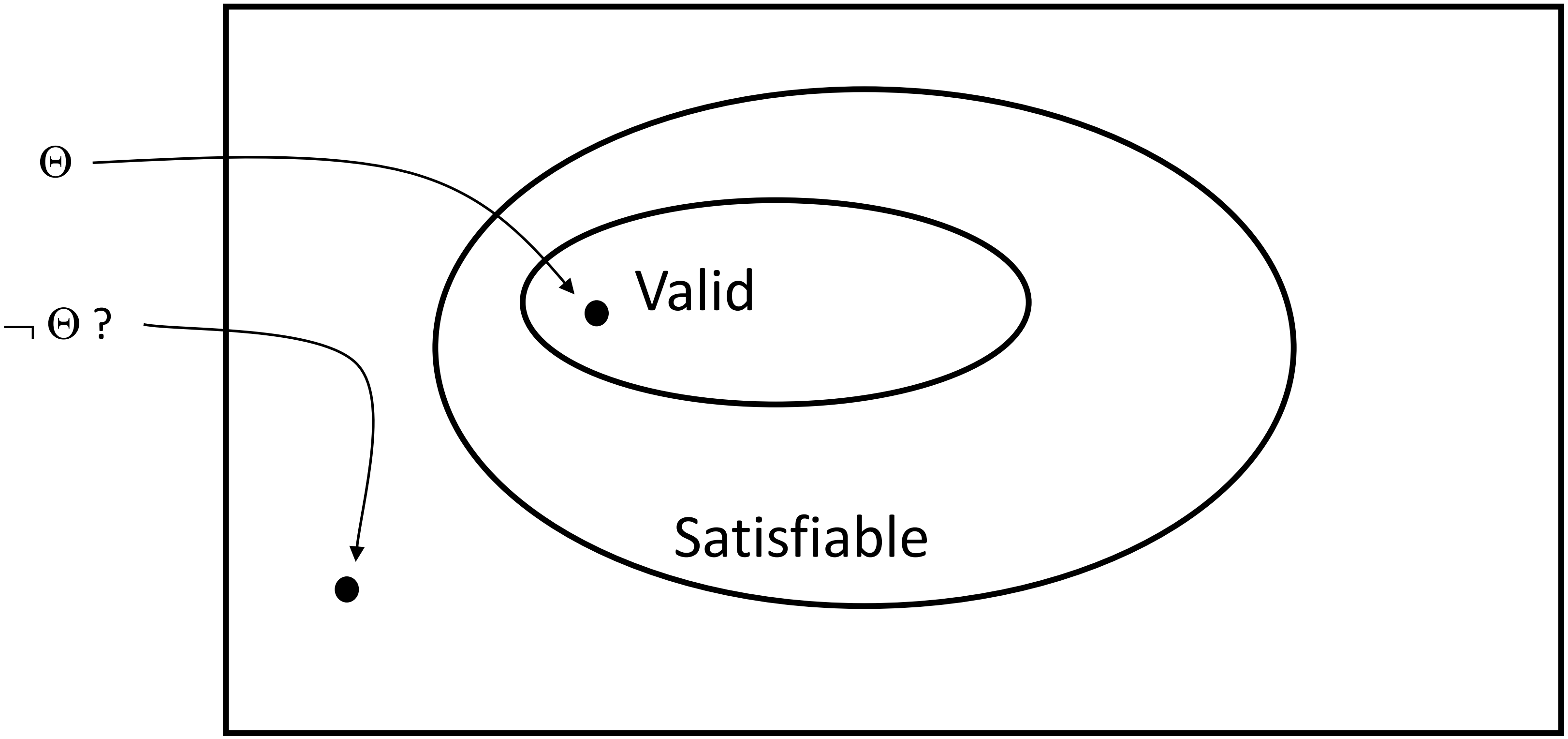
Can we derive a tautology?

Try proving $\mathbf{P} \vee \neg\mathbf{P}$

Negated Goal:

$\neg \mathbf{P} \wedge \mathbf{P}$

Negated Valid Sentence

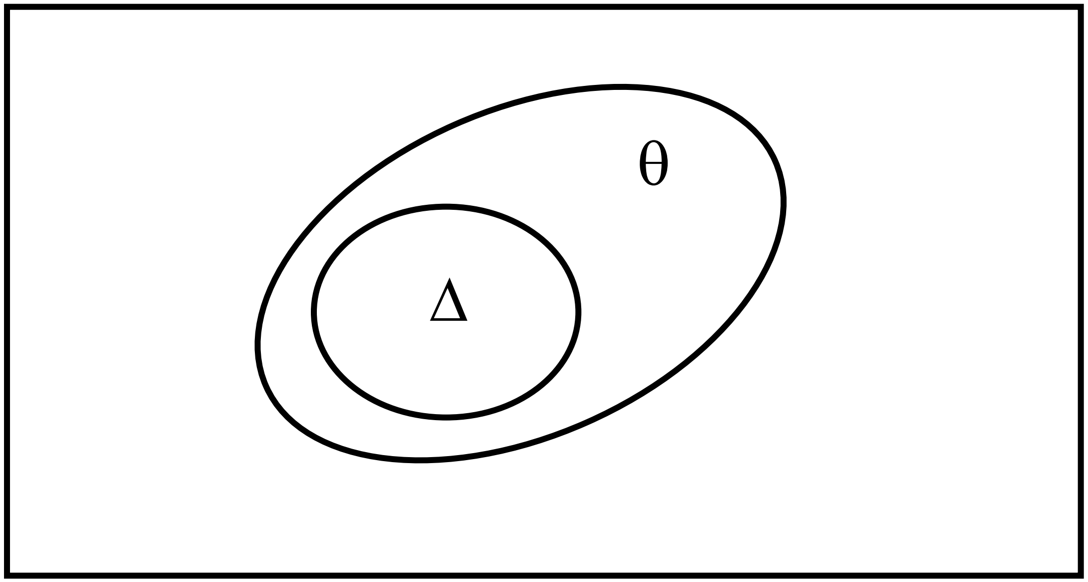


All WFFs

(**not** universe of possible worlds!)

What happens with possible worlds?

Suppose Δ is Satisfiable and Δ Entails θ



All possible worlds

What are the possible worlds of θ?

What are the possible worlds of $\neg\theta$?

What are the possible worlds of $(\Delta \wedge \neg\theta)$?

FOPC Resolution Refutation Example

- Anything that can read is literate
- Dolphins are illiterate
- Some dolphins are intelligent

- Is there some intelligent thing that does not read?

To Clause Form

1. Anything that can read is literate

    {¬Reads(x1), Literate(x1)}

2. Dolphins are illiterate

    {¬Dolphin(x2), ¬Literate(x2)}

3. Some dolphins are intelligent

    {Dolphin(Dolph21)}

    {Intelligent(Dolph21)}

Axioms in Clause Form

1. $\{\neg Reads(x1), Literate(x1)\}$
2. $\{\neg Dolphin(x2), \neg Literate(x2)\}$
3. $\{Dolphin(Dolph21)\}$
4. $\{Intelligent(Dolph21)\}$

Resolution

$$\frac{\alpha \vee \beta \qquad \neg\beta \vee \gamma}{\alpha \vee \gamma} \qquad\qquad \frac{\{\alpha, \beta\} \qquad \{\neg\beta, \gamma\}}{\{\alpha, \gamma\}}$$

To improve generality and efficiency we use set operations (member, subset, etc.) to implement the inference procedure:

Find two sets in which an element of one unifies with the negated element of the other.

With the unifier applied, let β be the element, let Θ be one set, let Φ be the other set

suppose: $\beta \in \Theta$ **and** $\neg \beta \in \Phi$

Infer: Union($\Theta - \{\beta\}$ **,** $\Phi - \{\neg\beta\}$ **)**

remember to standardize variables apart in every new sentence and every new use of an existing sentence

Negate Goal

- Is there some intelligent being that does not read?

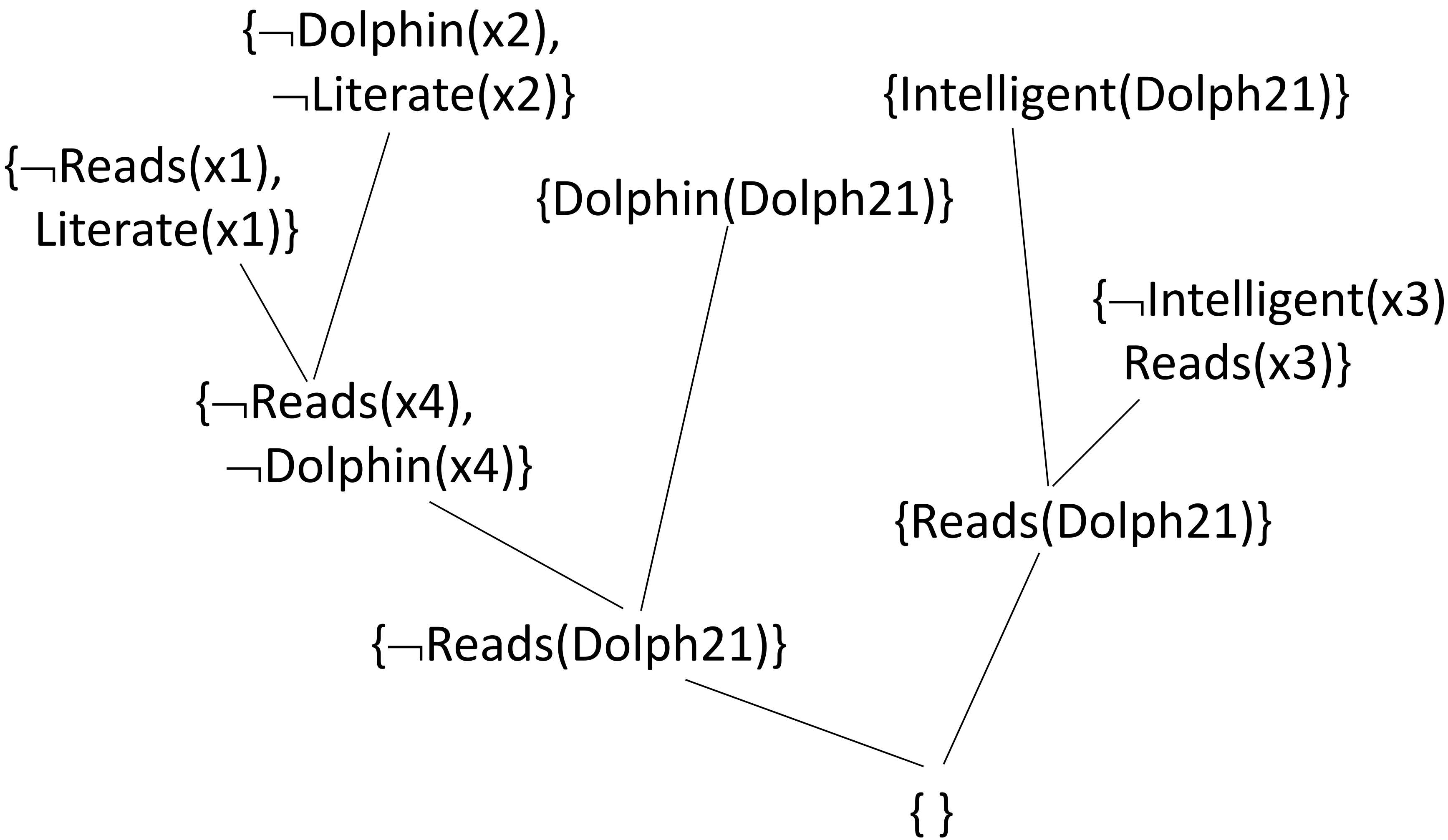
$\exists x\ [\text{Intelligent}(x) \land \neg \text{Reads}(x)]$	goal
$\forall x\ [\neg \text{Intelligent}(x) \lor \text{Reads}(x)]$	neg goal
$\{\neg \text{Intelligent}(x3), \text{Reads}(x3)\}$	clause

Derivation of { }

1.	{¬Reads(x1), Literate(x1)}	Δ
2.	{¬Dolphin(x2), ¬Literate(x2)}	Δ
3.	{Dolphin(Dolph21)}	Δ
4.	{Intelligent(Dolph21)}	Δ
5.	{¬Intelligent(x3), Reads(x3)}	Neg Goal
6.	{Reads(Dolph21)}	4 & 5
7.	{¬Reads(x4), ¬Dolphin(x4)}	1 & 2 stand vars apart!
8.	{¬Reads(Dolph21)}	3 & 7
9.	{}	6 & 8

Q.E.D.

Resolution Proof Tree



Issues

- The role of search
- What if we guess wrong about resolvents?
- Are there heuristics?
  - Input / Unit Resolution
  - Horn Clauses - PROLOG
- Completeness and refutational inference

From Unsatisfiability to Returning Answers

Suppose that instead of

3. Some dolphins are intelligent

we have

3. Flipper is an intelligent dolphin

and we want to know
Is there some intelligent being that does not read?
If so, who?

We get the same axiom set but with an object constant instead of a zero-argument skolem function:
Flipper3 instead of Dolph21

but the result is the same: {}

Answer Literal

Replace each negated query sentence

{θ(x,y,z)}

with

{θ(x,y,z), Answer(x,y)}

- "Answer" is a new predicate, x & y are terms of interest
- The augmented axiom set is no longer inconsistent (why?)
- Can no longer derive {}; no ¬Answer literal to resolve with
- Terminate with any sentence of only an Answer atom
- Bindings of the Answer atom specifies sufficient requirements
- Our query: {¬Intelligent(x3), Reads(x3), Answer(x3)}
- At termination: {Answer(Flipper3)}

What can we do with it?

- Deductive databases
- Semantic Web
- Planning (next)
- Many other applications…

Deductive databases

- Directed advertising using database
- Query: Age(JohnSmith, < 10) ?
  - DB returns nothing
  - DB does not contain his age
- But DB includes other info on John Smith
  - DB does have his birth date
  - Relating DOB to age is an inference

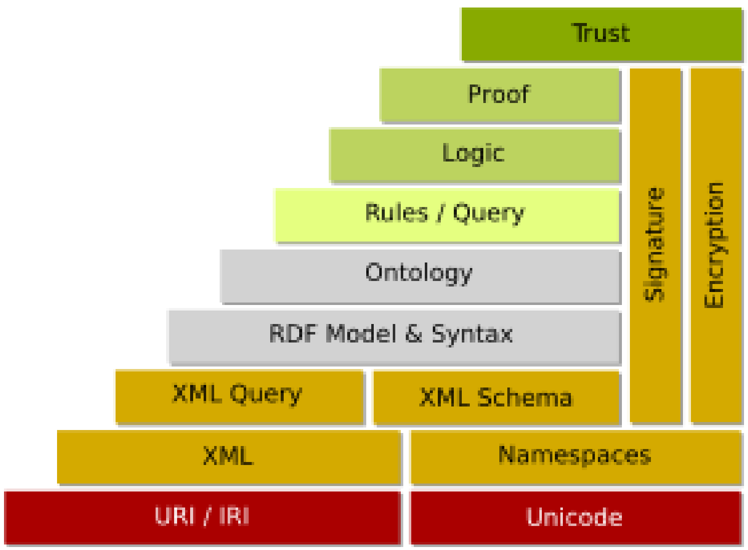
Deductive databases

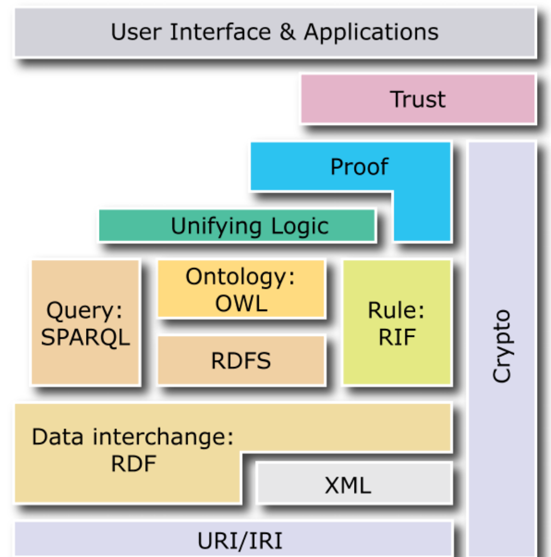
- Directed advertising using database
- Query: Age(JohnSmith, < 10) ?
  - DB returns nothing
  - DB does not contain his age
- But DB includes other info on John Smith
  - has an Illinois drivers license
  - in Illinois, minimum driving age is 17
- Deductive DB: use all / more relevant knowledge to answer queries

Semantic Web

- Current Web
  - Lots of information
  - Well hidden in sematically ill-defined pages
  - Automated server-side cleverness
  - Client-side cleverness relies on human
- Semantic Web
  - Standardize representations
  - Automate client-side cleverness
  - Need a model for web pages
    - RDF resource description framework
    - OWL Web Ontology Language

Semantic Web





Old and New Semantic Web "Layer Cakes"

How Hard is Theorem Proving?

$$\Delta \overset{?}{\models} \theta$$

Depends on the expressiveness of the language

Logic

• Propositional	Decidable (recursive)
• Description	Depends on choices
• First Order	Semi-decidable (thms recursively enumerable)
• Second Order	Undecidable