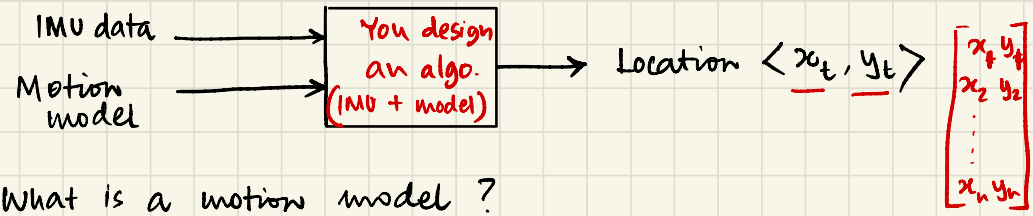
BAYESIAN FILTERING

- IMU based motion tracking will diverge for sure.
  But perhaps additional information (motion models) could keep the divergence in check

$\Downarrow$

How do you combine / fuse IMU with motion models

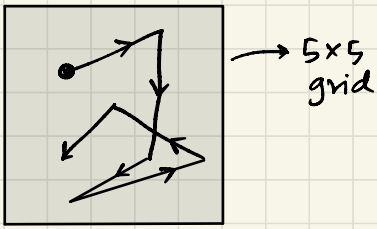


- What is a motion model ?
  ↳ Behaviors or properties of the motion described in mathematical terms.

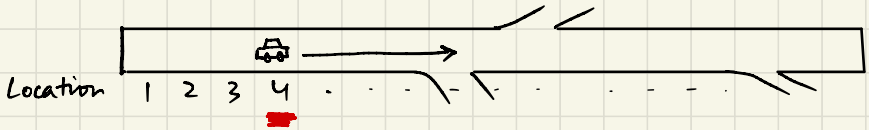
Ex ① Brownian motion model :

$$P(L_i \rightarrow L_j) = \frac{1}{25}$$

Particle goes to any of the 25 boxes with equal probability (uniform dist.)



Ex ② What would be a car's motion model on the highway?



- High probability of going straight around speed limit = 0.7
- Low probability of far below or above speed limit = 0.05
- Very small probability of driving backwards = 0.001
- Small prob. of stopping = 0.01

■ Motion model as a transition probability matrix

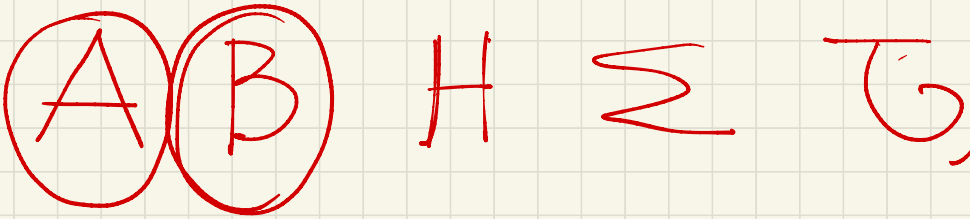
Current Loc. \ Next Loc.	1	2	3	4	5	6	7	8	9
1	0.05	0.2	0.3	0.2	0.1	.05	.05	0.05	0
2	0	0.05	0.2	0.3	0.2	.			
3	0	0	0.05	0.2	0.3	0.2	.	.	.
4				0.05	0.2				
5					0.05	0.2			
6						0.05	0.2		
7							.	.	
8								.	.
9									.

→ Each element is the probability of transitioning from one location to another.

⊕ We intend to combine IMU measurements + motion model to prevent divergence of object location or trajectory

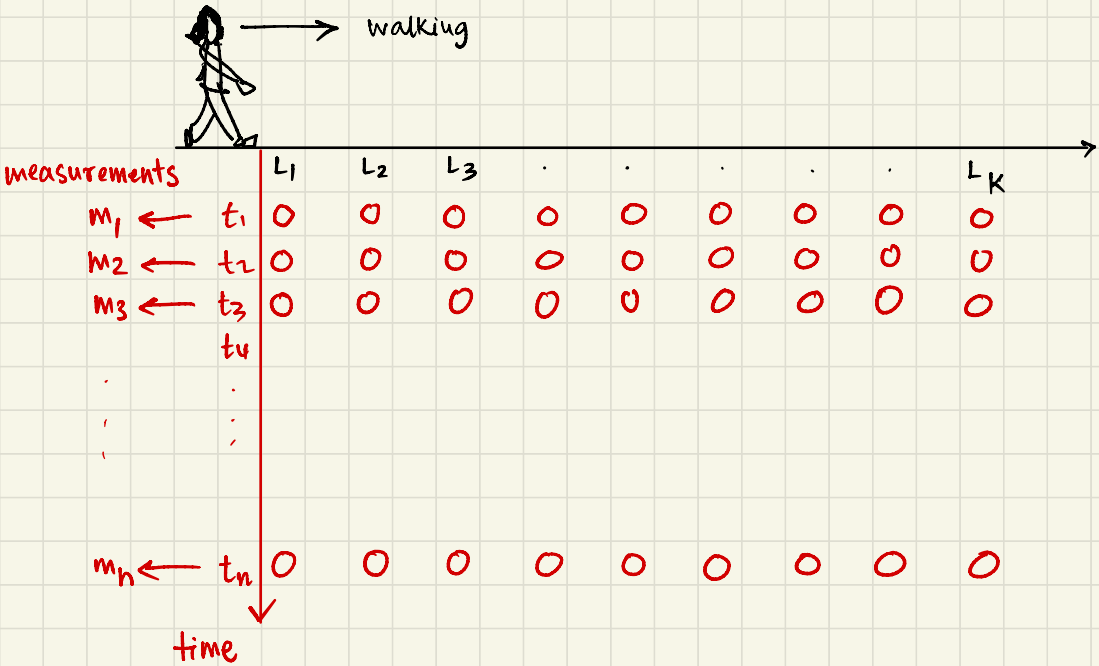
⇓

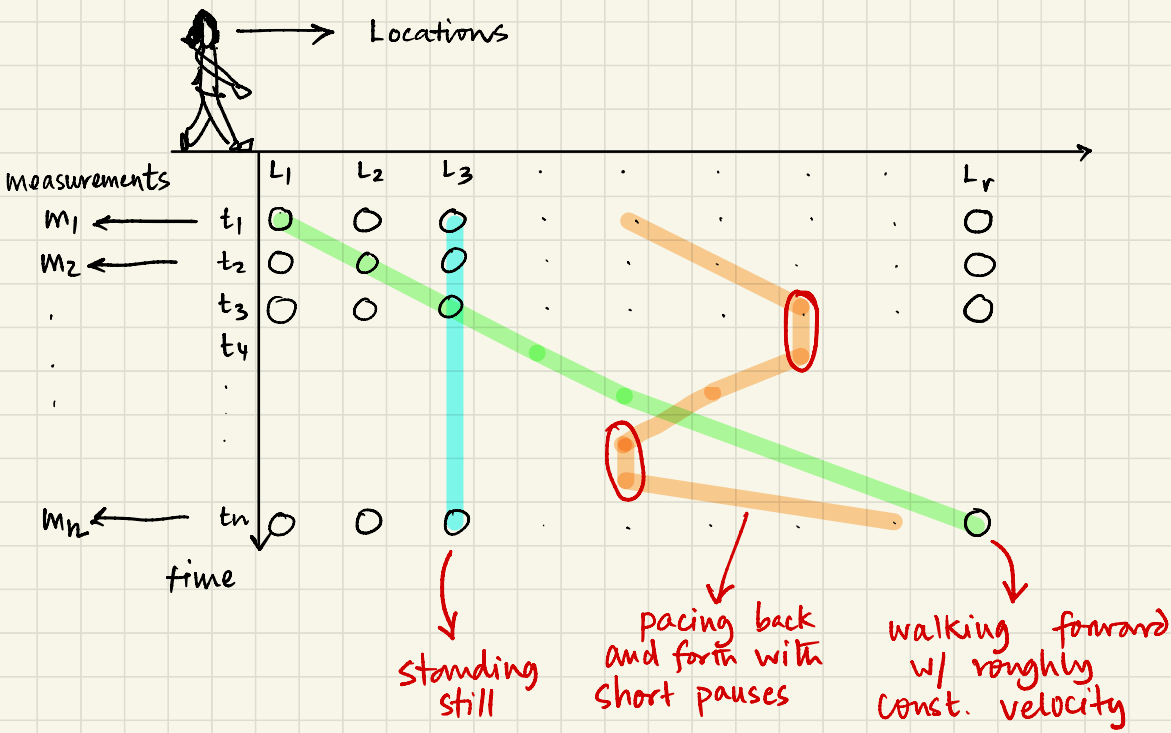
Hidden Markov Models (HMM)

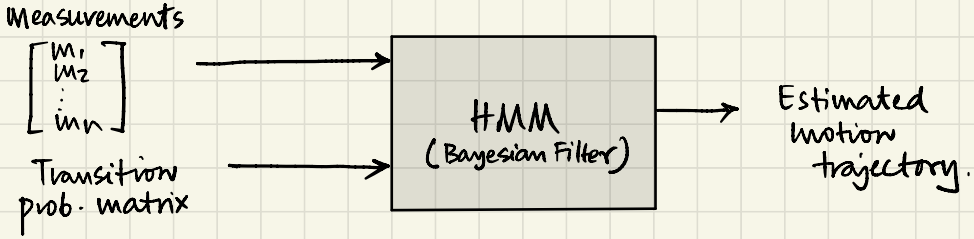


Lang. model + Camera measured pixel

HIDDEN MARKOV MODELS (HMM)







Formulating the state transition diagram :

Let S_K denote the state of the object at time K

- S_K is a random variable, i.e., $S_K = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix}$ Loc r
- S_K called the "state variable"

n time

The human's walking motion is captured in

$$S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow \cdots \rightarrow S_n$$

And the measurement and motion model is available for each state

Human at location L_i at time t_1 ← $S_1 = L_i$

$$\begin{array}{ccccccccc} S_1 & \xrightarrow{P_{ij}} & S_2 & \rightarrow & S_3 & \rightarrow \cdots & S_K & \cdots \rightarrow & S_n \\ \downarrow & & \downarrow & & \downarrow & & & & \downarrow \\ m_1 & & m_2 & & m_3 & & & & m_n \end{array}$$

$$S_1 = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_r \end{bmatrix} \quad S_2 = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_r \end{bmatrix} \quad S_3 = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_r \end{bmatrix} \quad S_K = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_r \end{bmatrix} \quad S_n = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ L_r \end{bmatrix}$$

Key Question: Where is/was the human at time t_K?

$P(S_K \mid m_{1:K}) = P(S_K \mid m_1, m_2, m_3 \ldots m_K)$ or $P(S_K \mid m_{1:n})$

Is this posterior or likelihood ?

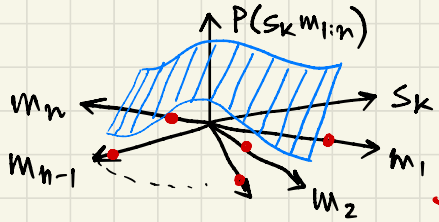
Do you have an intuitive feel for $P(s_k \mid m_{1:n})$?

If not, fall back on visualizing them as vectors

$$P\left(s_k = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \;\middle|\; m_1 = \overset{\text{GPS}}{\begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix}} \quad m_2 = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \cdots \quad m_n = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \right)$$

$$\overset{\text{Bayes' Rule}}{=} \frac{P\left(s_k, \overset{\text{acc} = \begin{bmatrix} 0 \\ 2 \\ \vdots \\ 15 \end{bmatrix}}{m_{1:n}}\right)}{P(m_{1:n})} =$$

$$P(A \mid B) = \frac{P(A, B)}{P(B)}$$



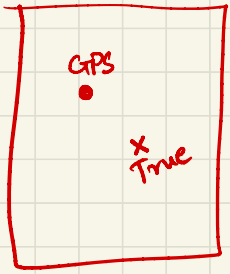
Joint distribution of $(n+1)$ R.V.s.

$$\rightarrow P\left(m_1 = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \quad m_2 = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \cdots \quad m_n = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix} \right)$$

- From this **joint dist.** (in numerator) you want to know which value of $s_k = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix}$ has the **max** probability given the $m_{1:n}$ measurements **you already have.**

- The denominator is **same** for all $s_k = \begin{bmatrix} L_1 \\ L_2 \\ \vdots \\ L_r \end{bmatrix}$, so **only numerator** matters. But we don't know that joint distribution.

- Turn this **posterior** to **likelihood** : $\dfrac{P(m_{1:n} \mid s_k)}{P(m_{1:n})} P(s_k)$



Likelihood... and that is not hard because it's the sensor's **measurement quality**

Who cares! We want to compare the numerator for different values of s_k

Hmmm! This depends on **where I was last.** So its f^n of s_{k-1}

Let's do this mathematically now. We want $P(s_k \mid m_{1:n})$ ← offline

Let's start with a basic result for $P(s_{1:n} \mid m_{1:n})$

→ trajectory given all measurements

$$P(s_{1:n} \mid m_{1:n}) = \frac{P(s_{1:n}, m_{1:n})}{P(m_{1:n})} \propto P(s_{1:n}, m_{1:n})$$

$$P(s_{1:n}, m_{1:n}) \overset{\text{chain rule}}{=} P(m_n \mid m_{1:n-1}, s_{1:n})\, P(m_{n-1} \mid m_{1:n-2}, s_{1:n}) \cdots$$

$\mathbb{R}^{2n}$

$$\cdots P(m_1 \mid s_{1:n})\, P(s_n \mid s_{1:n-1}) \cdots P(s_2 \mid s_1) P(s_1)$$

$P(ABC) = P(A \mid BC)\, P(B \mid C) P(C)$

Markovian

$$= P(m_n \mid s_n)\, P(m_{n-1} \mid s_{n-1}) \cdots P(m_1 \mid s_1)\, P(s_n \mid s_{n-1}) \cdots P(s_2 \mid s_1) P(s_1)$$

$$\boxed{P(s_{1:n}, m_{1:n}) = P(m_1 \mid s_1) P(s_1) \prod_{i=2}^{n} P(m_i \mid s_i)\, P(s_i \mid s_{i-1})}$$

→ which trajectory & measurements are in agreement.

sensor error

Transition prob. matrix