Announcement

- Homework 1 due Thursday

More Examples

Birds fly.

Some birds fly.

Room 1404 Siebel is empty.

Some Ford is better than any Buick.

Someone on the basketball team is taller than anyone on the football team.

“Birds Fly”

$\forall x\ [Bird(x) \Rightarrow Flies(x)]$

$\forall x\ [B(x) \Rightarrow F(x)]$ where B means “is a bird” and F means “can fly”

We can also think about the meaning as
“There is no bird that cannot fly”

$\neg\exists x\ [Bird(x) \wedge \neg Flies(x)]$

These are equivalent: the two predicate calculus sentences have the same meaning although they look quite different.

Some birds fly.

$\exists x$ [Bird(x) $\wedge$ Flies(x)]

Note: in logic "some" traditionally means "at least one"

Room 1404 Siebel is empty. [taken to mean empty of people]

Really Bad: P

Poor: Empty(Room1404SC)

Better: $\forall x$ [Person(x) $\Rightarrow$ Different(Location-of(x), Room1404SC)]

Still Better: $\forall x \forall y$ [(Person(x) $\wedge$ Location(y) $\wedge$ At(x,y)) $\Rightarrow$ Different(y, Room1404SC)]

Completely Wrong: (why?)

$\forall x$ [Person(x) $\Rightarrow$ At(x, $\neg$Room1404SC)]

NOTE: functions (like Location-of) are partial…

Some Ford is better than any Buick.

$\exists x\ [\text{Ford}(x) \wedge \forall y\ [\text{Buick}(y) \Rightarrow \text{Better}(x,y)]]$

Better(x,y) means "x is better than y"

(probably better to postulate some measurable quantity)

Someone on the basketball team is taller than anyone on the football team.

$\exists x\ [\text{Member}(x,\text{BBallTeam}) \wedge$

$\forall y\ \forall z\ \forall w\ [(\text{Height}(x,z) \wedge \text{Member}(y,\text{FBallTeam}) \wedge \text{Height}(y,w))$

$\Rightarrow \text{Greater}(z,w)]]$

Greater(x,y) means "x is larger than y"

Semantics

- **WFFs are truth-valuable; they may hold or they may not (depending on the state or configuration of the world).**
- **There are many ways the world might be.**
- **For each, there are many distinct denotational correspondences.**
- **A *possible world*, for us, is a world and a denotation.**
- **There is some Universe of all possible worlds.**
- **The *meaning* of a WFF is the subset of possible worlds in which it holds.**

(Lecture & text use “possible world” for the more standard but less intuitive word “interpretation”)

SEMANTICS

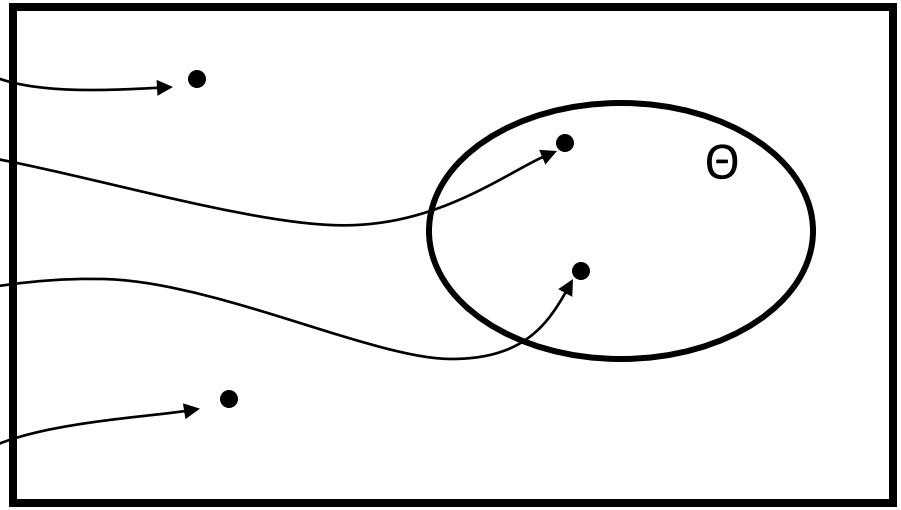
Θ: $\forall x$ [Student(x) $\Rightarrow$ Happy(x)]

Intuitive meaning
in our world

Student means “is a student”
Happy means “is happy”
and all students are joyful in this world

Student means “is a giraffe”
Happy means “has a short neck”
and there are no giraffes in this world

Student means “can drive”
Happy means “can swim”
in our world



Universe of Possible Worlds

A WFF is

Satisfiable if it holds

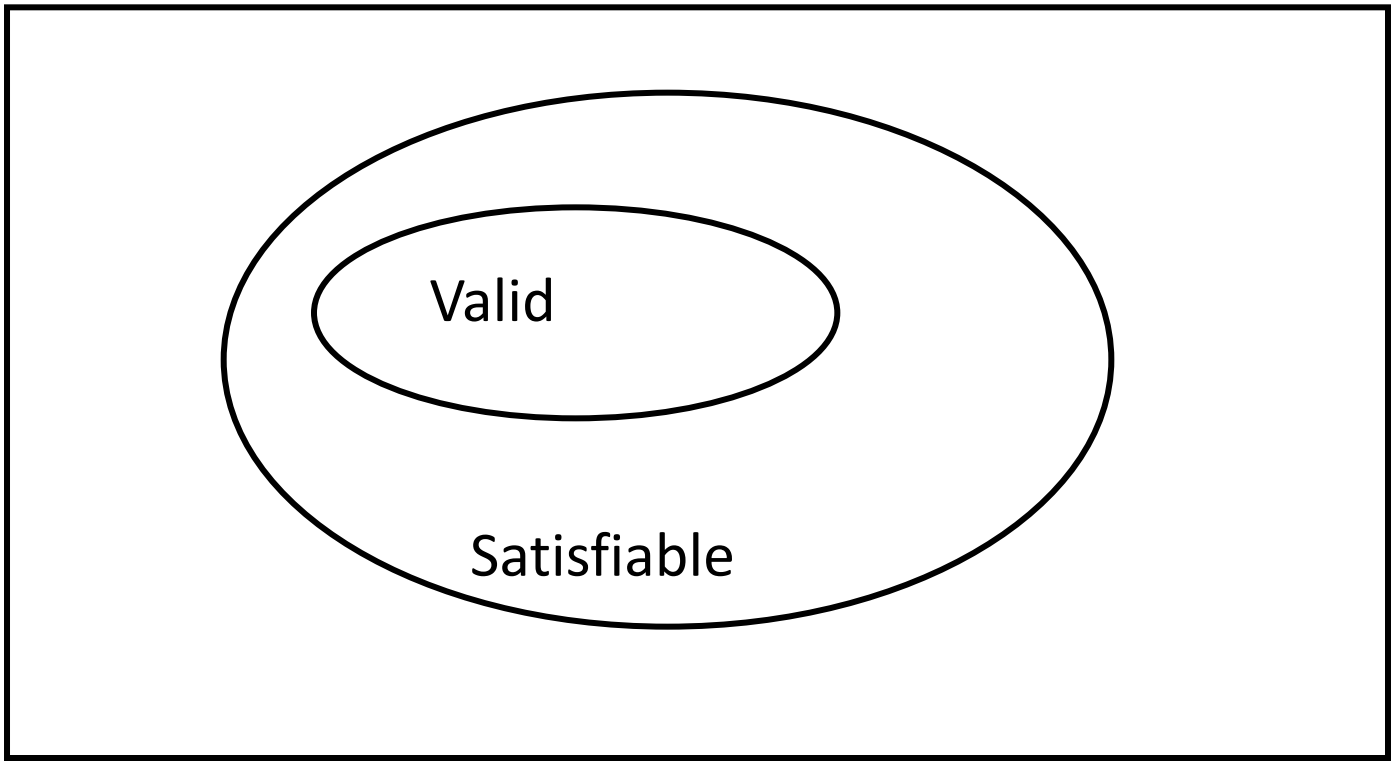
for **some** denotational correspondence
and **some** world

Valid if it holds

for **any** denotational correspondence
and **any** world

In Venn diagrams (actually Euler circles)…

Validity and Satisfiability



All WFFs

(**not** universe of possible worlds!)

“CS440 is my favorite class”

There is some amount that I like CS440
and I like all other classes less

$$\exists z\ \exists w\ [\text{Class}(w) \wedge \text{Name}(w,\text{“CS440”}) \wedge \text{Likes}(\text{Me},w,z) \wedge$$
$$\forall x\ \forall y\ \{[\text{Class}(x) \wedge \text{Likes}(\text{Me},x,y) \wedge \text{Different}(x,w)]$$
$$\Rightarrow \text{Greater}(z,y)\}]$$

Likes(a,b,c) means “a likes b by amount c”
Greater(a,b) means “a is larger than b”

There is no class that I like as much as CS440

(many others)

Give a WFF that is

1. Valid $P \vee \neg P$ $R \Rightarrow R$
2. Satisfiable P
3. Not valid P
4. Not satisfiable $P \wedge \neg P$
5. Satisfiable but not valid P
6. Valid but not satisfiable Not possible

Componential Semantics

- WFFs express constraints
- Meaning of a WFF is the set of possible worlds that satisfy
- $\wedge$ Intersect sets
- $\vee$ Union sets
- $\neg$ Complement set