Physical Layer

CS 438: Spring 2014

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http://courses.engr.illinois.edu/cs438/

Course Outline

~ Apr 16	**L7**	**Application**
~ Mar 21	**L4**	**Transport**
~ Feb 26	**L3**	**Network**
~ Feb 15	**L2**	**Data link**
Today	**L1**	**Physical**

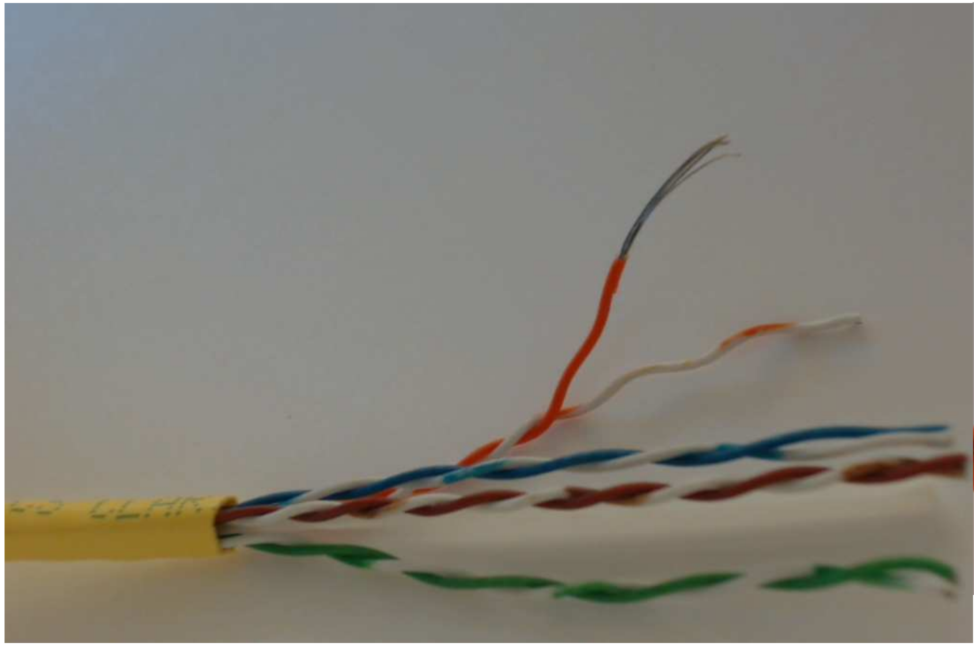
Outline for Today

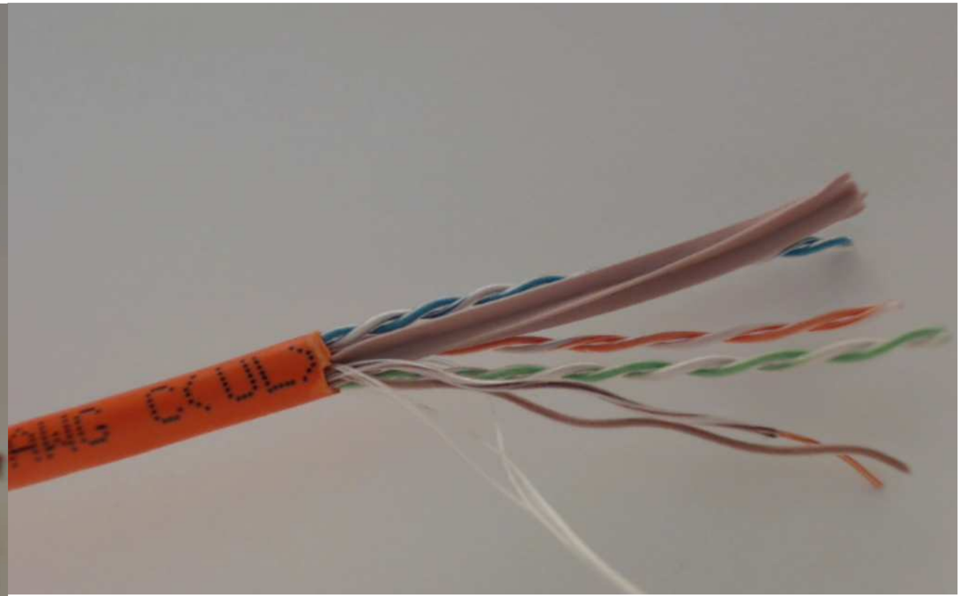
- Today: The Physical Layer

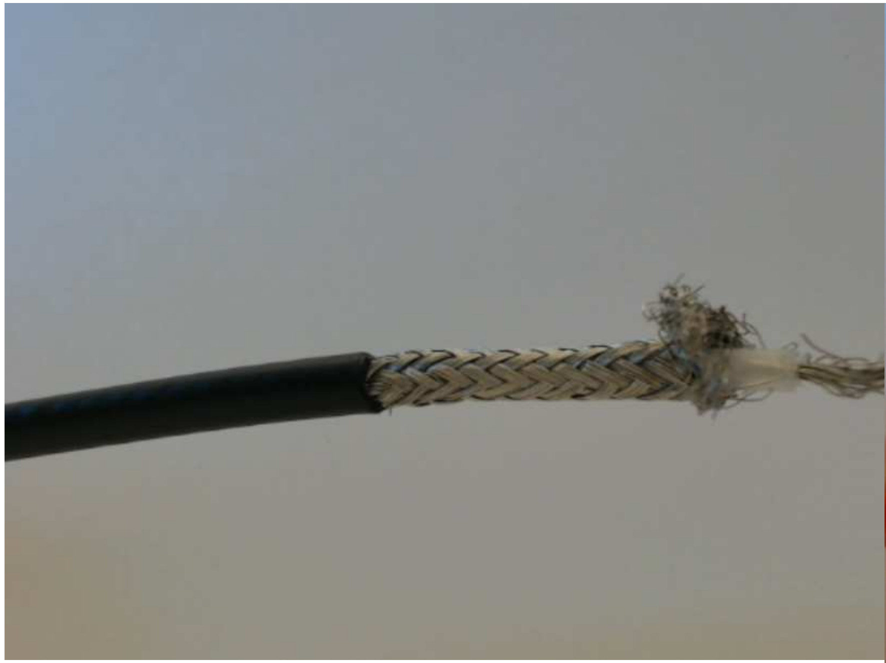
- How to encode data over a link

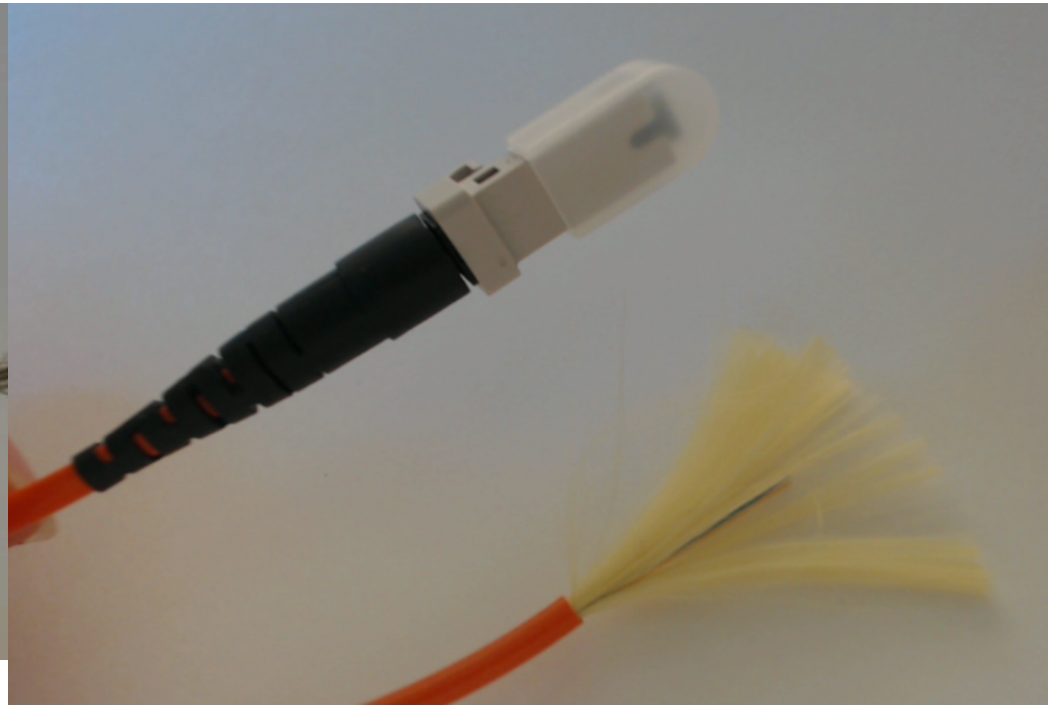
- How to detect and correct errors

A Brief Overview of Physical Media




AWG C(UL)





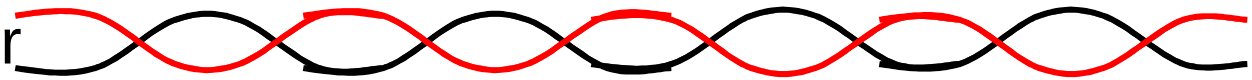
Links - Copper

- Copper-based Media

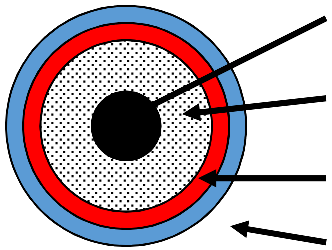
Category 3 Twisted Pair	10-100Mbps	100m
Category 5 Twisted Pair	10-100Mbps	200m
ThinNet Coaxial Cable	10-100Mbps	500m
ThickNet Coaxial Cable		

more twists, less crosstalk, better signal over longer distances

twisted pair



coaxial cable (coax)

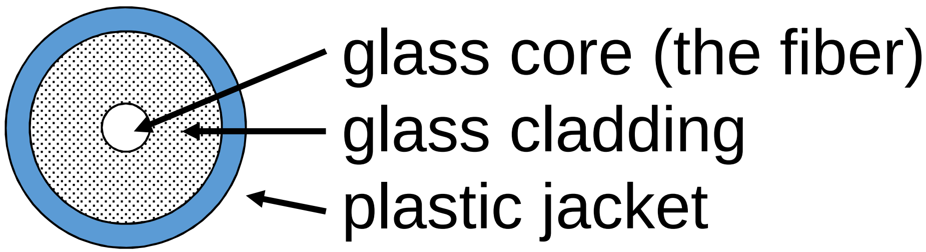


More expensive than twisted pair
High bandwidth and excellent noise immunity

Links - Optical

- Optical Media
  - Multimode Fiber 100Mbps 2km
  - Single Mode Fiber 100-2400Mbps 40km

optical
fiber



Links - Optical

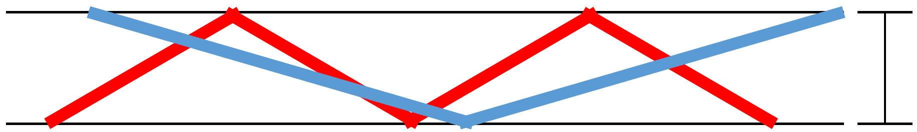
- Single mode fiber
  - Expensive to drive (Lasers)
  - Lower attenuation (longer distances) ≤ 0.5 dB/km
  - Lower dispersion (higher data rates)

- Multimode fiber
  - Cheap to drive (LED's)
  - Higher attenuation
  - Easier to terminate

core of single mode fiber

~1 wavelength thick = ~1 micron

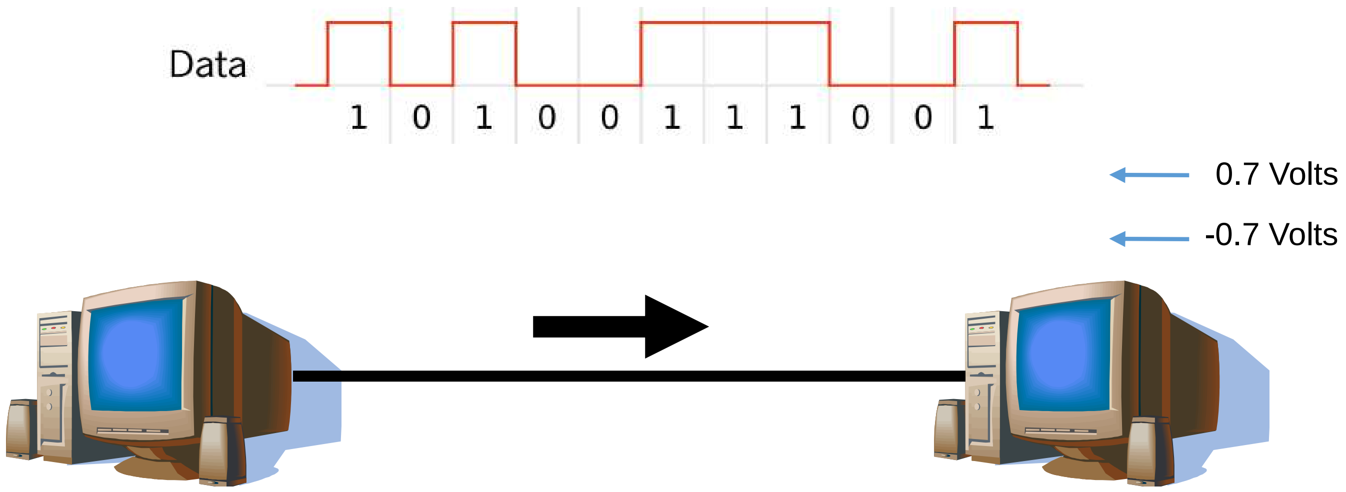
core of multimode fiber (same frequency; colors for clarity)



O(100 microns) thick

Encoding

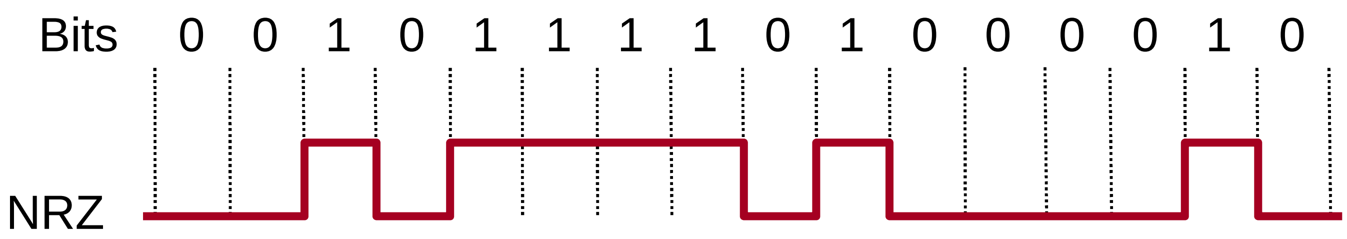
How can two hosts communicate?



- Encode data as variations in electrical/light/EM
  - Phase, frequency, and signal strength modulation, and combinations thereof
  - Simple scheme: voltage encoding
    - Encode 1's and 0's as variations in voltage
    - How to do that?

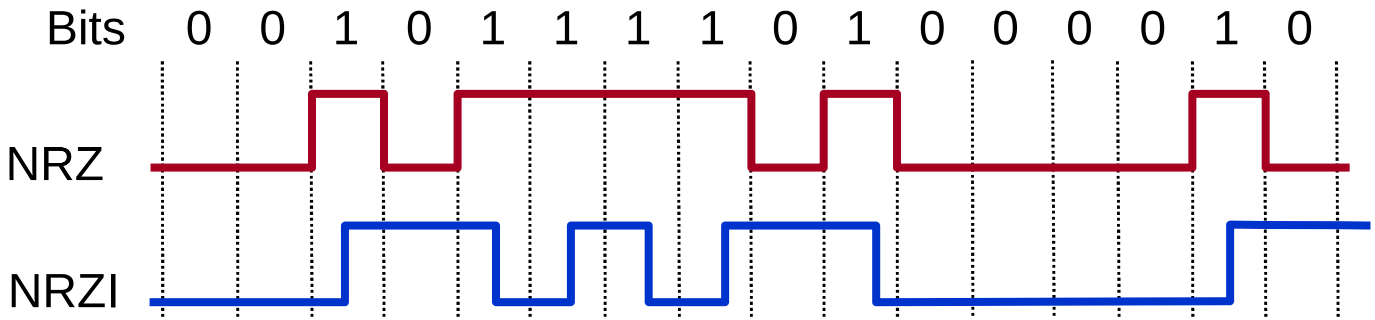
Non-Return to Zero (NRZ)

- Signal to Data
  - High ⇨ 1
  - Low ⇨ 0
- Comments
  - Transitions maintain clock synchronization
  - Long strings of 0s confused with no signal
  - Long strings of 1s causes baseline wander
  - Both inhibit clock recovery



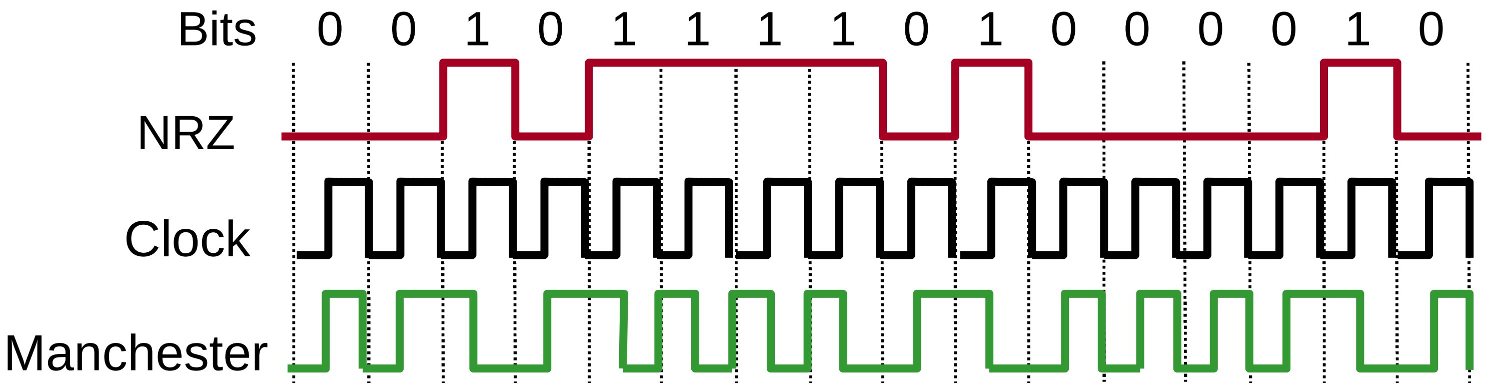
Non-Return to Zero Inverted (NRZI)

- Signal to Data
  - Transition ⇨ 1
  - Maintain ⇨ 0
- Comments
  - Solves series of 1s, but not 0s



Manchester Encoding

- Signal to Data
  - XOR NRZ data with clock
  - High to low transition ⇨ 1
  - Low to high transition ⇨ 0
- Comments
  - Used by old 10Mbps Ethernet
  - Solves clock recovery problem
  - Only 50% efficient (½ bit per transition)



4B/5B

- Signal to Data
  - Encode every 4 consecutive bits as a 5 bit symbol
- Symbols
  - At most 1 leading 0
  - At most 2 trailing 0s
  - Never more than 3 consecutive 0s
  - Transmit with NRZI
- Comments
  - 16 of 32 possible codes used for data
  - At least two transitions for each code
  - 80% efficient
  - Used by old 100Mbps Ethernet
  - Variation (64B/66B) used by modern 10Gbps Ethernet

4B/5B – Data Symbols

At most 1 leading 0

- 0000 ⇒ 11110
- 0001 ⇒ 01001
- 0010 ⇒ 10100
- 0011 ⇒ 10101
- 0100 ⇒ 01010
- 0101 ⇒ 01011
- 0110 ⇒ 01110
- 0111 ⇒ 01111

At most 2 trailing 0s

- 1000 ⇒ 10010
- 1001 ⇒ 10011
- 1010 ⇒ 10110
- 1011 ⇒ 10111
- 1100 ⇒ 11010
- 1101 ⇒ 11011
- 1110 ⇒ 11100
- 1111 ⇒ 11101

4B/5B – Control Symbols

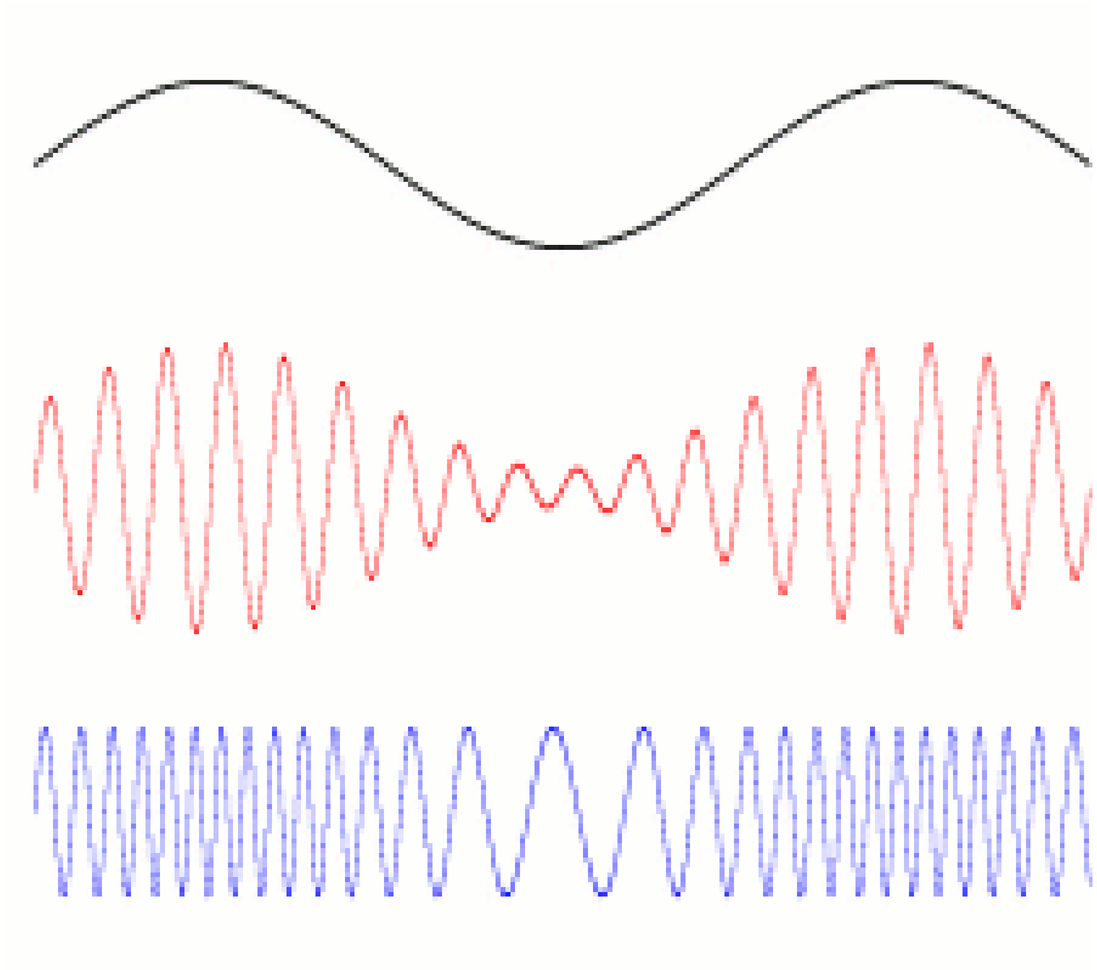
- 11111 $\Rightarrow$ idle
- 11000 $\Rightarrow$ start of stream 1
- 10001 $\Rightarrow$ start of stream 2
- 01101 $\Rightarrow$ end of stream 1
- 00111 $\Rightarrow$ end of stream 2
- 00100 $\Rightarrow$ transmit error
- Other $\Rightarrow$ invalid

Binary Voltage Encodings

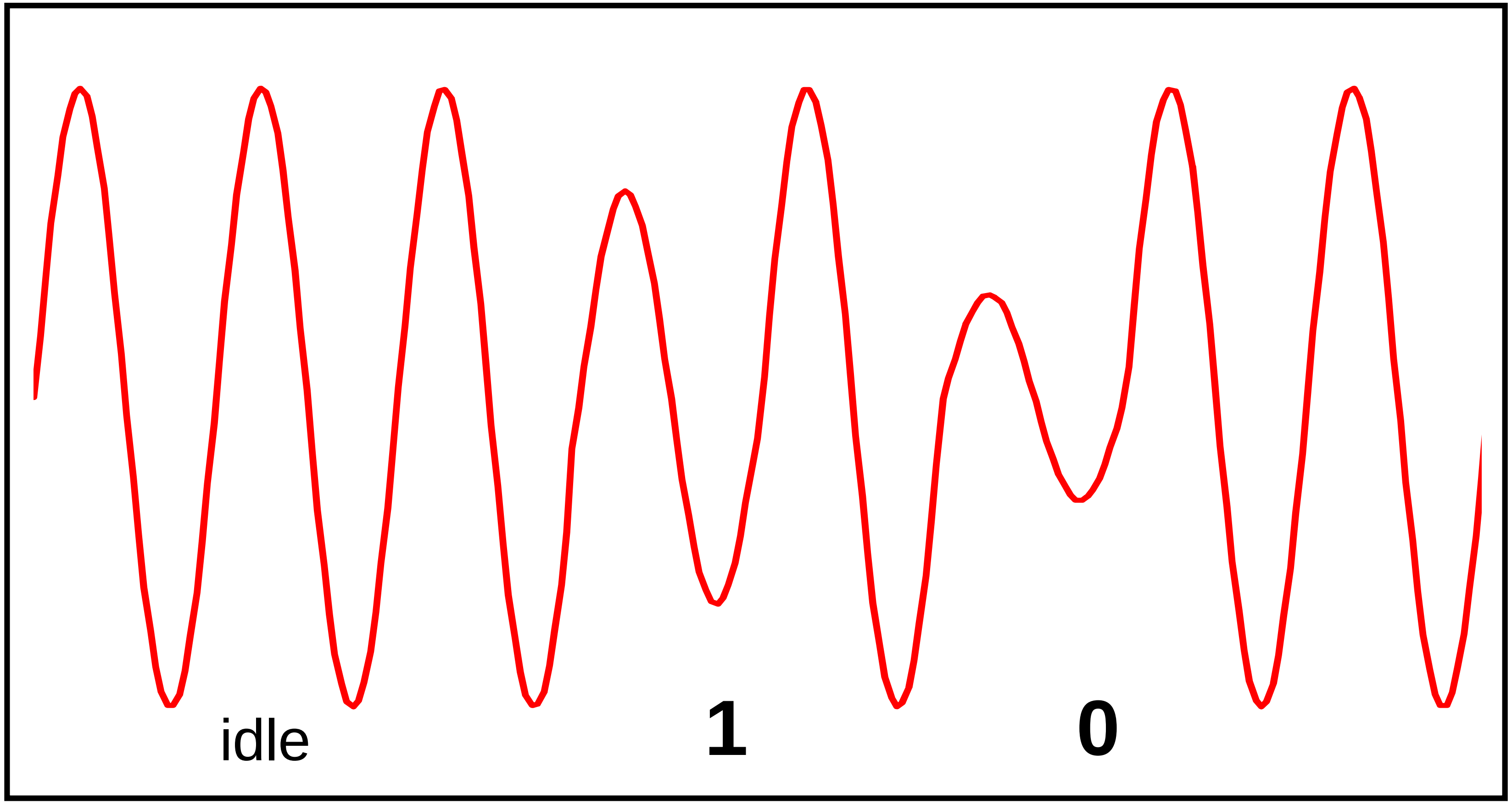
- Problem with binary voltage (square wave) encodings
  - Wide frequency range required, implying
    - Significant dispersion
    - Uneven attenuation
  - Prefer to use narrow frequency band (carrier frequency)
- Types of modulation
  - Amplitude (AM)
  - Frequency (FM)
  - Phase/phase shift
  - Combinations of these
  - Used in wireless Ethernet, optical communications

Example:
AM/FM for continuous signal

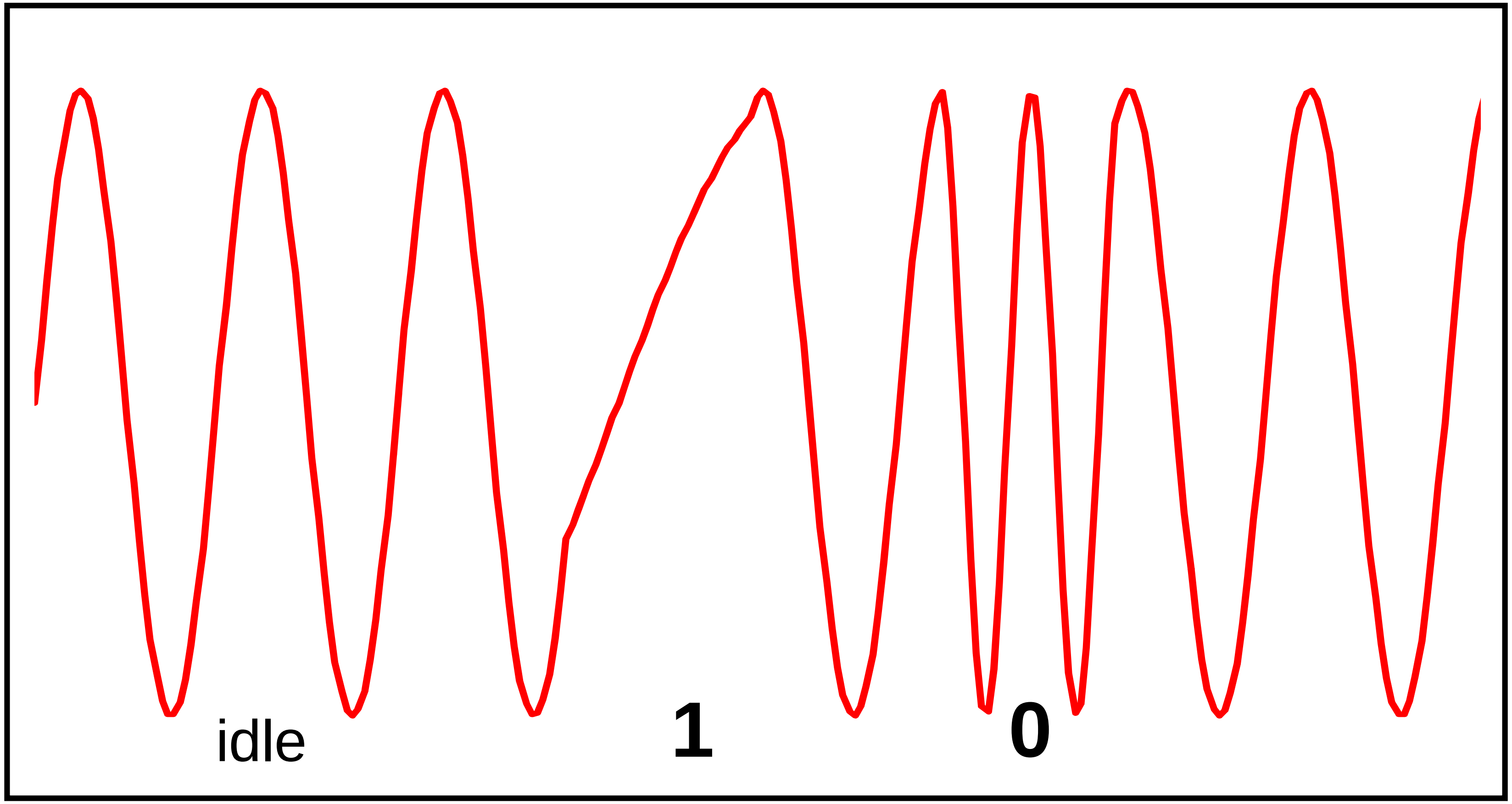
- Original signal
- Amplitude modulation
- Frequency modulation



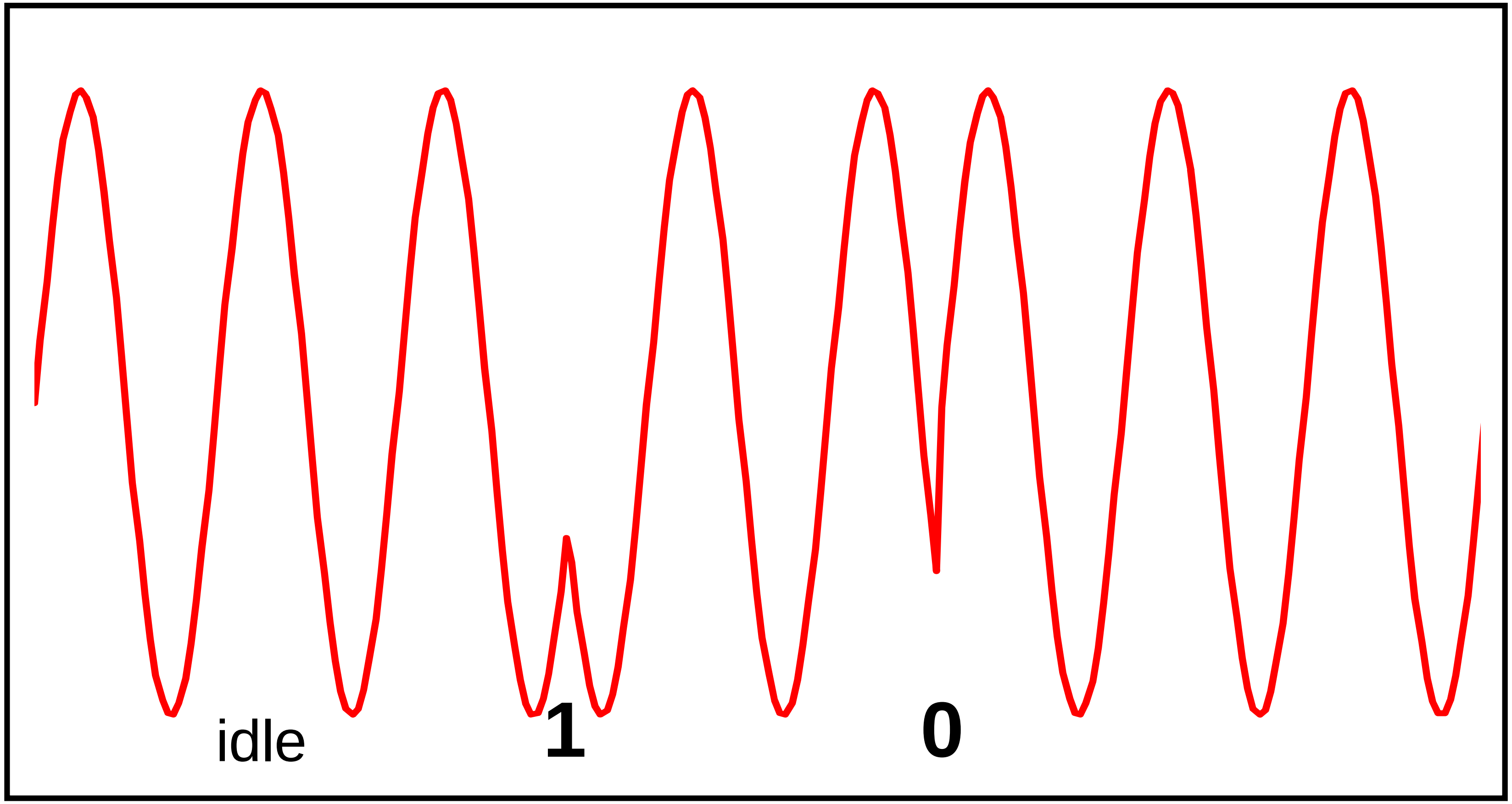
Amplitude Modulation



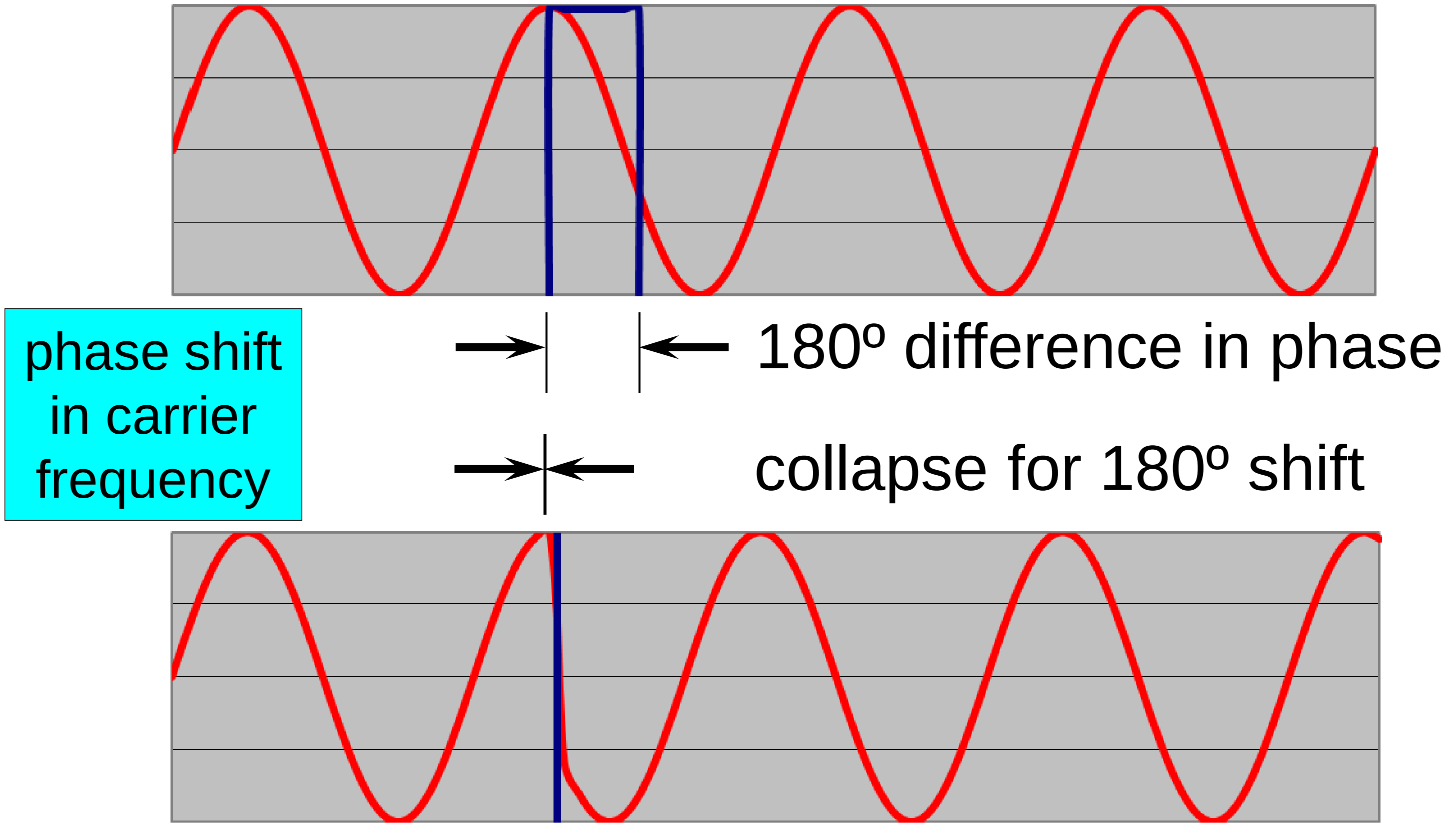
Frequency Modulation



Phase Modulation



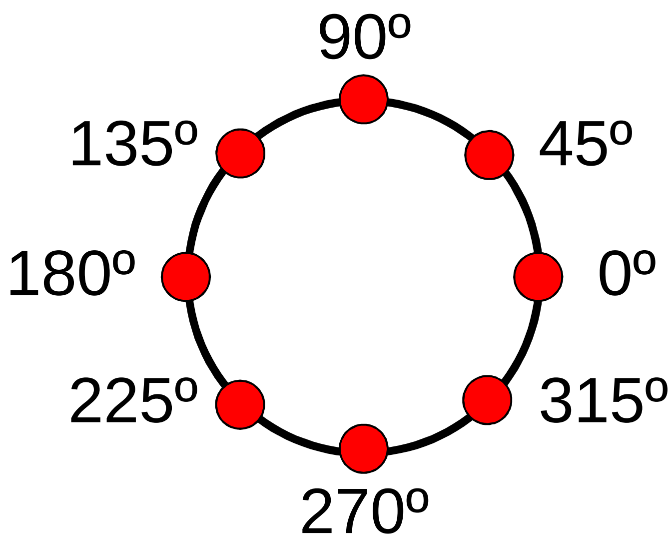
Phase Modulation



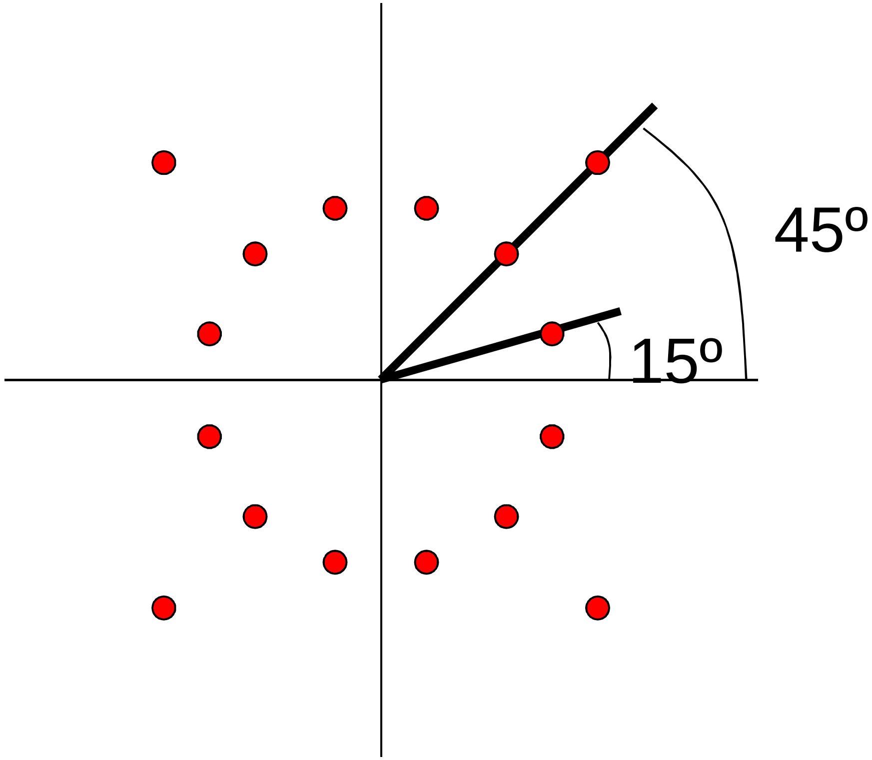
Phase Modulation Algorithm

- Send carrier frequency for one period
  - Perform phase shift
  - Shift value encodes symbol
    - Value in range [0, 360º)
    - Multiple values for multiple symbols
    - Represent as circle

8-symbol example



You can combine modulation schemes



Example: QAM (Quadrature Amplitude Modulation)

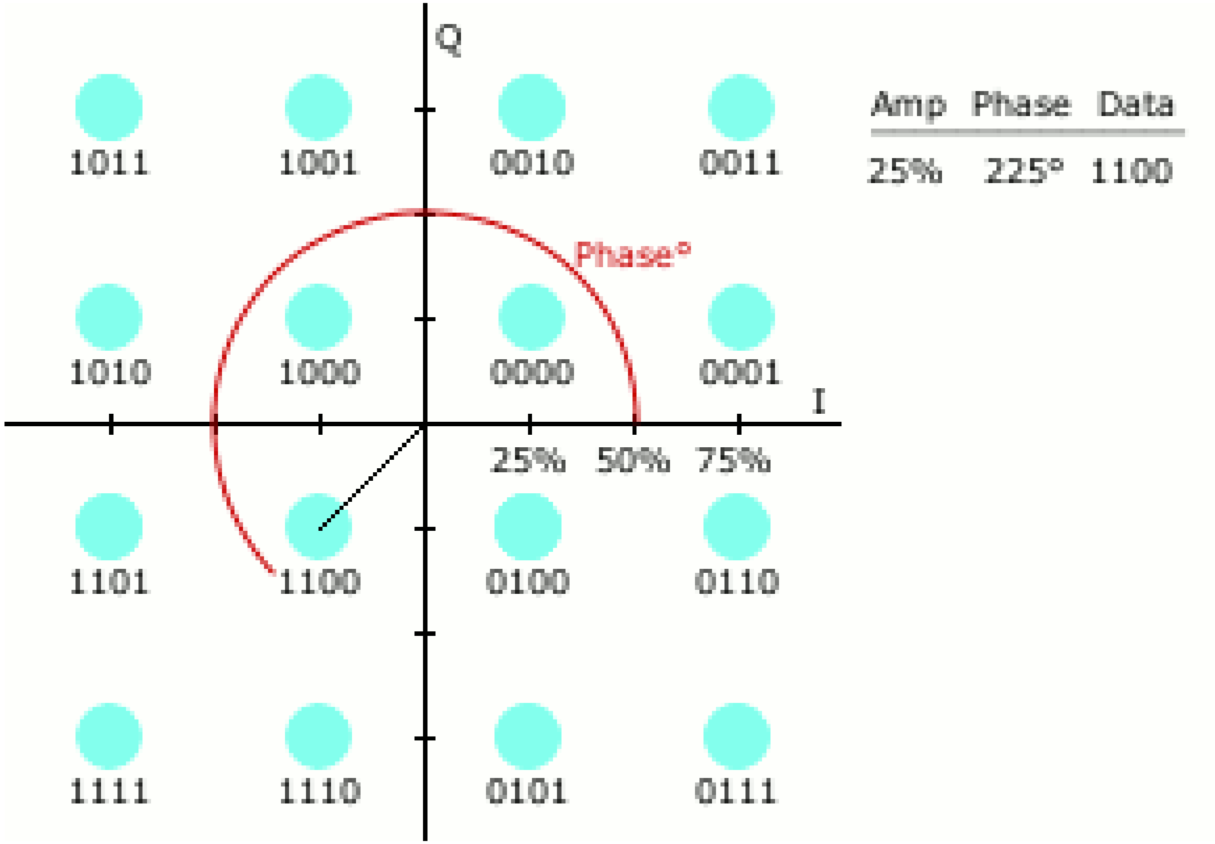
For a given symbol:

- Perform phase shift and change to new amplitude

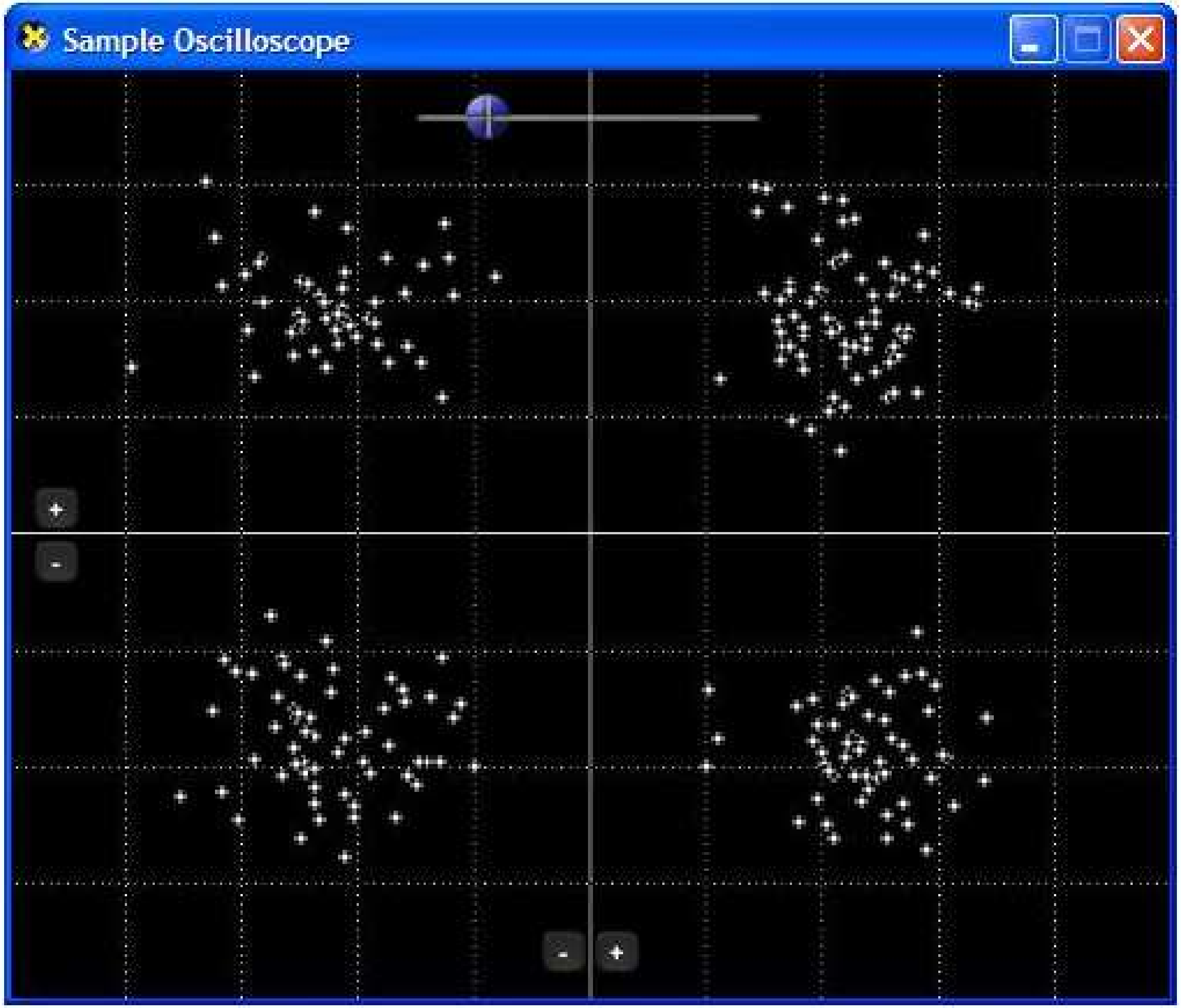
2-dimensional representation:

- Angle is phase shift
- Radial distance is new amplitude

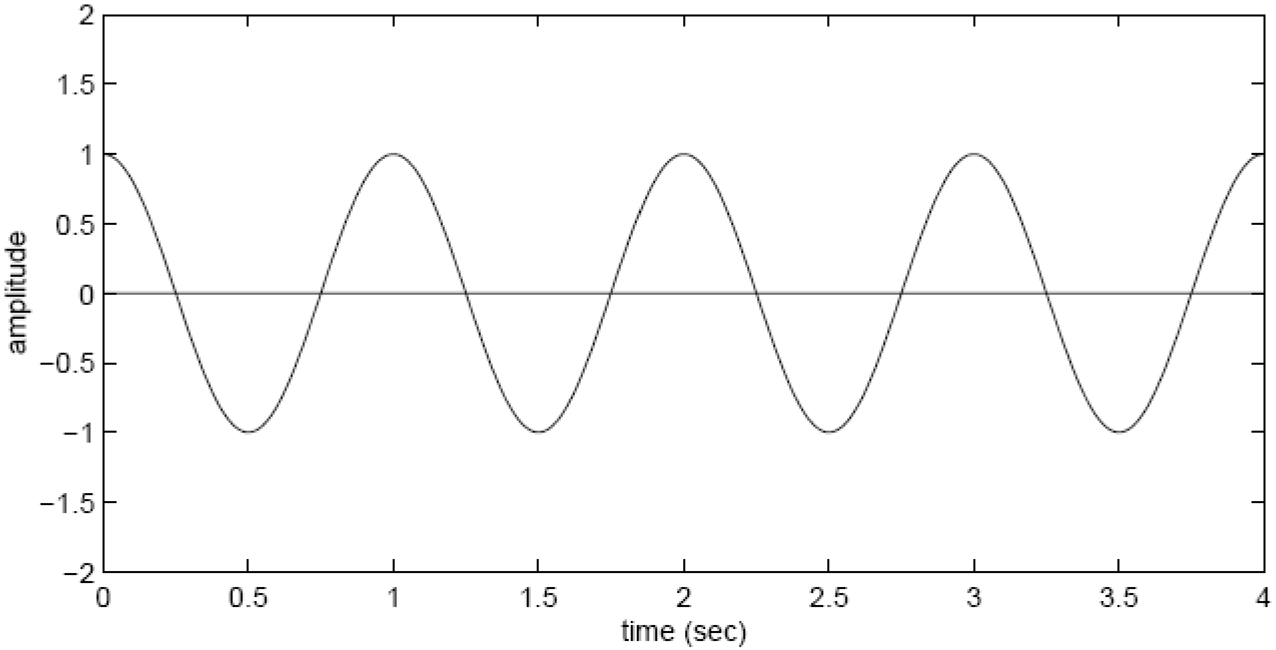
QAM: Example transmission



Real constellation with noise

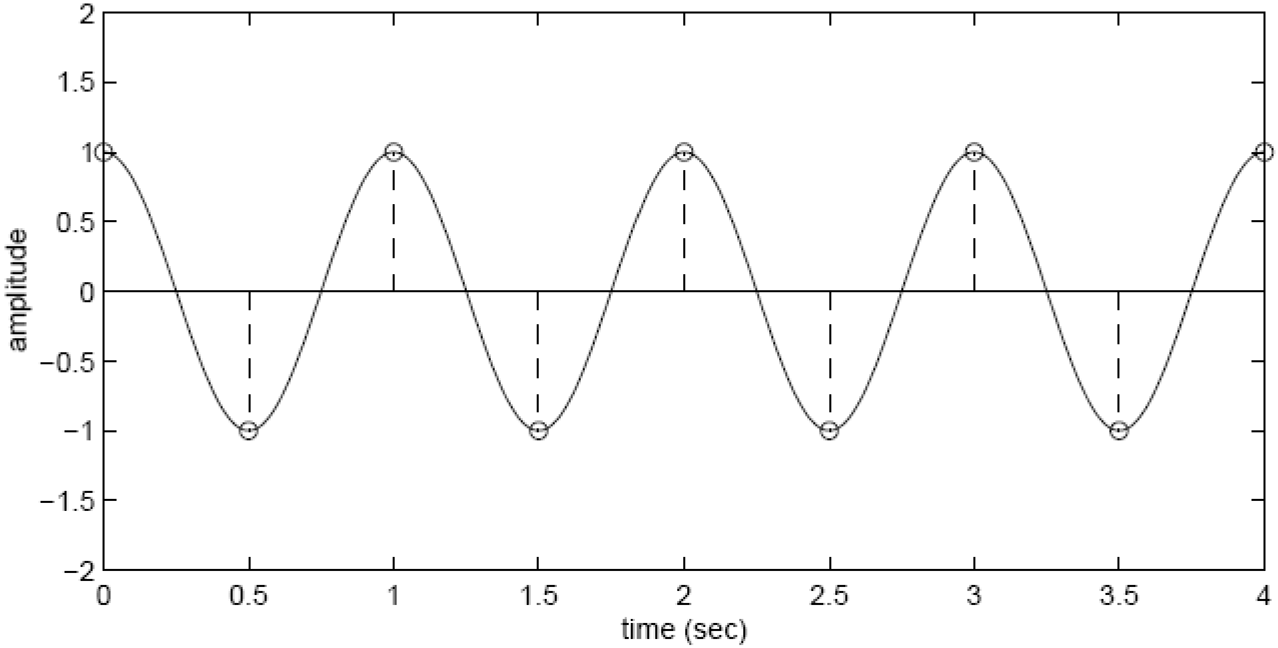


Sampling



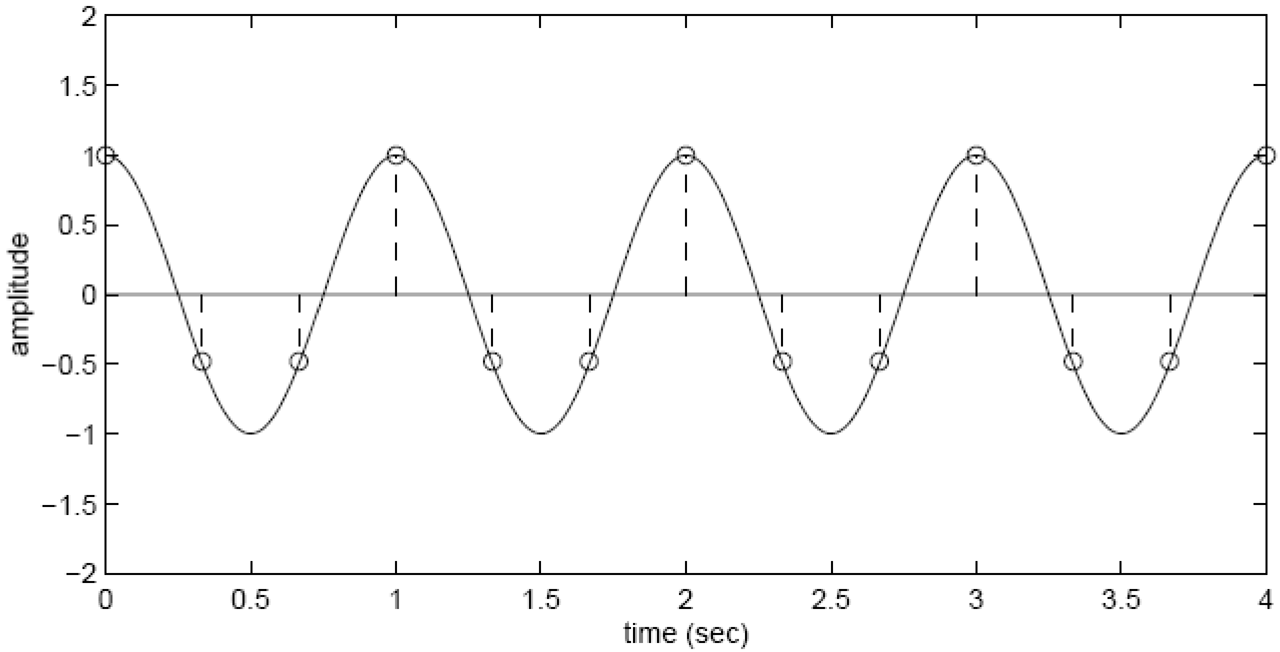
- Suppose you have the following 1Hz signal being received
- How fast to sample, to capture the signal?

Sampling



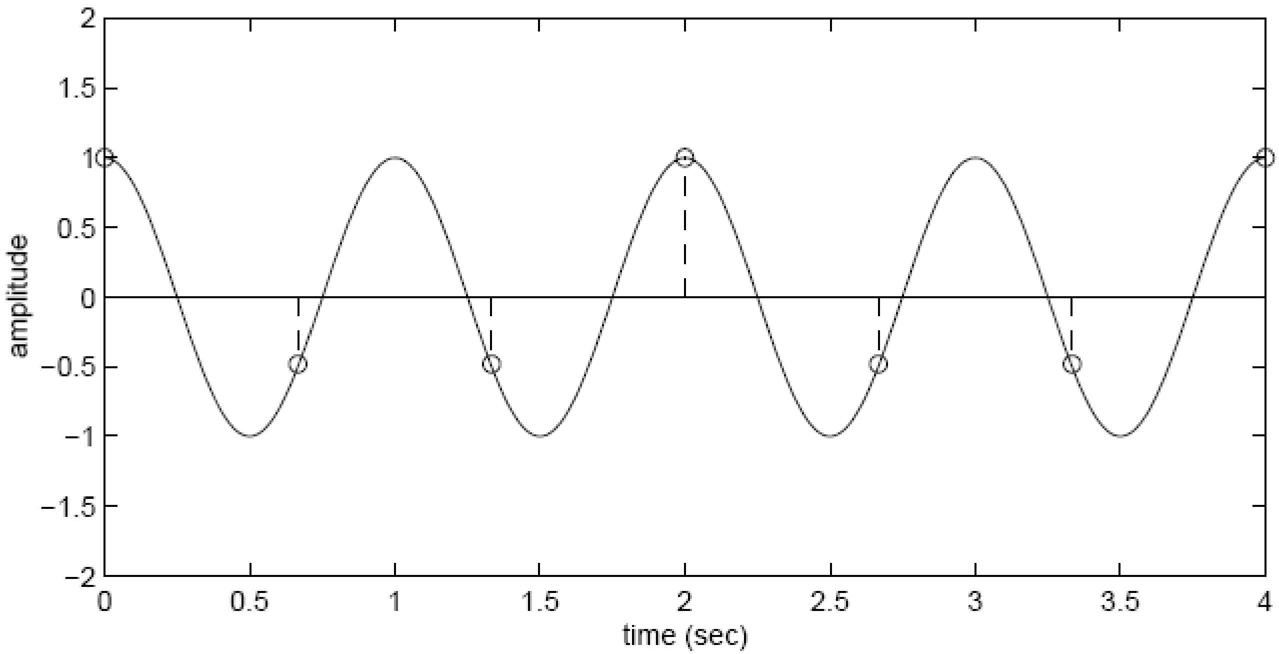
- Sampling a 1 Hz signal at 2 Hz is enough
  - Captures every peak and trough

Sampling

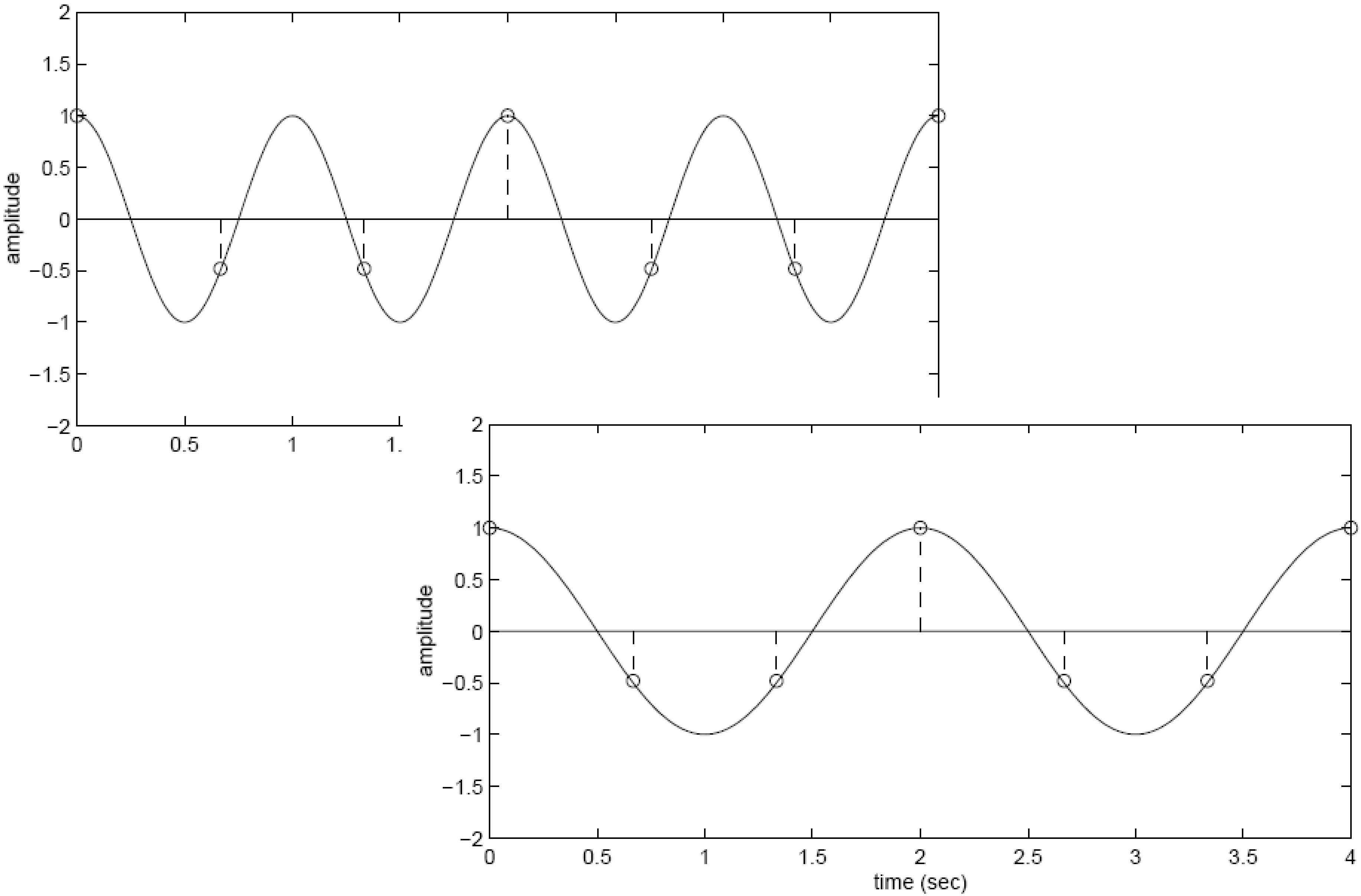


- Sampling a 1 Hz signal at 3 Hz is also enough
  - In fact, more than enough samples to capture variation in signal

Sampling

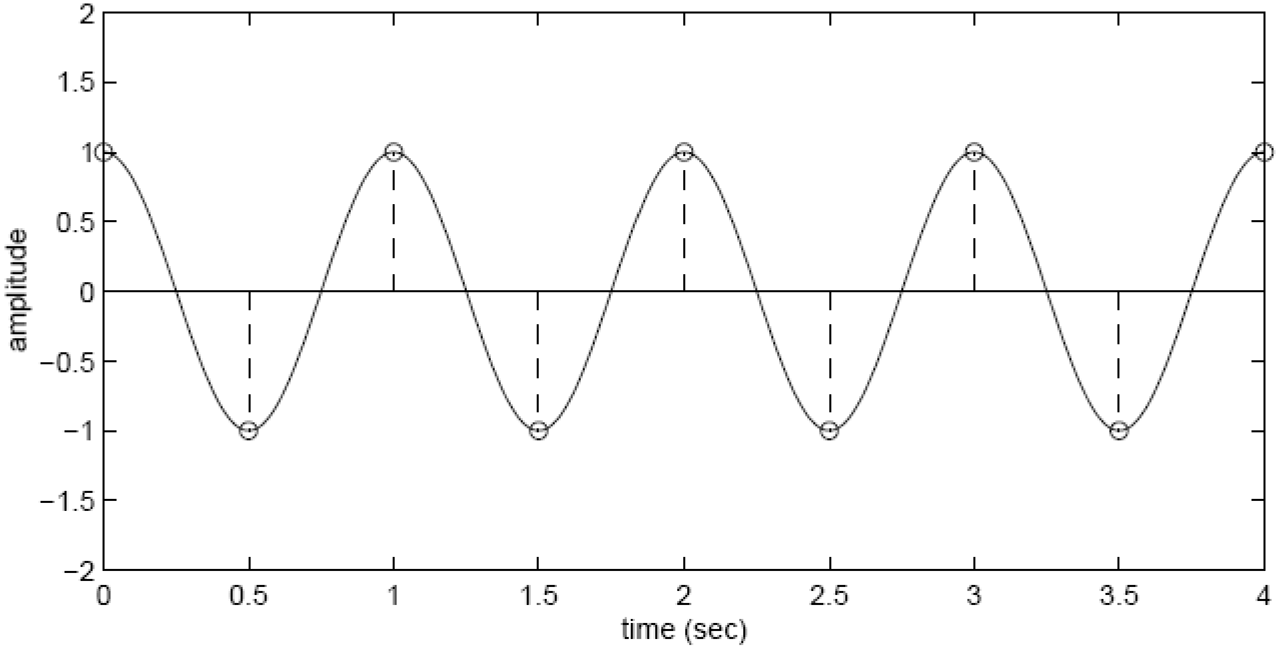


- Sampling a 1 Hz signal at 1.5 Hz is not enough
  - Why?



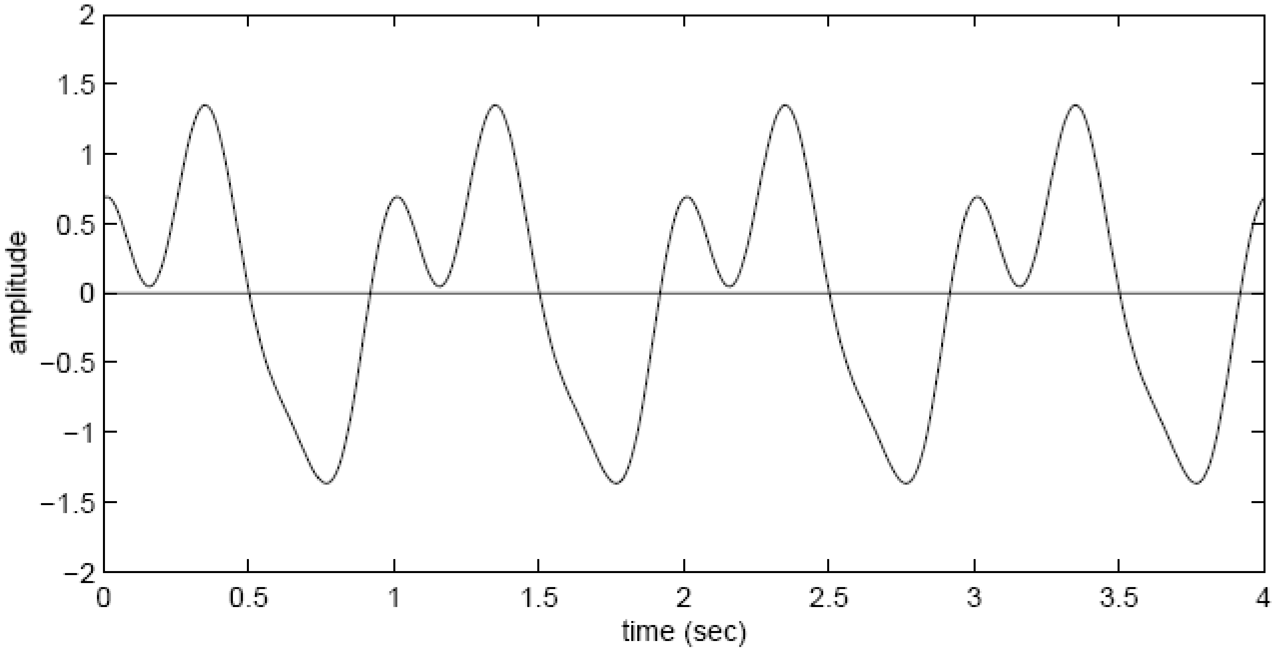
- Sampling a 1 Hz signal at 1.5 Hz is not enough
    - Not enough samples, can't distinguish between multiple possible signals

In general



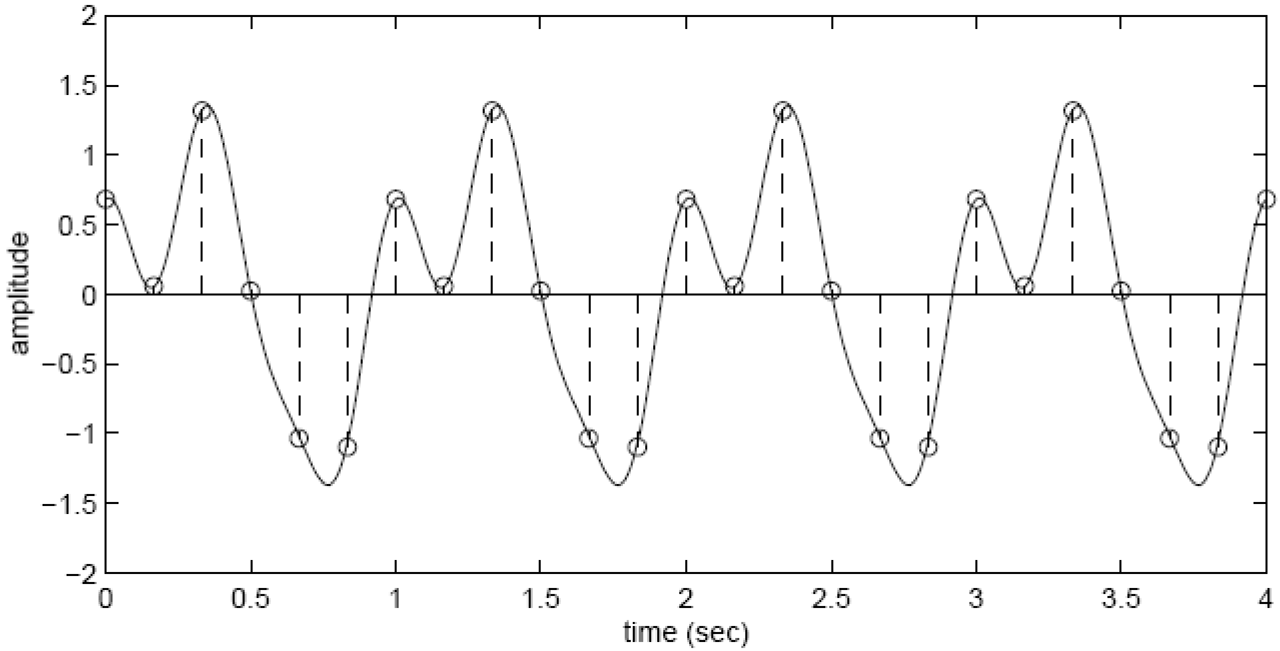
- Sampling a 1 Hz signal at 2 Hz is both necessary and sufficient
- In general: sampling twice rate of signal is enough

What about more complex signals?



- Fourier's theorem: any continuous signal can be decomposed into a sum of sines and cosines at different frequencies
- Example: Sum of 1 Hz, 2 Hz, and 3 Hz sines
  - How fast to sample?

What about more complex signals?



- Fourier’s theorem: any continuous signal can be decomposed into a sum of sines and cosines at different frequencies
- Example: Sum of 1 Hz, 2 Hz, and 3 Hz sines
  - How fast to sample?
  - Answer: Twice rate of fastest signal (bandwidth): **6 Hz**

Nyquist–Shannon sampling theorem

- If a function $x(t)$ contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced $1/(2B)$ seconds apart

- In other words:
  - If the bandwidth of your channel is B
  - Your sampling rate should be 2B
  - Higher sampling rates are pointless
  - Lower sampling rates lead to aliasing/distortion/error

Related Question: How much data can you pack into a channel?

- If I sample at a rate of 2B, I can precisely determine the signal of bandwidth B
- If I have data coming in at rate 2B, I can encode it in a channel of rate B
  - Similar argument to above, but in reverse
  - Instead of “reading” a sample, we “write” a sample
- More generally:
  - Transmitting N distinct signals over a noiseless channel with bandwidth B, we can achieve at most a data rate of
  - 2B log2 N

Noiseless Capacity

- Nyquist's theorem: $2B \log_2 N$
- Example 1: sampling rate of a phone line
  - B = 4000 Hz
  - 2B = 8000 samples/sec.
    - sample every 125 microseconds
- Example 2: noiseless capacity
  - B = 1200 Hz
  - N = each pulse encodes 16 levels
  - $C = 2B \log_2 (N) = D \times \log_2 (N)$

    $= 2400 \times 4 = 9600$ bps.

What can Limit Maximum Data Rate?

- Noise
  - E.g., thermal noise (in-band noise) can blur symbols
- Transitions between symbols
  - Introduce high-frequency components into the transmitted signal
  - Such components cannot be recovered (by Nyquist's Theorem), and some information is lost
- Examples
  - Phase modulation
    - Single frequency (with different phases) for each symbol
    - Transitions can require very high frequencies

How does Noise affect these Bounds?

- In-band (thermal, not high-frequency) noise
  - Blurs the symbols, reducing the number of symbols that can be reliably distinguished.
- Claude Shannon (1948)
  - Extended Nyquist's work to channels with additive white Gaussian noise (a good model for thermal noise)

$$\text{channel capacity } C = B \log_2 (1 + S/N)$$

B is the channel bandwidth

S/N is the ratio between

the average signal power and

the average in-band noise power

Noisy Capacity

$$\mathrm{SNR(dB)} = 10\log_{10}\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right)$$

- Telephone channel
  - 3400 Hz at 40 dB SNR
  - C = B $\log_2$ (1+S/N) bits/s
  - SNR = 40 dB

    40 =10 $\log_{10}$ (S/N)

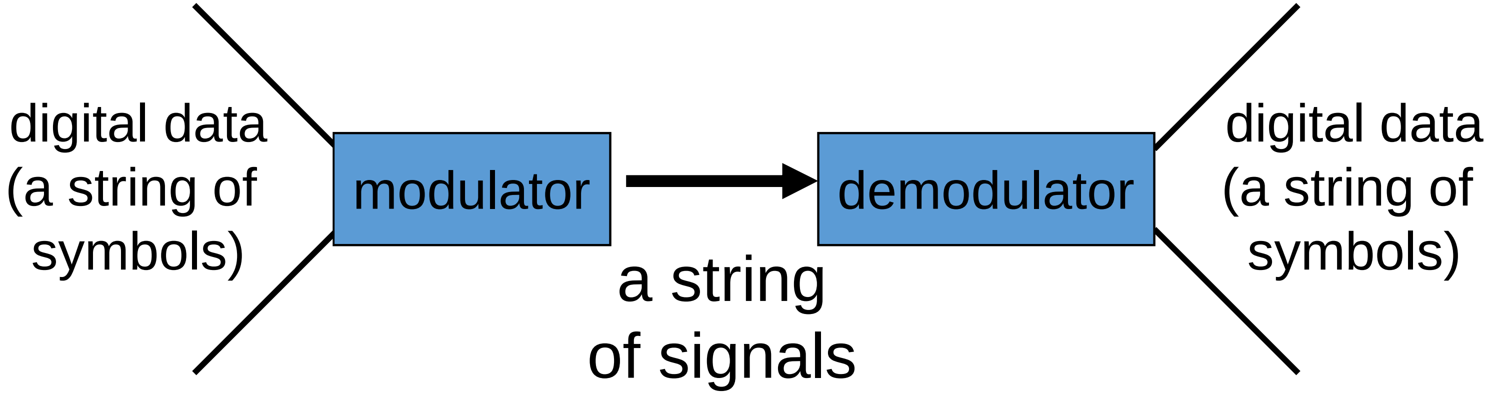
    S/N =10,000
  - C = 3400 $\log_2$ (10001) = 44.8 kbps

Summary of Encoding

- Problems
  - Attenuation, dispersion, noise
- Digital transmission allows periodic regeneration
- Variety of binary voltage encodings
  - High frequency components limit to short range
  - More voltage levels provide higher data rate
- Carrier frequency and modulation
  - Amplitude, frequency, phase, and combinations
  - Quadrature amplitude modulation: amplitude and phase, many signals
- Nyquist (noiseless) and Shannon (noisy) limits on data rates

Error Detection/Correction

Error Detection



- **Encoding** translates symbols to signals
- **Framing** demarcates units of transfer
- **Error detection** validates correctness of each frame

Error Detection

- Key idea: Add redundant information that can be used to determine if errors have been introduced, and potentially fix them

- Errors checked at many levels
  - Demodulation of signals into symbols (analog)
  - Bit error detection/correction (digital)—our main focus
    - Within network adapter (CRC check)
    - Within IP layer (IP checksum)
    - Possibly within application as well

Error Detection

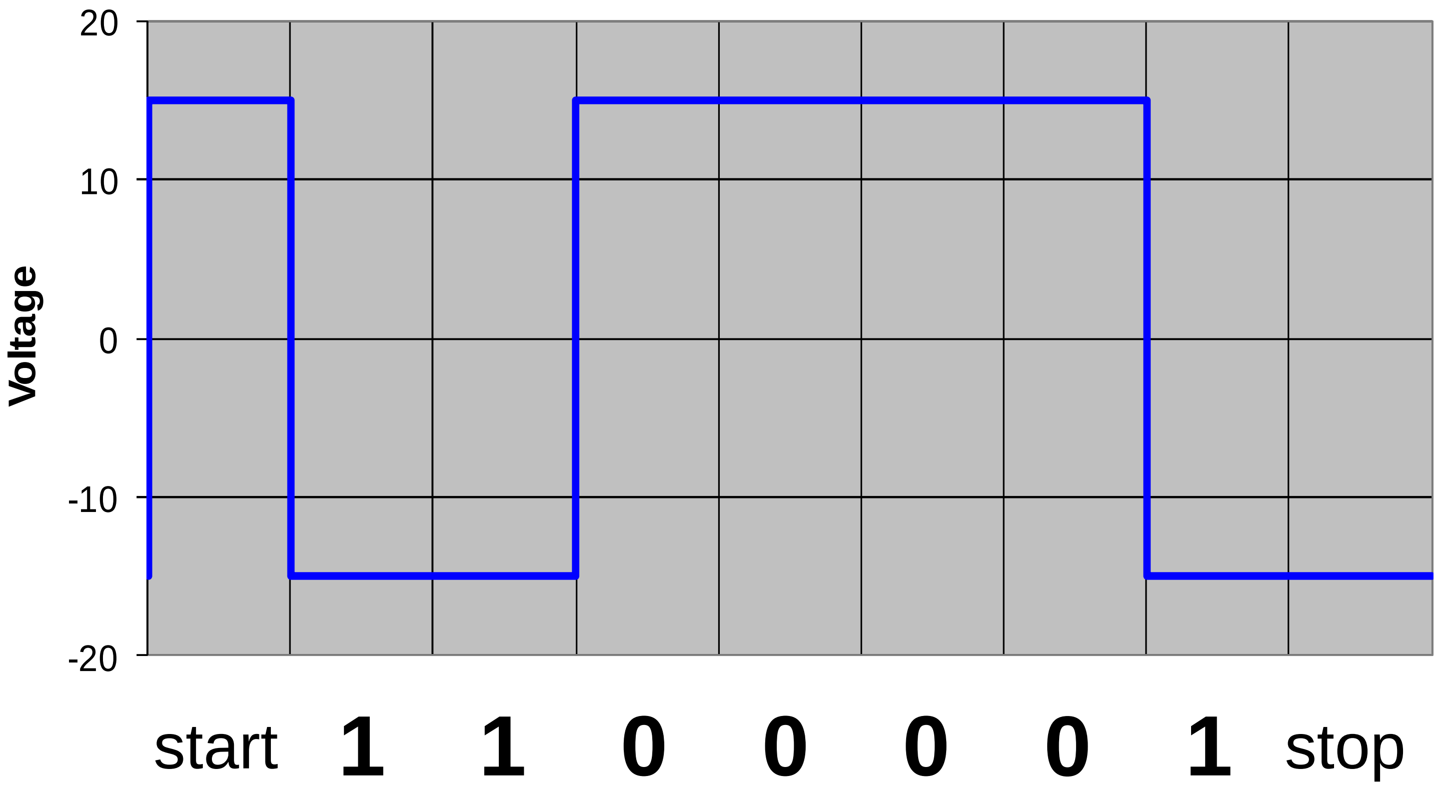
- Analog Errors
  - Example of signal distortion
- Hamming distance
  - Parity and voting
  - Hamming codes
- Error bits or error bursts?
- Digital error detection
  - Two-dimensional parity
  - Checksums
  - Cyclic Redundancy Check (CRC)

Analog Errors

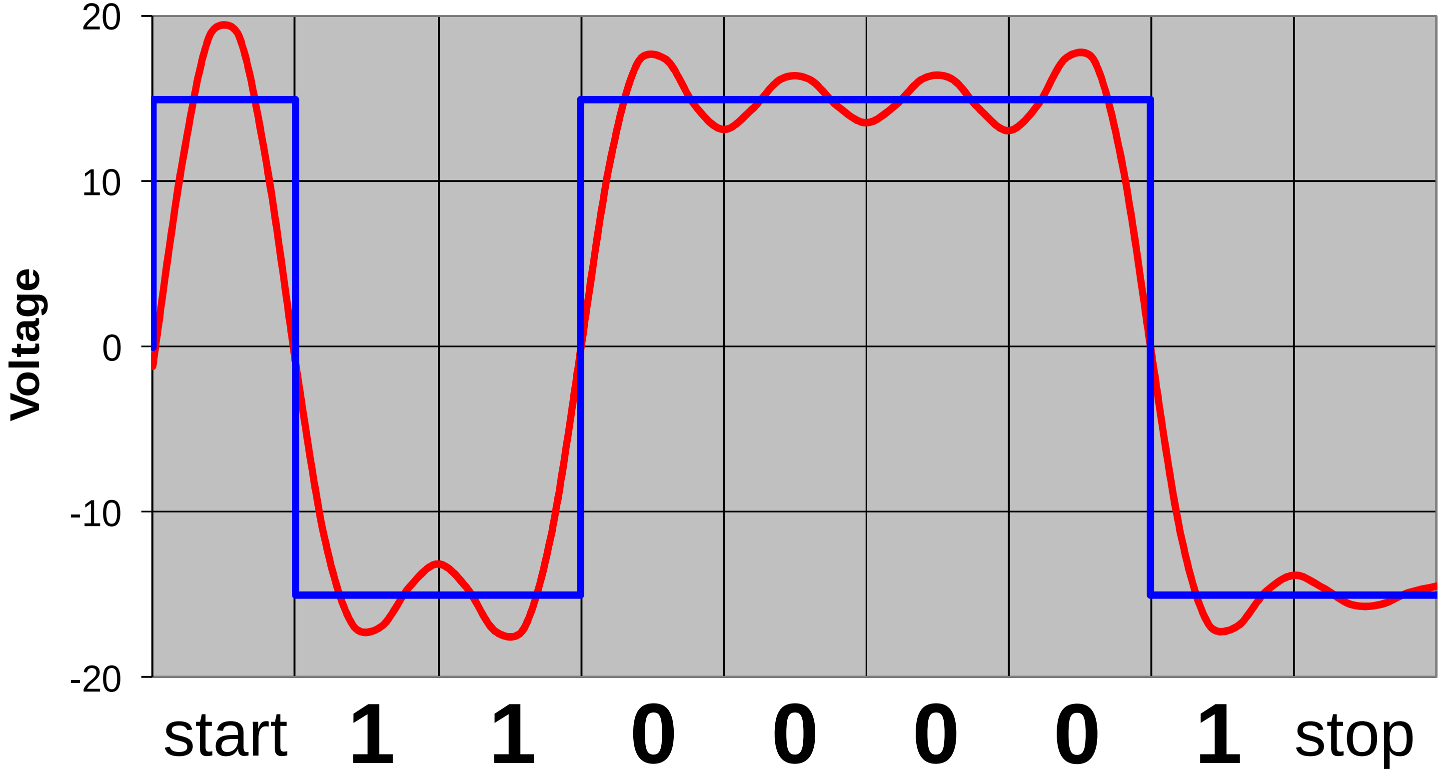
- Consider RS-232 encoding of character ‘Q’
  - ASCII Q = 1100001

- Assume idle wire (-15V) before and after signal

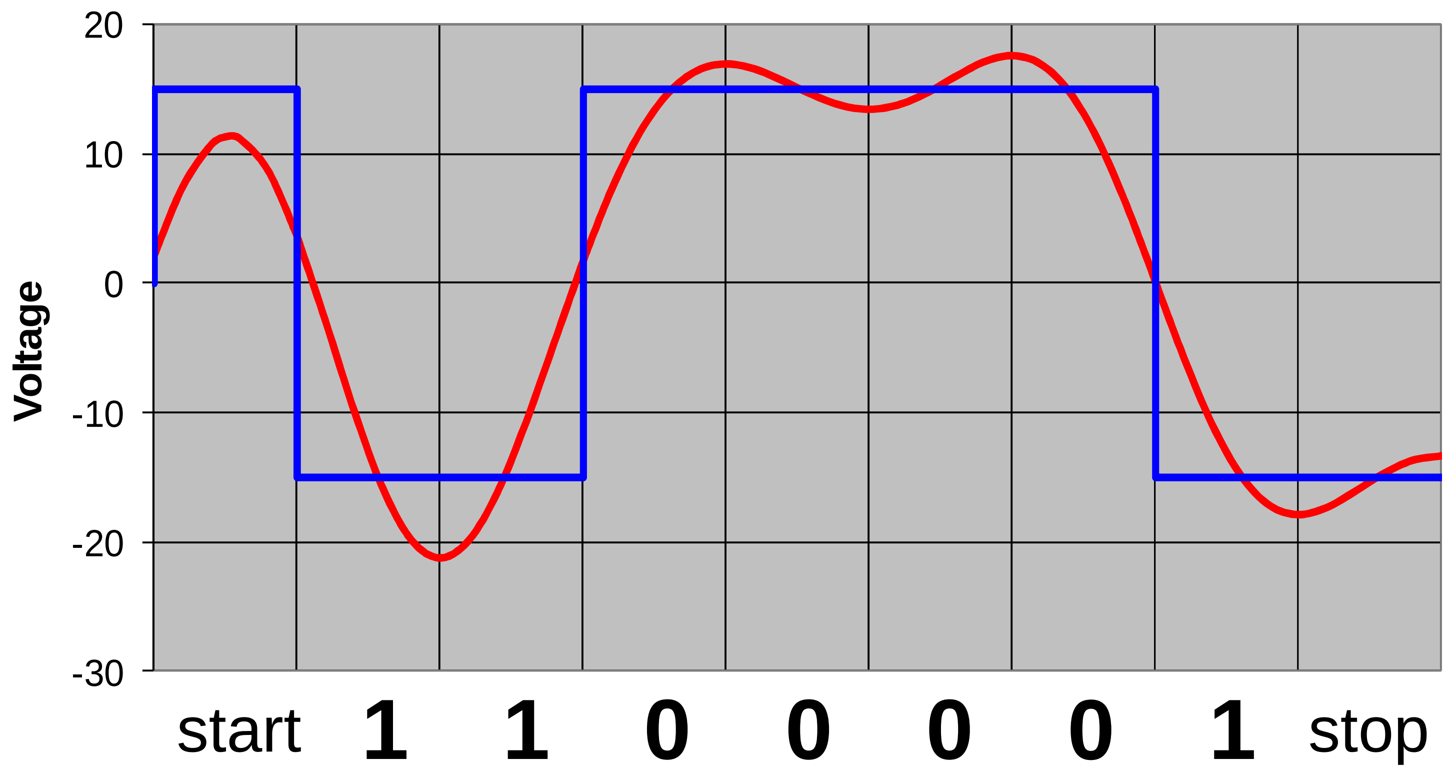
RS-232 Encoding of 'Q'



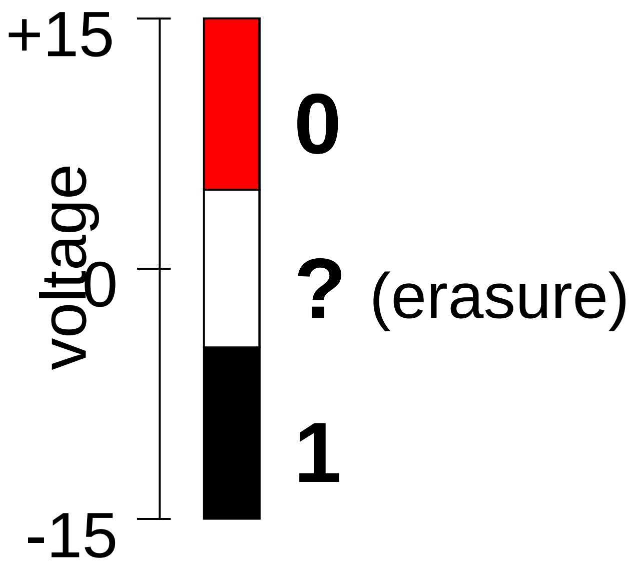
Limited-Frequency Signal Response (bandwidth = baud rate)



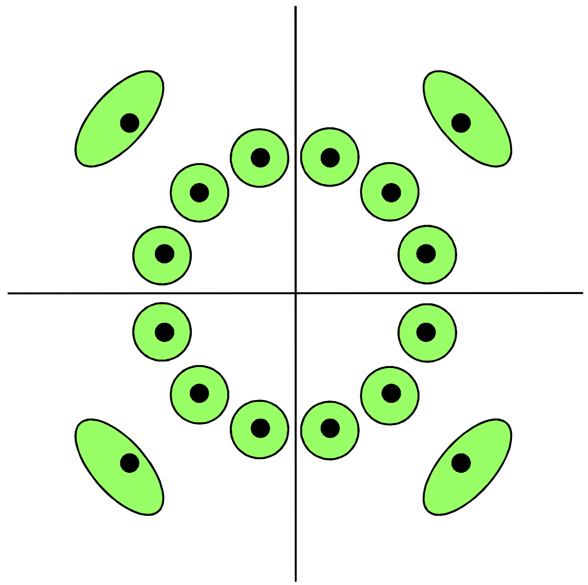
Limited-Frequency Signal Response (bandwidth = baud rate/2)



Symbols



possible binary voltage encoding symbol neighborhoods and erasure region



possible QAM symbol neighborhoods in green; all other space results in erasure

Symbols

- Inputs to digital level
  - valid symbols
  - erasures
- Hamming distance
  - Definition
  - 1-bit error-detection with parity
  - 1-bit error-correction with voting
  - 2-bit erasure-correction with voting
  - Hamming codes (1-bit error correction)

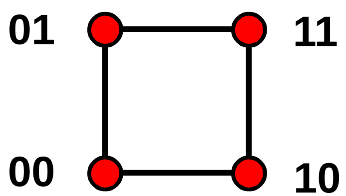
Hamming Distance

- The Hamming distance between two code words is the minimum number of bit flips to move from one to the other
  - Example:
  - 00101 and 00010
  - Hamming distance of 3

Detecting bit flips with *Parity*

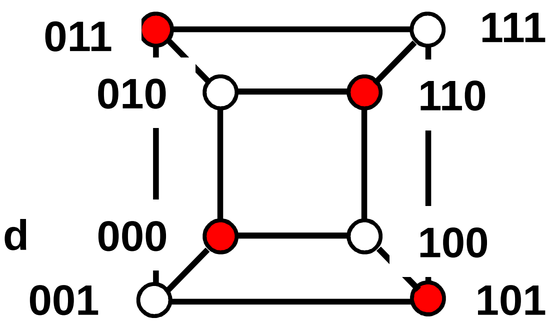
- 1-bit error detection with parity
  - Add an extra bit to a code to ensure an even (odd) number of 1s
  - Every code word has an even (odd) number of 1s

Valid code words



Parity Encoding:

White – invalid (error)



Correcting bit flips with *Voting*

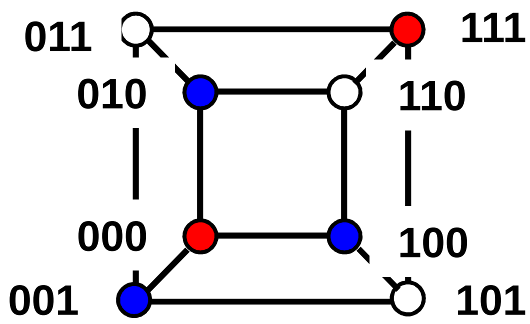
- 1-bit error correction with voting
  - Every codeword is transmitted n times

Valid code words

Voting:

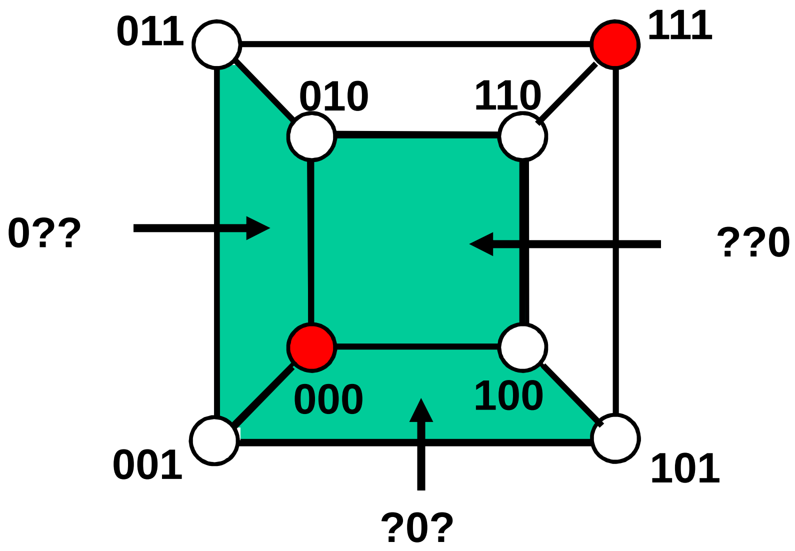
White – correct to 1

Blue - correct to 0



2-bit Erasure Correction with Voting

- Every code word is copied 3 times



2-erasure planes in green
remaining bit not ambiguous

cannot correct 1-error and 1-erasure

Minimum Hamming Distance

- The minimum Hamming distance of a code is the minimum distance over all pairs of codewords
  - Minimum Hamming Distance for parity
    - 2
  - Minimum Hamming Distance for voting
    - 3

Coverage

- N-bit error detection
  - No code word changed into another code word
  - Requires Hamming distance of N+1
- N-bit error correction
  - N-bit neighborhood: all codewords within N bit flips
  - No overlap between N-bit neighborhoods
  - Requires hamming distance of 2N+1

Hamming Codes

- Linear error-correcting code, Named after Richard Hamming
  - Simple, commonly used in RAM (e.g., ECC-RAM)
- Can detect up to 2 simultaneous bit errors
- Can correct single-bit errors
- Construction
  - number bits from 1 upward
  - powers of 2 are check bits
  - all others are data bits
  - Check bit j is XOR of all bits k such that (j AND k) = j
- Example: 4 bits of data, 3 check bits

1	2	3	4	5	6	7	8	
C	C	D	C	D	D	D	C	…

Hamming Codes

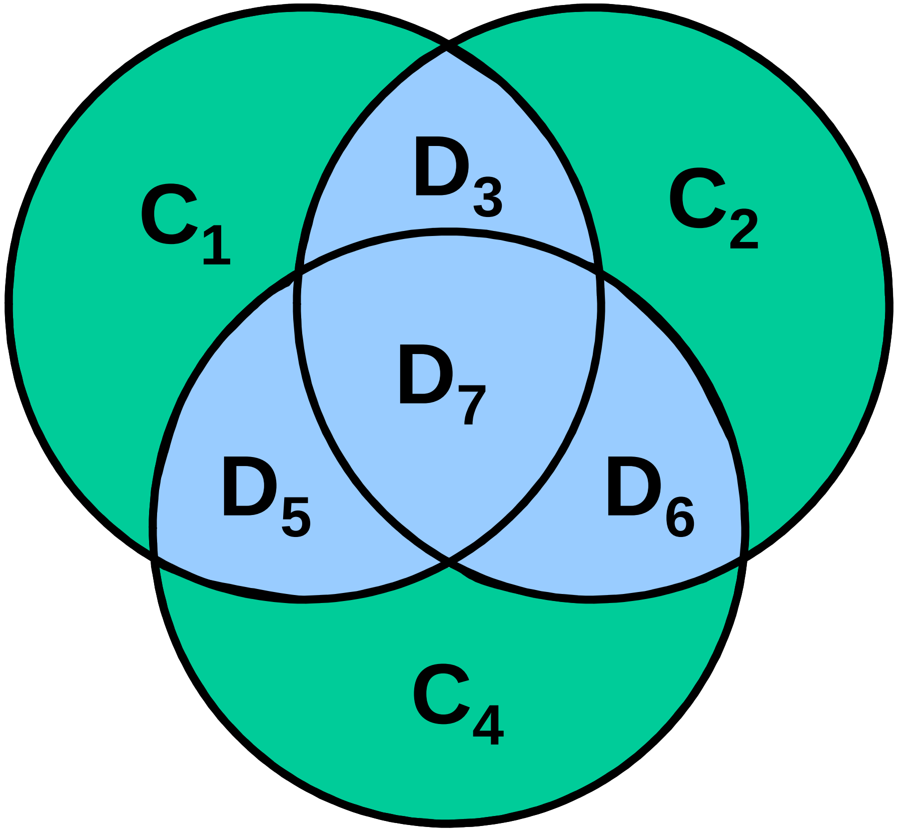
1	2	3	4	5	6	7
C_1	C_2	D_3	C_4	D_5	D_6	D_7

C1 = D3 XOR D5 XOR D7

C2 = D3 XOR D6 XOR D7

C4 = D5 XOR D6 XOR D7

Hamming Codes



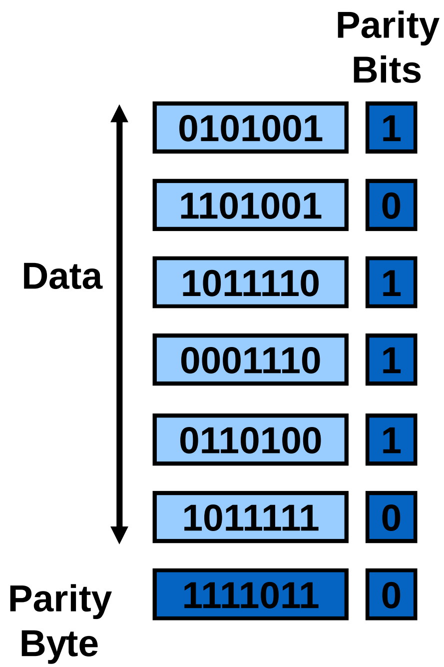
Error Bits or Bursts?

- Common model of errors
  - Probability of error per bit
  - Error in each bit independent of others
  - Value of incorrect bit independent of others
- Burst model
  - Probability of back-to-back bit errors
  - Error probability dependent on adjacent bits
  - Value of errors may have structure
- Why assume bursts?
  - Appropriate for some media (e.g., radio)
  - Faster signaling rate enhances such phenomena

Digital Error Detection Techniques

- Two-dimensional parity
  - Detects up to 3-bit errors
  - Good for burst errors
- IP checksum
  - Simple addition
  - Simple in software
  - Used as backup to CRC
- Cyclic Redundancy Check (CRC)
  - Powerful mathematics
  - Tricky in software, simple in hardware
  - Used in network adapter

Two-Dimensional Parity



- Use 1-dimensional parity
  - Add one bit to a 7-bit code to ensure an even/odd number of 1s
- Add 2nd dimension
  - Add an extra byte to frame
    - Bits are set to ensure even/odd number of 1s in that position across all bytes in frame
- Comments
  - Can **detect** and **correct** any 1-bit error
  - Can **detect** any 1-, 2- and 3-bit, and most 4-bit errors

Two-Dimensional Parity

0	1	0	0	0	1	1	1		0
0	1	1	0	1	1	1	1		0
0	1	1	0	1	1	1	1		0
0	1	1	0	0	1	0	0		1
0	0	1	0	0	0	1	1		1

What happens if…

Can detect exactly which bit flipped
Can also correct it!

0	1	~~0~~ 1	0	0	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	0	1	0	0	1
0	0	1	0	0	0	1	1	1

What happens if…

Can detect the two-bit error,
But can't tell which bits are flipped,
so can't correct

No longer a problem here

0	1	~~0~~ 1	0	0	~~1~~ 0	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	0	1	0	0	1
0	0	1	0	0	0	1	1	1

What happens if…

Suppose these four parity bits don't match
Which bits could be in error?

Could be the blue pair, OR, could be the orange pair. So, can't correct.

0	1	0	0	0	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	0	1	0	0	1

0	0	1	0	0	0	1	1	1

What about 3-bit errors?

Can detect exactly which bit flipped
You can correct in this case

0	1	~~0~~ 1	0	0	~~1~~ 0	1	~~1~~ 0	0
0	1	1	0	1	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	0	1	0	0	1
0	0	1	0	0	0	1	1	1

What about 3-bit errors?

Can detect exactly which bit flipped
But you can't correct (eg if orange bits got flipped instead of the blue ones)

0	1	~~0~~ 1	0	0	~~1~~ 0	1	~~1~~ 0	0
0	1	1	0	1	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	0	0	1	0	0	1
0	0	1	0	0	0	1	1	1

What about 4-bit errors?

Are there any 4-bit errors this scheme *can* detect?

Can you think of a 4-bit error this scheme can’t detect?

~~0~~ 1	1	~~0~~ 1	0	0	1	1	1	0
0	1	1	0	1	1	1	1	0
~~0~~ 1	1	~~1~~ 0	0	1	1	1	1	0
0	1	1	0	0	1	0	0	1
0	0	1	0	0	0	1	1	1

Internet Checksum

- Idea: Add up all the words, transmit the sum
- Internet Checksum
  - Use 1’s complement addition on 16bit codewords
  - Example

• Codewords:	-5	-3
• 1’s complement binary:	1010	1100
• 1’s complement sum	1000	

- Comments
  - Small number of redundant bits
  - Easy to implement
  - Not very robust

IP Checksum

```
u_short cksum(u_short *buf, int count) {
    register u_long sum = 0;
    while (count--) {
        sum += *buf++;
        if (sum & 0xFFFF0000) {
        /* carry occurred, so wrap around */
                sum &= 0xFFFF;
                sum++;
        }
    }
    return ~(sum & 0xFFFF);
}
```

Cyclic Redundancy Check (CRC)

- Non-secure hash function based on cyclic codes

- Idea
  - Add **k** bits of redundant data to an **n**-bit message
  - **N**-bit message is represented as a **n**-degree polynomial with each bit in the message being the corresponding coefficient in the polynomial
  - Example
    - Message = 10011010
    - Polynomial

$$= \mathbf{1} * x^7 + \mathbf{0} * x^6 + \mathbf{0} * x^5 + \mathbf{1} * x^4 + \mathbf{1} * x^3 + \mathbf{0} * x^2 + \mathbf{1} * x + \mathbf{0}$$

$$= x^7 + x^4 + x^3 + x$$

Overly simplified CRC-like protocol, using regular numbers

- Both endpoints agree in advance on a divisor value C=3
- Sender wants to send a message M=10
- Sender computes a value P=M+X=10+2=12 that is evenly divisible by C
- Sender sends P and M to receiver
- Receiver checks to make sure P=12 is evenly divisible by C=3
  - If it is not, then there's error(s)
  - If it is, then there are probably no errors

- CRC is vaguely like this, but uses polynomials instead of numbers
  - CRC can reconstruct M from P and C, so just needs to send P

CRC Approach

- Given
  - Message M(x) 10011010
  - Represented as $x^7 + x^4 + x^3 + x$

1. Select a divisor polynomial C(x) with degree k
   - Example with k = 3:
     - $C(x) = x^3 + x^2 + 1$
     - Represented as 1101
2. Transmit a polynomial P(x) that is evenly divisible by C(x)
   - P(x) = M(x) + k bits

How can we determine these k bits?

Properties of Polynomial Arithmetic

- Divisor
  - Any polynomial B(x) can be divided by a polynomial C(x) if B(x) is of the same or higher degree than C(x)
- Remainder
  - The remainder obtained when B(x) is divided by C(x) is obtained by subtracting C(x) from B(x)
- Subtraction
  - To subtract C(x) from B(x), simply perform an XOR on each pair of matching coefficients

- For example: $(x^3+1)/(x^3+x^2+1) = \boxed{?}$

CRC - Sender

- Given
  - M(x) = 10011010 = $x^7 + x^4 + x^3 + x$
  - C(x) = 1101 = $x^3 + x^2 + 1$
- Steps
  - $T(x) = M(x) * x^k$ (add zeros to increase degree of M(x) by k)
  - Find remainder, R(x), from T(x)/C(x)
  - $P(x) = T(x) – R(x) \Rightarrow$ M(x) followed by R(x)
- Example
  - T(x) = 10011010000
  - R(x) = 101
  - P(x) = 10011010101

CRC - Receiver

- Receive Polynomial P(x) + E(x)
  - E(x) represents errors
  - (if no errors then E(x) = 0)
- Divide (P(x) + E(x)) by C(x)
  - If result = 0, either
    - No errors (E(x) = 0, and P(x) is evenly divisible by C(x))
    - (P(x) + E(x)) is exactly divisible by C(x), error will not be detected

CRC – Example Encoding

$C(x) = x^3 + x^2 + 1 = 1101$ **Generator**

$M(x) = x^7 + x^4 + x^3 + x = 10011010$ **Message**

```
1101 | 10011010000     <- Message plus k zeros
       1101
       ----
        1001
        1101
        ----
         1000
         1101
         ----
          1011
          1101
          ----
           1100
           1101
           ----
            1000
            1101
            ----
             101
```

k + 1 bit check sequence c, equivalent to a degree-k polynomial

Remainder

m mod c

Result:

Transmit message followed by remainder:

10011010101

CRC – Example Decoding – No Errors

$C(x) = x^3 + x^2 + 1 = 1101$ **Generator**

$P(x) = x^{10} + x^7 + x^6 + x^4 + x^2 + 1 = 10011010101$ **Received Message**

```
1101 | 10011010101
       1101
       ----
        1001
        1101
        ----
         1000
         1101
         ----
          1011
          1101
          ----
           1100
           1101
           ----
             1101
             1101
             ----
                0
```

k + 1 bit check sequence c, equivalent to a degree-k polynomial

Received message, no errors

Remainder

m mod c

Result:

CRC test is passed

CRC – Example Decoding – with Errors

C(x) =	$x^3 + x^2 + 1$	**= 1101**	**Generator**
P(x) =	$x^{10} + x^7 + x^5 + x^4 + x^2 + 1$	**= 10010110101**	**Received Message**

```
1101 | 10010110101
        1101
        ----
         1000
         1101
         ----
          1011
          1101
          ----
           1101
           1101
           ----
            0101
```

Received message

Two bit errors

k + 1 bit check sequence c, equivalent to a degree-k polynomial

Remainder

m mod c

Result:

CRC test failed

CRC Error Detection

- Properties
  - Characterize error as E(x)
  - Error detected unless C(x) divides E(x)
    - (*i.e.*, E(x) is a multiple of C(x))

Example of Polynomial Multiplication

- Multiply
  - 1101 by 10110
  - $x^3 + x^2 + 1$ by $x^4 + x^2 + x$

```
      1011
     10110
    ------
     1101
    1101
   1101
   -------
00011111110
```

This is a multiple of c, so that if errors occur according to this sequence, the CRC test would be passed

On Polynomial Arithmetic

- Polynomial arithmetic
  - A fancy way to think about addition with no carries.
  - Helps in the determination of a good choice of C(x)
  - A non-zero vector is not detected if and only if the error polynomial E(x) is a multiple of C(x)
- Implication
  - Suppose C(x) has the property that C(1) = 0 (i.e. (x + 1) is a factor of C(x))
  - If E(x) corresponds to an undetected error pattern, then it must be that E(1) = 0
  - Therefore, any error pattern with an odd number of error bits is detected

CRC Error Detection

- What errors can we detect?
  - All single-bit errors, if x^k and x^0 have non-zero coefficients
  - All double-bit errors, if C(x) has at least three terms
  - All odd bit errors, if C(x) contains the factor (x + 1)
  - Any bursts of length < k, if C(x) includes a constant term
  - Most bursts of length $\geq$ k

Common Polynomials for C(x)

CRC	C(x)
CRC-8	$x^8 + x^2 + x^1 + 1$
CRC-10	$x^{10} + x^9 + x^5 + x^4 + x^1 + 1$
CRC-12	$x^{12} + x^{11} + x^3 + x^2 + x^1 + 1$
CRC-16	$x^{16} + x^{15} + x^2 + 1$
CRC-CCITT	$x^{16} + x^{12} + x^5 + 1$
CRC-32	$x^{32} + x^{26} + x^{23} + x^{22} + x^{16} + x^{12} + x^{11} + x^{10} + x^8 + x^7 + x^5 + x^4 + x^2 + x^1 + 1$