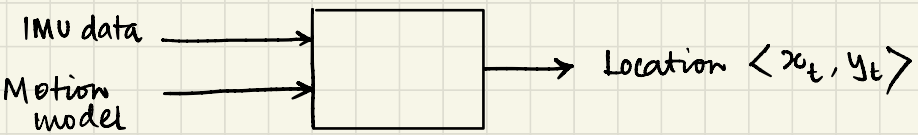
BAYESIAN FILTERING

- IMU based motion tracking will diverge for sure.
  But perhaps additional information (motion models) could keep the

⇓

How do you — IMU with motion models

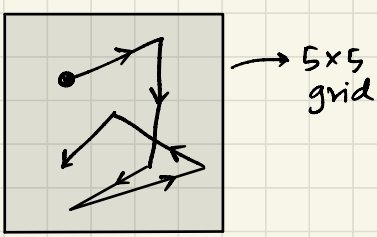


- What is a motion model ?
  ↳ or of the motion described in mathematical terms.

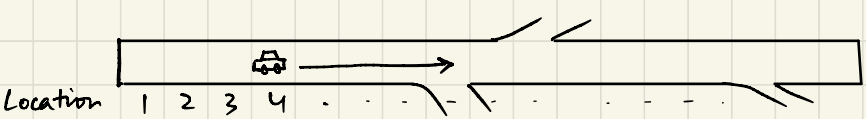
Ex ① Brownian motion model:

$$P(L_i \rightarrow L_j) =$$

Particle goes to any of the 25 boxes with probability

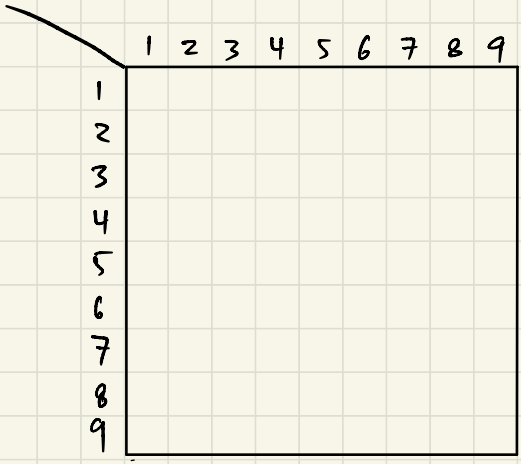


Ex ② What would be a car's motion model on the



- High probability of going straight around speed limit
- Low probability of far below or above speed limit
- Very small probability of driving backwards
- Small prob. of stopping

■ Motion model as a



→ Each element is the probability of from one location to another.

↖

⊕ We intend to combine IMU measurements + motion model to prevent divergence of object location or trajectory

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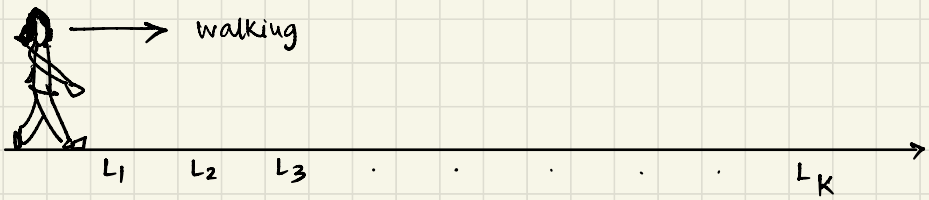
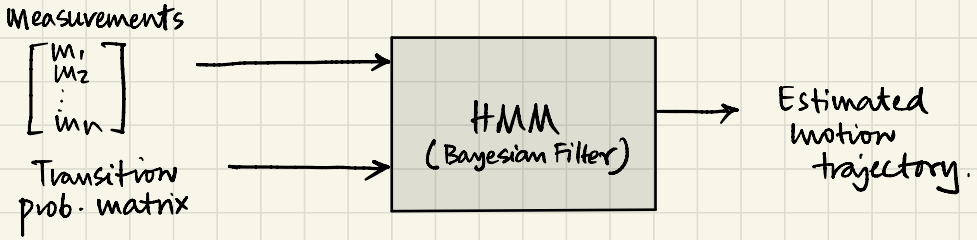
HIDDEN MARKOV MODELS (HMM)





① Formulating the state transition diagram :

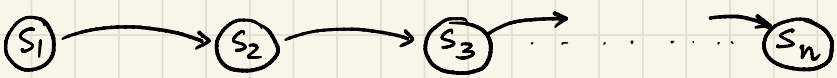
Let denote the state of the at time k

→ S_k is a , i.e., $S_k = \begin{bmatrix} \\ \end{bmatrix}$ $\forall k \in [1, n]$

→ S_k called the "

The human's walking motion is captured in

And the measurement and motion model is available for each state



② Key Question: Where the human at time t_k ?

Do you have an intuitive feel for $P(s_k \mid m_{1:n})$?
If not, fall back on visualizing them as vectors

$$P\left(s_k = \begin{bmatrix} \\ \\ \end{bmatrix} \;\middle|\; m_1 = \begin{bmatrix} \\ \\ \end{bmatrix} \quad m_2 = \begin{bmatrix} \\ \\ \end{bmatrix} \cdots \quad m_n = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix}\right)$$

$$= \frac{P(s_k, m_{1:n})}{P(m_{1:n})} = \frac{\qquad\qquad}{P\left(m_1 = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \quad m_2 = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \cdots \quad m_n = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix}\right)}$$

- From this (in numerator) you want to know which value of $s_k = \begin{bmatrix} \\ \\ \end{bmatrix}$ has the probability given the measurements
- The denominator is for all $s_k = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix}$, so matters. But we don't know that joint distribution.
- Turn this to : $\frac{P(\qquad)}{P(m_{1:n})} P(\ )$
  - ... and that is not hard because it's the sensor's
  - Hmmm! This depends on

- Let's do this mathematically now.
- Let's start with a basic result for $P(s_{1:n} \mid m_{1:n})$

$$P(s_{1:n} \mid m_{1:n}) = \frac{P(\quad)}{P(m_{1:n})} \propto P(\quad)$$

$$P(s_{1:n}, m_{1:n}) = P(\quad) P(\quad) \ldots$$

$$\ldots P(\quad) P(\quad) \ldots P(\quad) P(\quad)$$

$$= P(\quad) P(\quad) \ldots P(\quad) P(\quad)$$

$$\ldots P(\quad) P(\quad)$$

$$P(s_{1:n}, m_{1:n}) =$$

- Now we want $P(s_K \mid m_{1:n})$

$$P(s_K \mid m_{1:n}) \propto P(s_K, m_{1:n}) = P(s_K, \quad)$$

$$= P(m_{K+1:n} \mid \quad) P(\quad)$$

$$= P(m_{K+1:n} \mid \quad) P(s_K \mid m_{1:K}) P(\quad)$$

$$= P(\quad) . P(\quad)$$

↓ Probability that is at s_K = green st. given surveillance camera measurements of main street → wright street → 6th street

↓ Probability that 4th to 8th measurement are Neil st. → Kirby road → Lincoln drive → university avenue, given s_K = green street.

⊕ Let's look at the component $P(s_k | m_{1:k})$

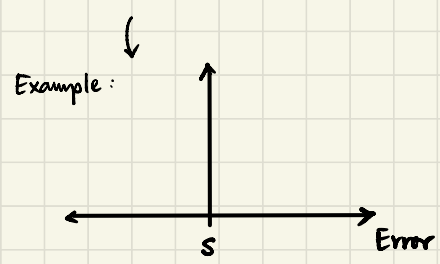
$$P(s_k | m_{1:k}) = \frac{P(s_k, m_{1:k})}{P(m_{1:k})} \propto P(s_k, m_{1:k})$$

By marginalizing RHS $= \sum_{s_{k-1}} P(\quad)$

$$= \sum_{s_{k-1}} P(m_k | \quad) P(\quad)$$

$$= \sum_{s_{k-1}} P(m_k | \quad) P(s_k | \quad) P(\quad)$$

$$P(s_k, m_{1:k}) = \sum_{s_{k-1}} P(m_k | s_k) P(s_k | \quad) P(s_{k-1}, m_{1:k-1})$$

$$\vdots = \sum_{s_{k-1}} P(m_k | s_k) P(s_k | s_{k-1})$$

⊕ Initial condition $P(s_1, m_1) =$ needs to be known.