IMU based motion tracking will diverge for sure. But perhaps additional information (motion models) could keep the divergence in check.

How do you combine/fuse IMU with motion models?

What is a motion model?
→ Behaviors or properties of the motion described in mathematical terms.

**Example 1: Brownian motion model**

\[
P(L_i \rightarrow L_j) = \frac{1}{25}
\]

Particle goes to any of the 25 boxes with equal probability (uniform dist.).

**Example 2: What would be a car's motion model on the highway?**

- High probability of going straight around speed limit = 0.7
- Low probability of far below or above speed limit = 0.05
- Very small probability of driving backwards = 0.001
- Small prob. of stopping = 0.01
### Motion Model as a transition probability matrix

<table>
<thead>
<tr>
<th>Current Loc.</th>
<th>Next Loc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>1</td>
<td>0.05 0.2 0.3 0.2 0.1 0.05 0.05 0.05 0.0</td>
</tr>
<tr>
<td>2</td>
<td>0 0.05 0.2 0.3 0.2</td>
</tr>
<tr>
<td>3</td>
<td>0 0 0.05 0.2 0.3 0.2 0.05 0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.05 0.2</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
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<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

Each element is the probability of transitioning from one location to another.

- We intend to combine IMU measurements + motion model to prevent divergence of object location or trajectory.

**Hidden Markov Models (HMM)**

- Language model
- Camera measured pixel
HIDDEN MARKOV MODELS (HMM)

Measurements

\[ \begin{align*}
  m_1 & \leftarrow t_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
  m_2 & \leftarrow t_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
  m_3 & \leftarrow t_3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
  \vdots & \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
  m_n & \leftarrow t_n \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
\end{align*} \]

Time

Measurements

\[ \begin{align*}
  m_1 & \leftarrow t_1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
  m_2 & \leftarrow t_2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
  m_3 & \leftarrow t_3 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\
  \vdots & \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\
  m_n & \leftarrow t_n \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0
\end{align*} \]

Locations

- Walking
- Locations
- Standing still
- Pacing back and forth with short pauses
- Walking forward w/ roughly const. velocity
Measurements

\[
\begin{bmatrix}
W_1 \\
W_2 \\
\vdots \\
W_n
\end{bmatrix}
\]

Transition prob. matrix

HMM (Bayesian Filter)

Estimated motion trajectory.

Formulating the state transition diagram:

Let \( S_k \) denote the state of the object at time \( k \).

\( S_k \) is a random variable, i.e.,

\[
S_k = \begin{bmatrix}
L_1 \\
L_2 \\
\vdots \\
L_r
\end{bmatrix}
\]

The human's walking motion is captured in

\[ S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow \cdots \rightarrow S_n \]

And the measurement and motion model is available for each state:

\[ P \left( S_k \mid W_{1:k} \right) = P(S_k \mid W_{1}, W_2, W_3, \ldots, W_k) \text{ or } P(S_k \mid W_{1:n}) \]

Key Question: Where is/was the human at time \( t_k \)?

Is this posterior or likelihood?
Do you have an intuitive feel for $P(S_k | m_1:n)$?
If not, fall back on visualizing them as vectors:

$$P(S_k = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \vdots \\ \mathbf{L}_r \end{bmatrix} | m_1 = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \vdots \\ \mathbf{L}_r \end{bmatrix}, m_2 = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \vdots \\ \mathbf{L}_r \end{bmatrix}, \ldots, m_n = \begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \vdots \\ \mathbf{L}_r \end{bmatrix})$$

**Bayes' Rule:**

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(S_k, m_1:n)}{P(m_1:n)}$$

1. From the joint distribution of $(n+1)$ R.V.s.
2. The denominator is same for all $S_k$, so only numerator matters.
3. Turn this posterior to likelihood:

$$P(m_1:n | S_k) \cdot P(S_k)$$

**Likelihood... and that is not hard because it's the sensor's measurement quality**

**Who cares?**

We want to compare the numerator for different values of $S_k$.
We want $P(s_k | m_{1:n}) \leftarrow \text{offline}$.

Let's start with a basic result for $P(s_{1:n} | m_{1:n})$.

$$P(s_{1:n} | m_{1:n}) = \frac{P(s_{1:n}, m_{1:n})}{P(m_{1:n})} \propto P(s_{1:n}, m_{1:n})$$

$$P(s_{1:n}, m_{1:n}) = P(m_n | m_{1:n-1}, s_{1:n}) P(m_{n-1} | m_{1:n-2}, s_{1:n}) \ldots$$

$$\ldots P(m_1 | s_{1:n}) P(s_n | s_{1:n-1}) \ldots P(s_2 | s_1) P(s_1)$$

Markovian:

$$= P(m_n | s_n) P(m_{n-1} | s_{n-1}) \ldots P(m_1 | s_1) P(s_n | s_{n-1}) \ldots P(s_2 | s_1) P(s_1)$$

$$P(s_{1:n} | m_{1:n}) = P(m_1 | s_1) P(s_1) \prod_{i=2}^{n} P(m_i | s_i) P(s_i | s_{i-1})$$

which trajectory and measurements are in agreement.